Technology, Investment, and Economic Fluctuations

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Abstract

Since 1854, the United States has experienced 32 business cycles. While the average length of these cycles (trough-to-trough) has been 51 months, there has been significant variation across different subperiods. This paper attempts to explore the relationship between capital accumulation, technology accumulation and the length of the business cycle.

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1 Introduction

The business cycle has been a well known phenomenon for well over a century. The length of such cycles, however, has not been constant. From 1869 until 1913, the US experienced 12 business cycles of an average length (trough from previous trough) of 47 months. During the period from the US experienced 8 peacetime cycles of an average length of 45 months. Finally, from 1972 until 1995, the US experienced four cycles averaging 64 months in length. The National Bureau of Economic Research has studied business cyclles since the 1920's and has discovered that every cycle differed in length from a low of 2.5 - 3 years and a high of 10 years. It is also interesting to note that productivity varied across these time periods as well. Gordon (2000) examines multifactor productivity from 1870 to 1999. He finds an alternating pattern of slow growth followed by fast growth.

Period	1870 − 1913	1913 − 1972	1972 − 1995	1995 − 1999
MFP (%$\triangle$)	.47	1.08	.02	1.25
Business Cycle Length	47	45	64	??

Gordon attributes this pattern of multifactor productivity to technological innovation. The Late 1800's produced many revolutionary inventions. For example, the electric light (1879), the internal combustion engine (1877),the telephone

(1876). Gordon attributes the fast growth 1913 - 1972 to such inventions. Similarly, the rapid expansion of the late 90's could be attributed to the invention of the microprocessor (first marketed by INTEL in 1971). Note that periods of rapid technological growth seem to correspond to shorter cycles while slower growth periods correspond to longer cycles. This paper attempts to explain this relationship.

In this paper the distinction is made between *process innovation* and *product innovations.* Process innovations relate to productivity improvements evolving out of organzational changes or human capital improvements. Process innovations allow the economy to produce more output per unit of capital. Product innovations , however, refer to productivity improvements as a result of the economy's ability to create better capital goods over time. Standard neoclassical models stress the importance of process innovations. Capital in standard models is considered purely homogeneous. The capital stock evolves smoothly over time in resopnse to exogenous changes in productivity (new process innovations).

Microeconomic evidence, however, shows that investment expenditures at the plant level are "lumpy"; occurring infrequently and in bursts. Doms and Dunne (1993) use a 12,000 plant study and find that, over a 15 year period , 25% of a plant's investment expenditures is concentrated in a single year - 50% is concentrated in a contiguous 3 year period. Further, recent studies suggest that investment specific technological change is a potentially important source of growth. Bahk and Gort (1993) find that a one year change in the age of capital is associated with a 2.5 to 3.5 % rise in output. Greenwood, Hercowitz and Krusell (1994) argue that 60% of post war growth can be attributed to investment specific technological progress. Additionally, there is evidence showing that the production of capital goods is becoming more efficient over time. Gordon (1990) documents that the relative price of capital in terms of consumption has declined in the US. economy.

Recently, information technology investments have become increasingly important. Since 1980, firms have shifted equipment spending away from heavy machinery and towards information processing equipment, particularly computers. Between 1970 and 1990, constant dollar investment in office and computing equipment grew at an annual rate of 18.1 percent. However, growth in other producer durables was only around 3.3 percent. Kriebel (1989) notes that roughly 50% of new corporate capital expenditures by major U.S. companies is in information technologies. Yorokoglu (1995) finds that information technology has several features that distinguish it from traditional capital. Among these are as follows:

- IT capital has a very high pace of technological improvement. For example, IBM introduced its Pentium PCs in the early 90's at the same price at which it introduced its 286 PC in the 1980's. That is, it took less than a decade to dramatically increase the speed and memory capabilities without increasing the cost.

- Fast technological improvements lead to standardization problems. Compatibility between capital stocks differing by even a few years in age might be large. Therefore, it may be difficult to improve or add to existing capital stocks.

These observations suggest that a vintage capital model where new technology is embodied in newer, more efficient capital goods might be a more adequate representation of the relationship between technology, capital, and economic fluctuations. Some of the earliest work on vintage capital was pioneered by Robert Solow. In Solow (1962), capital had a fixed lifetime and the amount of labor in a plant was fixed over its lifetime. Here, the representative firm owns several plants and is allowed to efficiently allocate labor across those plants. This allocation results in older plants employing less labor than plants with newer technology. This is consistent with the empirical observation that older plants are smaller than newer plants. Cooley, Greenwood, and Yorukoglu (1997) construct a vintage capital economy to examine the implications of changes in the tax treatment of capital. Among their findings is that the dynamics of capital accumulation are strikingly different from the standard models. Particularly, the dynamics of capital accumulation are more "sluggish" than in standard models.

In this paper, a vintage capital model is constructed to examine the relationship between process innovations, product innovations, long run growth and the length of business cycles. Product innovations will be represented by new, more productive capital goods becoming available each period. New capital goods are incompatable with older capital goods. Therefore, when a firm purchases new capital, it must scrap its old capital (capital is firm specific-therefore, no market exists for used capital). Depreciation in this model is economic rather than physical. Capital goods have a fixed lifetime at wich point they become obsolete relative to state of the art capital goods and must be scrapped. Further, the economy is hit by random process innovations that help all firms equally. It will be shown that as the rate of adoption increases (the lifetime of capital falls) process innovations will have a reduced impact on the economy in the sense that the economy will return to its balanced growth path much quicker. Although the adoption decision is not modelled here, it is reasonable to believe that as the underlying rate of growth in the productivity of new capital goods increases, the adoption rate will increase as well

2 The Economic Environment

A representative firm operates a portfolio of manufacturing plants of measure one. Plants are indexed by the age of the capital installed. Capital has a lifetime of N periods after which it is scrapped. Therefore, every period an age N plant is retired and must be replaced with a new plant. The firm must decide how to allocate labor across plants as well as how much capital to install in a new plant. Once the capital is installed, it is in place until it is retired in N periods. Capital goods become more productive over time, so as a plant ages its capital becomes less productive relative to new capital.

2.1 The Representative Firm's Problem

The representative firm owns a continuum of manufacturing plants distributed over the unit interval. A particular plant is identified by the age of the capital employed. Let ψ_i denote the measure of age i plants. Capital has a life of N years, after which it is unusable and, hence, has a value of zero. With a uniform age distribution, $\psi_i = 1/N$. Therefore, every period, an age N plant is scrapped and replaced by a new plant. The firm manager must decide the size of a new plant as well as how to allocate labor over the various plants. Consider an age i

plant. It has at its disposal k_i efficiency units of capital and employs l_i units of labor. It produces output according to the following technology.

$$y_i = A_t k_i^{\alpha} l_i^{\omega} \quad \alpha + \omega \leq 1 \tag{2.1}$$

$$A_{t+1} = \rho A_t + \varepsilon_{t+1} \tag{2.2}$$

$$\varepsilon \sim N(0,1) \tag{2.3}$$

Note that $\alpha + \omega \prec 1$ implies decreasing returns to scale and, hence, positive profits.The term, A, can be thought of as productivity changes from process improvements. This productivity shock affects all plants equally. Output can be used for either consumption or for investment in a new plant. Consumption goods can be produced from output on a one to one basis. However, as in Greenwood, et al (1994), the ability to produce new capital goods grows over time. Let q_t represent the time t state of technology for producing capital goods. This technology grows at an exogenous rate of γ_q. Note that this technological growth is not necessary for the enhanced propagation, but will allow the model to be consistent with the observation that newer plants are larger than older plants.

$$k_{1t+1} = q_t i_t \tag{2.4}$$

The static decision facing the firm manager concerns the allocation of labor across plants. Given k_i efficiency units of capital and taking the real wage rate w as given, the plant manager maximizes an age i plant profits.

$$\Pi_i(k_i, w) = \max_{l_i} \{Ak_i^{\alpha} l_i^{\omega} - wl_i\} \tag{2.5}$$

The first order condition associated with this problem is

$$w = \omega A k_i^{\alpha} l_i^{\omega - 1} \tag{2.6}$$

The dynamic decision facing the firm manager how much capital to place in the new plants. The firm pays wages and capital expenditures out of current revenues. Any remaining profits are payed out as dividends to households. This decision is in line with the following dynamic programming problem. Note that primed variables indicate time $t+1$ values.

$$V\left(k_1,...,k_N;A\right)=\max_{k_1'}\left\{\sum_{i=1}^{N}\psi_i\Pi_i\left(k_i,w\right)-\psi_1\left(\frac{1}{q}\right)k_1'+E_t\left\{\frac{V\left(k_1',...,k_N';A'\right)}{1+r'}\right\}\right\} \tag{2.7}$$

subject to

$$k_{i+1}' = k_i \tag{2.8}$$

$$k_i' = 0\ \forall\ i\geq N \tag{2.9}$$

Equation (2.9) is the rule for capital accumulation. Age i capital today will be age $i+1$ capital tomorrow. The first order condition associated with this problem is

$$\frac{1}{q}=E_t\left\{\frac{V_1\left(k_1',...,k_N';A'\right)}{1+r'}\right\} \tag{2.10}$$

with

$$V\left(k_1', ..., k_N'; A'\right) = \psi_i' \Pi_i \left(k_i', w'\right) + E_{t+1} \left\{ \frac{V_{i+1}\left(k_1'', ..., k_N''; A''\right)}{1 + r''} \right\} \tag{2.11}$$

Equation (2.11) determines the amount of capital to be placed in a new plant. This can be solved forward to yield the following.

$$\frac{1}{q} = E_t \sum_{i=1}^{N} \left\{ \prod_{j=1}^{i} \left(1 + r_{t+j}\right)^{-1} \right\} \Pi_{i1}\left(k_{t+i}, w_{t+i}\right) \tag{2.12}$$

This expression states that today's marginal cost of capital must equal the present value of value marginal products over its lifetime - significantly different from the standard first order condition for capital which only depends on the marginal product one period forward.

2.2 Households

Households have preferences defined over random streams of consumption and leisure represented by the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t W\left(c_t, 1 - l_t\right) \tag{2.13}$$

$$W\left(c, l; \lambda\right) = \ln\left(c - \lambda \frac{\Theta l^{1+\nu}}{1+\nu}\right)$$

where c represents consumption , l represents labor, $\beta \prec 1$ is the discount rate and E_0 represents the conditional expectation based on information available at time 0. The form of the utility function is justified by Greenwood, Rogerson, and Wright (1994) as being consistent with household production theory. The term λ represents the state of technology in the household production sector. Households recieve wage income as well as any profits of the firm.

$$y_t = w_t l_t + \Pi_t^p \tag{2.14}$$

Income can be allocated for consumption purposes or can be saved. Savings earns the real rate of interest.

$$c_t + s_{t+1} = w_t l_t + (1 + r_t)\, s_t + \Pi_t^p \tag{2.15}$$

the household's decision problem is to choose a contingency plan for $\{c_t, , l_t, s_{t+1}\}_{t=0}^{\infty}$ that maximizes expected lifetime utility subject to the budget constraint. The consumers problem can be written in the following recursive formulation.

$$J(s; A) = \underset{c,l,s'}{Max} \left\{ \begin{array}{c} W(c, l) + \beta E_t J(s'; A') \\ +\lambda_1 \left(wl + (1 + r)\, s + \Pi_t^p - c - s'\right) \end{array} \right\} \tag{2.16}$$

This results in the following first order conditions.

$$W_1(c, l) = \lambda_1 \tag{2.17}$$

$$-W_2(c, l) = w\lambda_1 \tag{2.18}$$

$$\beta E J_1(s'; e') = \lambda_1 \tag{2.19}$$

Along with the following envelope condition

$$J_1(s;A) = \lambda_1(1+r) \tag{2.20}$$

Equation (2.17) is the efficiency condition for consumption. The multiplier represents the marginal utility of wealth. Equation (2.18) in the first order condition for labor. The marginal disutility of labor $W_2(c,l)$ converts the hours into utility. Equation (2.19) is the efficiency condition for savings.

3 Equilibrium

The model is completed by a description of the state of the world. This is given by the vector $\{k_1, ..., k_N, A, \}$. Given the definition of the state, the competitive equilibrium can be defined as a set of decision rules $\{c, l, s, k_1'\}$ and a set of pricing functions $\{r, w\}$ such that

1) Consumers optimize, taking interest rates, wages, and prices as given, resulting in decisions for consumption, labor, and savings given by c, l, s,

2) The representative firms and all plants maximize profits taking interest rates, wages, and prices as given. The resulting decisions are represented by k_1', l_i.

3) Given the behavior of consumers and producers, prices adjust such that

markets clear.

$$s = i \tag{3.21}$$

$$c + i = \sum_{i=1}^{N} \psi_i k_i^{\alpha} l_i^{\omega} \tag{3.22}$$

$$\sum_{i=1}^{N} \psi_i l_i = l \tag{3.23}$$

4 Calibration

The next step in the analysis is to choose values for the models parameters. The values come from either *a priori* information or so that along the model's balanced growth path various endogenous variables assume the long run values seen in the US. data. A time period is chosen to correspond to one year. Over the post war period, labor's share of income has averaged .65. This implies that $\omega = .65$. The value of $1/\nu$ corresponds to the elasticity of labor supply. Following Greenwood, Hercowitz and Huffman (1988), a value of .6 was chosen. This implies a value of 1.7 for the labor supply elasticity, which is an average found by earlier researchers. The average growth rate of output per hour was 1.24 percent between $1954-90$. Finally, the average ratio of hours to non sleeping hours of the working age population is .25. Using these restrictions implies the following parameter values

$$\alpha = .2$$

$$\gamma_q = 1.05$$

$$\beta = .9633$$

$$\Theta = .476$$

5 The Capital Replacement Decision

Clearly, the decision on when to replace capital should be endogenous. For simplicity, this decision is not modeled explicitly here and N should be thought of as a closed form solution to that decision. For example, N could be thought of as a function of taxes and subsidies aimed at capital formation as in Cooley, et al (1997). An increase in the lifetime of capital could be the result of decreases in investment tax credits. However, consider the effect of a purely exogenous increase in the lifetime of capital. The age distribution of plants in this economy is uniform. Therefore, the measure of plants with age i capital is $1/N$ (the rate of capital depreciation for the aggregate economy). Each period, the age N plant is retired and all its capital is lost. Therefore, the aggregate capital stock falls by k_N/N. As the number of vintages increases, two there are two opposing effects. If a new plant has a longer lifetime, more capital is placed in that plant. However, with a larger number of vintages, the measure of a new plant is smaller and therefore has a smaller impact on the aggregate capital stock. It turns out that the for small values of N, the first effect dominates and for large values, the second effect dominates. Therefore, as N increase, investment initially rises, then falls. However, investment as a fraction decreases monotonically as N increases.

6 Transitional Dynamics

The experiment run in this section is an unexpected one standard deviation increase in the exogenous technology shock. In particular, the purpose of the experiment is to see how much the vintage capital structure of the economy enhances the propagation mechanism of capital accumulation. To compute the dynamics, the model is linearized around the steady state. The system of difference equations characterizing the model's dynamics has N eigenvalues with modulus less than one. This corresponds to the model's N state variables $k_1, ..., k_N$. Therefore, the transition path is stable and unique. The dynamics of the vintage capital are strikingly different from standard models. Figures 1.1 − 1.4 present the dynamic transition paths for investment in a new plant k_1, employment in a new plant l_1, output y and consumption following a one standard deviation shock to multifactor productivity when the lifetime of capital is set at 20 years. Note that this implies an aggregate depreciation rate of 5% annually. For comparison purposes, an identical experiment is run using a standard real business cycle model with the depreciation rate set at 5% annually. These results are shown in figures 2.1 − 2.4. The initial impact is similar in each model. The improved technology increases the demand for capital and thus raises investment. It is in the periods following the shock that the vintage capital structure alters the dynamic paths returning the economy to the steady state. Notice that the persistence mechanism of a technology shock is greatly enhanced. In the standard model, the economy returns to its steady state after 10 periods. By contrast, the vintage structure remains above the steady state for over 25 periods. Similar experiments are calculated for various capital lifetimes.

		Length of Expansion	
Lifetime of Capital	*Effective Aggregate Depreciation*	*Vintage Model*	*Standard Model*
5	*20%*	*10*	*5*
10	*10%*	*16*	*7*
20	*5%*	*25*	*10*

As the lifetime of capital increases (which would correspond to periods with a lower rate of technological growth), the length of the expansion caused by deviation to disembodied technology growth increase proportionally. In the standard model, the length of the cycle is largely unrelated to the rate of depreciation.

7 Intuition

7.0.1 A simple example: No labor

Suppose that output is produced using only capital. The lifetime of capital is N years. Therefore, the output of an age i plant is output is $y_i = k_i^{\alpha}$. The total measure of plants is one. Therefore, each vintage plant is of measure $\left(\frac{1}{N}\right)$. Aggregate output is the sum over all vintages of plant output.

$$y_t = \sum_{i=1}^{N} \left(\frac{1}{N}\right) k_{it}^{\alpha} \tag{7.24}$$

The economy is inhabited by a representative agent with standard CES preferences.

$$U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \tag{7.25}$$

Output can be used for consumption or for investment in new capital.

$$\begin{aligned} c_t + I_t &= y_t \\ I_t &= \left(\frac{1}{N}\right) k_{1t+1} \end{aligned} \tag{7.26}$$

First, consider the steady state of this economy. The first order condition for capital requires that the marginal cost of new investment (in terms of utility) must equal the discounted lifetime utility value of the marginal product of that capital. Note that there is no physical depreciation of capital. Therefore, an investment in a new plant of size k_1 provides k_1^α units of output for N periods. Let λ_t be the marginal utility of consumption at time t. The first order condition for capital can be written as follows.

$$\lambda_t = \sum_{i=1}^{N} \left\{ \beta^i \lambda_{t+i} \right\} \alpha k_{1t+1}^{a-1} \tag{7.27}$$

In the steady state, all variables are constant. Therefore, $\lambda_{t+i} = \overline{\lambda} \; \forall \; i$. Therefore, if steady state capital in a new plant is given by k_{ss},

$$k_{ss} = \left\{ \alpha \sum_{i=1}^{N} \left(\beta^i \right) \right\}^{\left(\frac{1}{1-\alpha} \right)} \tag{7.28}$$

Note that as the lifetime of capital increases, the steady state size of a new plant increases as well. This is because increasing the lifetime of capital raises the discounted lifetime value. However, as the capital per plant increases, the measure of each plant falls as N increases. Steady state investment, and consumption can be written as follows.

$$
\begin{aligned}
I_{ss} &= \Omega(N)\, y_{ss} \\
c_{ss} &= [1-\Omega(N)]\, y_{ss}
\end{aligned} \tag{7.29}
$$

where $\Omega(N)$ represents the fraction of output devoted to investment.

$$
\Omega(N) = \frac{\alpha \sum_{i=1}^{N} \beta^i}{N} \tag{7.30}
$$

Note that $\Omega(N)$ is strictly decreasing in N. For low values of N the effect of larger plant size dominates over the effect of the declining measure of the plant with respect to the total. Therefore, for small values of N, an increase in the lifetime of capital raises steady state investment. However, investment as a fraction of steady state output is decreasing and monotone in N.

Now, consider the following experiment. Give the vintage capital economy a "gift" of ε units of capital and look at the ensuing transitional dynamics back to the steady state relative to a traditional model of capital accumulation.

First, consider the standard neoclassical model without labor. The first order condition is

$$\lambda_t = \beta\lambda_{t+1}\left[\alpha\left(k_{t+1}\right)^{\alpha-1} + (1-\delta)\right] \tag{7.31}$$

where λ is the marginal utility of consumption. The corresponding steady state condition for capital is as follows.

$$1 = \beta\left[\alpha\left(k_{ss}\right)^{\alpha-1} + (1-\delta)\right] \tag{7.32}$$

Note that λ_t is a decreasing function of the capital stock k_t. That is, $\lambda_t = \lambda\left(k_t\right)$ with $\lambda'\left(k_t\right) \leq 0$. Suppose that following the gift of ε units of capital, the economy's investment immediately returns to the steady state. Consider the first order condition governing capital accumulation.

$$\begin{aligned} \lambda\left(k_{ss}+\varepsilon\right) &\lesseqgtr \beta\lambda\left(k_{ss}+(1-\delta)\,\varepsilon\right)\left[\alpha\left(k_{ss}+(1-\delta)\,\varepsilon\right)^{\alpha-1} + (1-\delta)\right] \\ \frac{\lambda\left(k_{ss}+\varepsilon\right)}{\lambda\left(k_{ss}+(1-\delta)\,\varepsilon\right)} &\lesseqgtr \beta\left[\alpha\left(k_{ss}+(1-\delta)\,\varepsilon\right)^{\alpha-1} + (1-\delta)\right] \end{aligned} \tag{7.33}$$

The left side of this expression is less than one. If investment returns to the steady state, next period's capital stock is smaller than today's capital stock. Therefore, consumption is smaller than today and hence the marginal utility of consumption is larger. The degree to which the left hand side is less that one depends on the curvature of the utility function. The right hand side of this expression represents the (discounted) marginal product of capital. This will also be less than one again because capital is above the steady state. The degree to which it is less than one depends on the curvature of the production function. Therefore, it depends on the relative curvature of production and preferences to determine whether steady state investment is too small or too large. In standard models, investment drops below steady state which implies that the left hand side is larger than the right.

Now, consider the same example, but with the vintage model. That is, suppose that the economy is given a "gift" of ε units of capital and then returns to the steady state. The resulting efficiency condition for capital would be as follows.

$$\lambda\left(k_{ss}+\varepsilon, k_{ss}, ., k_{ss}\right) \lesseqgtr \alpha k_{ss}^{a-1}\left\{\beta\lambda\left(k_{ss}, k_{ss}+\varepsilon, k_{ss}, ., k_{ss}\right)+...+\beta^{N}\lambda\left(k_{ss}, ., k_{ss}+\varepsilon\right)\right\} \tag{7.34}$$

Also, note that λ is the marginal utility of consumption. All plants have the same technology. Therefore, aggregate consumption and, hence, marginal utility doesn't depend on the location of capital. Therefore, $\lambda\left(k_{ss}+\varepsilon,k_{ss},..,k_{ss}\right)=\lambda\left(k_{ss},k_{ss}+\varepsilon,..,k_{ss}\right)=...=\lambda\left(k_{ss},k_{ss},..,k_{ss}+\varepsilon\right)$. This suggests that the λ in the above expression cancel out and the equation is reduced to the following.

$$1\lesseqgtr\alpha k_{ss}^{a-1}\left\{\beta+\beta^{2}+...+\beta^{N}\right\} \tag{7.35}$$

Notice that this is the condition for steady state capital. This suggests two things: first, in a vintage model, returning to the steady state level of investment immediately following the shock is optimal and, second, this is regardless of the curvature of utility. This primarily due to the fact that, unlike traditional models of capital accumulation, new investment is not added to the aggregate capital stock but, rather, placed in a new location. Therefore, the marginal productivity of new investment expenditures is independent of the existing capital stock. However, suppose that the economy did go immediately to the steady state level of investment and stayed there indefinitely. Consider the efficiency condition for capital an time $t+N$ (the period that the "gift" wears out).

$$\lambda_{t+N} = \alpha k_{ss}^{\alpha-1}\left\{\beta\lambda_{t+N+1} + ... + \beta^N\lambda_{t+2N}\right\} \tag{7.36}$$

Where $\lambda_{t+N} = \lambda\left(k_{ss}, ..., k_{ss} + \varepsilon\right)$ and $\lambda_{t+N+i} = \lambda\left(k_{ss}, ..., k_{ss}\right) \; \forall \; i$. This says at the period the capital wears out, consumption experiences a large drop hence the marginal utility of consumption experiences a large increase. This would make the left hand side smaller that the right and dictate an increase in investment. Suppose that investment remained at the steady state level until time $t+N$. Then the efficiency condition for capital at time t would read as follows.

$$1 \lesseqgtr \alpha k_{ss}^{a-1}\left\{\beta + \beta^2 + ... + \beta^N\left(\frac{\lambda_{t+N}}{\lambda_t}\right)\right\} \tag{7.37}$$

Therefore, the curvature of utility does come into play as in the standard case, however it is discounted significantly. This strategy doesn't eliminate the drop in utility from the gift wearing out, it only diminishes the effect by spreading it out over two periods. An investment strategy (in fact, the correct strategy) would be to start preparing for it at time t and spreading it out over N periods. The condition for this strategy is given by the following.

$$1 \lesseqgtr \alpha k_{ss}^{a-1} \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \beta + \left(\frac{\lambda_{t+2}}{\lambda_t} \right) \beta^2 + ... + \beta^N \left(\frac{\lambda_{t+N}}{\lambda_t} \right) \right\} \tag{7.38}$$

Suppose that the investment strategy is chosen to spread out the changes in marginal utility evenly over time so that

$\left(\frac{\lambda_{t+1}}{\lambda_t}\right) = \left(\frac{\lambda_{t+2}}{\lambda_t}\right) = ... = \left(\frac{\lambda_{t+N}}{\lambda_t}\right)$. Then the above equation can be written as

$$\frac{\lambda_t}{\lambda_{t+1}} \left(1 + \beta + ... + \beta^N\right)^{-1} \prec \frac{\lambda_t}{\lambda_{t+1}} \lesseqgtr \beta \left[\alpha k_{ss}{}^{\alpha-1}\right] \tag{7.39}$$

When compared to the efficiency condition of the standard model, two things can be noticed. First, the effect of the curvature of utility is greatly reduced. All else equal, a vintage model would behave like a standard model with a much lower curvature of utility, which would imply a greater degree of persistence. Second, note that unlike the standard model, the marginal utility of new investment is independent of the aggregate capital stock (the RHS is independent of ε). This tells us that the marginal utility of new investment is everywhere larger in the vintage model than in the standard model. This also acts to increase the persistence following an increase in the capital stock. Figures $3.1 - 3.4$ show the resulting dynamics of one percent shock to new capital from the steady state. Figure 3.1 shows the transition path for new investment. Initially, following the gift of one percentage increase in the capital in a new plant, investment nearly returns to the steady state. However, as the retirement date of the extra capital draws closer, the discounted marginal utility loss of that decrease in capital gets larger, and hence, investment begins to increase. Therefore, from period 1 to period 12, the aggregate capital stock, output and consumption rise. At period 12, the capital "gift" is retired and consumption, and output experience a drop. The size of that drop depends on the curvature of utility. As the parameter σ(dictating the curvature of utility) is increased, investment rises more following the shock, and, hence, subsequent the drop in output and consumption on the retirement date is smaller.

7.0.2 A simple model with exogenous labor

Now, consider adding labor to the above vintage model. Technology can now be represented by the following.

$$y_t = \sum_{i=1}^{N} k_{it}^{\alpha} l_{it}^{\omega} \qquad \alpha + \omega \leq 1 \tag{7.40}$$

This only difference from the no labor version is that the first order condition for capital now becomes as follows.

$$\lambda_t = \sum_{i=1}^{N} \left\{ \beta^i \lambda_{t+i} l_{it+i}^{\omega} \right\} \alpha k_{1t+1}^{a-1} \tag{7.41}$$

Also, all plants face a common wage. This requires that the representative firm manager allocate labor across plants such that, given the distribution of capital, the marginal product of labor is equated across plants.

$$\omega k_{it}^{\alpha} l_{it}^{\omega-1} = \omega k_{jt}^{\alpha} l_{jt}^{\omega-1} = w_t \ , \forall \ i, j \tag{7.42}$$

Allocating labor across plants so that the marginal products are equated results in the maximum current period output given the capital distribution. To understand the effect on the first order condition for capital, think of the no labor model as an exogenous labor model where labor *cannot* be allocated across plants. For example, suppose that the distribution of labor is uniform across plants so that $l_i = \frac{l}{N}$. In the exogenous labor model with reallocation, this choice is always available. Therefore, if it isn't chosen, it must be because there is a better alternative available. That is, with the ability to reallocate labor, the lifetime marginal product of capital is at least as big as the no labor model for every value of capital. This should act to improve the persistence of shocks to the capital stock. Figures 4.1-4.4 show the result of a one percentage shock to new capital in a vintage model with exogenous labor and reallocation of labor between plants. Note that new investment is consistently higher following the shock now that labor reallocation is allowed. Adding endogenous labor supply only increases the persistence even more. This is because above steady state investment builds up the capital stock which raises real wages above the steady state. Higher wages increases labor supply.

7.1 Vintage Effects versus Irreversibility Effects

An interesting question to ask at this point is as follows. The economic environment presented here has two major deviations from the traditional approaches:

1) *Vintages are independent*: Each plant is of one vintage only rather than combining several vintages.

2) *The irreversibility effect:* Once new investments have been made, they cannot be reversed for N periods when the plant is scrapped and replaced with a new plant.

Therefore, is the enhanced propagation mechanism a result of one of the two effects alone, or are both effects needed to produce the desired result? While ultimately this is a question that needs to be decided through quantitative testing, some observations can be noted here. Consider the following two modifications two model.

7.1.1 Irreversibility without separate vintage plants

Consider a model with one plant. All new investment is placed in that one location. Once the capital is in place, it can't be removed for N periods. The first order condition for capital can therefore be written as follows.

$$\lambda_t = \sum_{i=1}^{N} \left\{ \beta^i \lambda_{t+i} \right\} \alpha k_{t+1}^{a-1} \tag{7.43}$$

Note that this is the same condition as in the previous section. Steady state plant investment is , therefore, the same as the previous case and is an increasing function of N. The difference, however, is in the dynamics. Consider a deviation from the steady state.

$$\lambda\left(k_{ss}+\varepsilon\right)=\sum_{i=1}^{N}\left\{\beta^{i}\lambda\left(k_{ss}+\varepsilon\right)_{i}\right\}\alpha\left(k_{ss}+\varepsilon\right)^{\alpha-1} \tag{7.44}$$

Now, new investments are added to the existing stock of capital. Suppose that the lifetime of capital were increased by one period, how does this affect the incentives for new investment? A longer lifetime of capital raises the steady state capital stock and, hence, lowers the marginal product of any new investment placed on top of that. Therefore, even though the returns to new investment can be enjoyed for an extra period, the returns per period are smaller. In short, as the length of the irreversibility increases, naturally, the amount of persistence increases because that capital is stuck in place for a longer period of time. However, without the vintage effect, the marginal product of new investment is lower and so the *magnitude* of changes in investment should be smaller. Adding vintages allows the productivity of new investment to be independent of the steady state capital stock and hence the persistence can be increased without sacrificing magnitude.

7.1.2 Vintages without irreversibility

Now, consider a model with N plants. Each period, an age N plant is retired and replaced with a new plant. However, now the size of each plant can be adjusted through positive or negative investment each period. Now, there are N first order conditions for capital (one for each plant) rather that just one. The first order condition facing an age i plant is as follows.

$$\lambda_t = \beta\lambda_{t+1}\left[\alpha\left(k_{it+1}\right)^{\alpha-1}\right] \tag{7.45}$$

In fact, the first order condition facing each plant is the same (except for the trivial condition facing the age N plant) and this condition looks essentially the same as the standard neoclassical model. Without irreversibility, a vintage model would behave essentially the same as the standard framework. In short, the persistence is lost, but the magnitude is recovered.

8 Conclusions

There seems to be a relationship between the growth rate of the economy and the average length of business cycles. Periods of faster growth seem to coincide with periods of shorter cycles. The view taken here is that economic disturbances can be classified as process disturbances (the ability to produce more output per unit of capital) and process improvements (the ability to produce more productive capital goods over time). Standard models concentrate on process innovations rateher than product innovations. Capital is homogeneous and adjusts smoothly to exogenous process innovations. . The ability of this class of model to replicate the relation between cycle length and trend growth hinges on the statistical properties of the inderlying disturbances to productivity.

This paper contrructs a vintage capital model which stresses the importance of product innovations. It is shown that process improvements have a smaller economies with faster adoption rates for new capital in the sense that the economy returns much faster to its balanced growth path.

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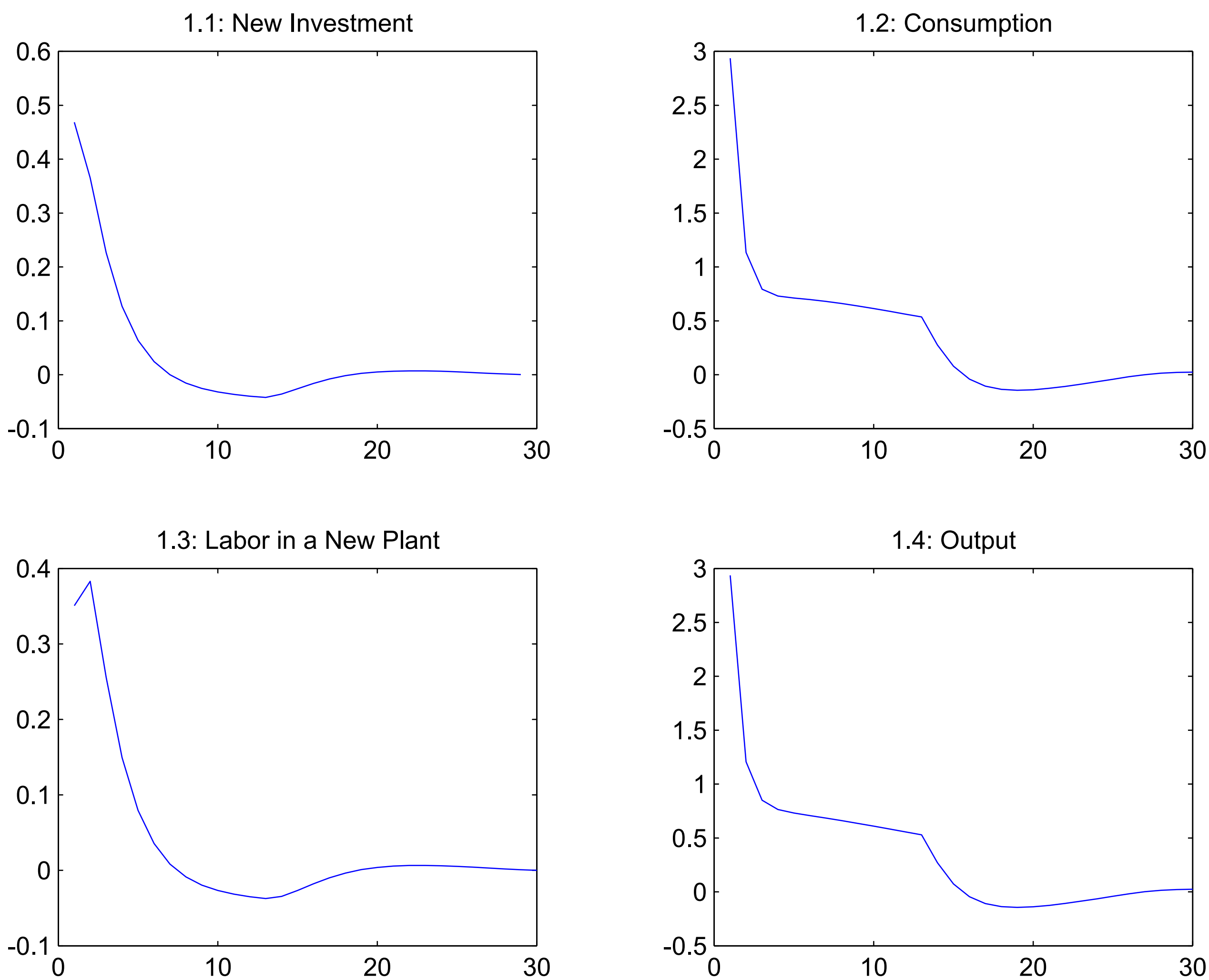
1.1: New Investment
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
10
20
30
1.2: Consumption
3
2.5
2
1.5
1
0.5
0
-0.5
0
10
20
30
1.3: Labor in a New Plant
0.4
0.3
0.2
0.1
0
-0.1
0
10
20
30
1.4: Output
3
2.5
2
1.5
1
0.5
0
-0.5
0
10
20
30

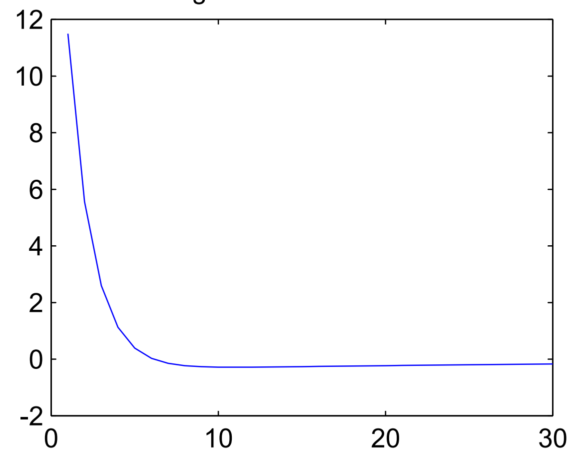

Figure 2.1: Investment

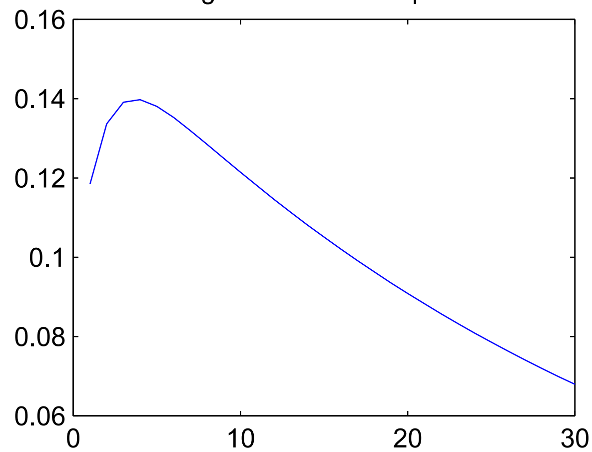

Figure 2.2: Consumption

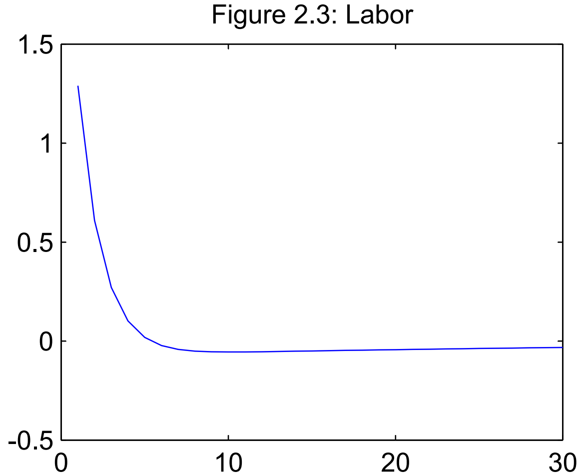

Figure 2.3: Labor

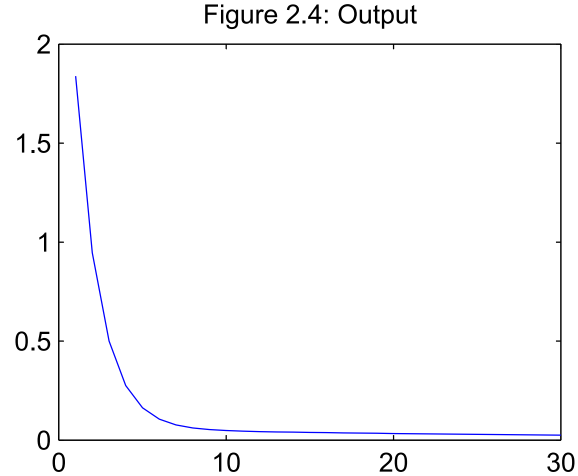

Figure 2.4: Output

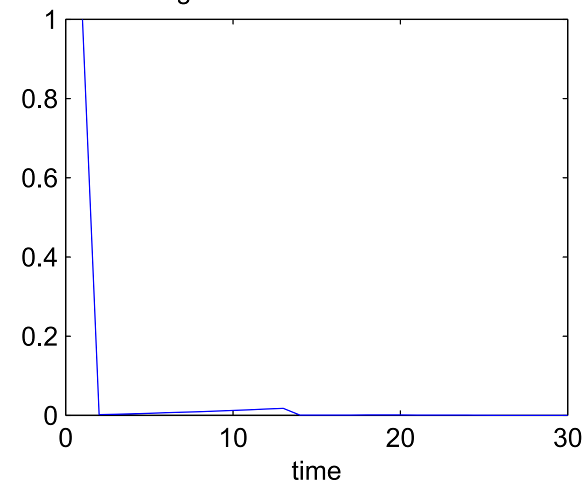

Figure 3.1: New Investment

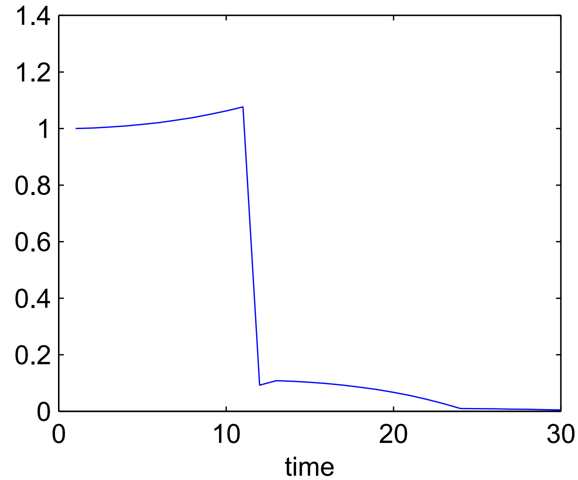

Figure 3.2: Capital Stock

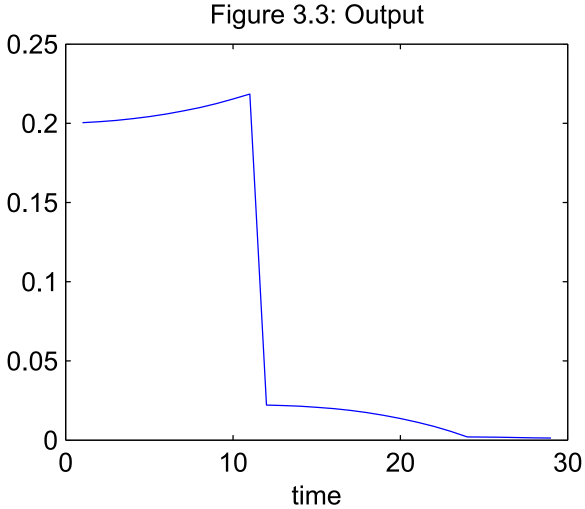

Figure 3.3: Output

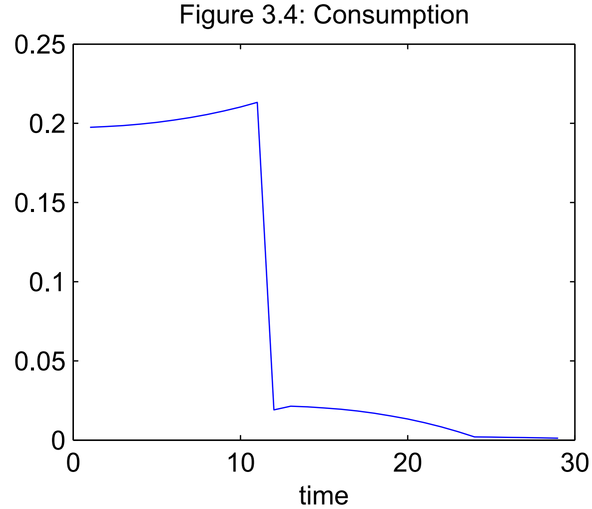

Figure 3.4: Consumption

Figure 4.1: New Investment

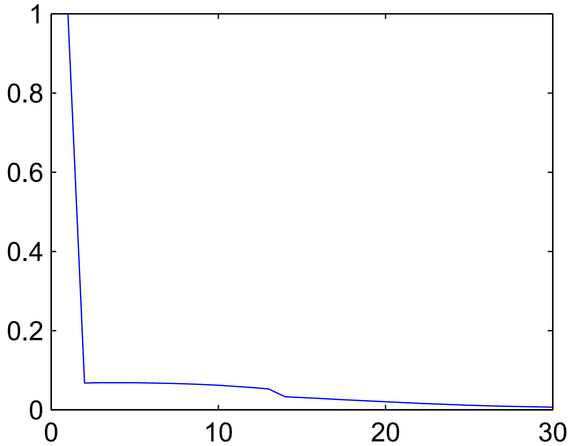

Figure 4.2: Consumption

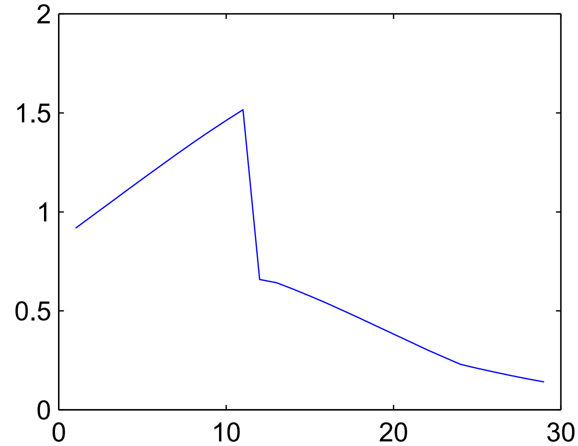

Figure 4.3: Labor in a New Plant

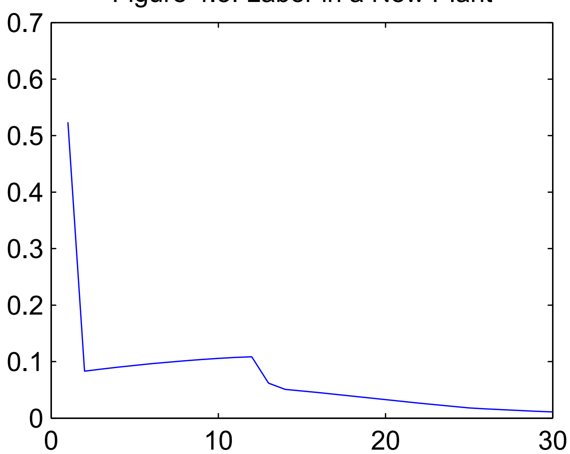

Figure 4.4: Output