Finance 462
Solutions to Practice Midterm #2

1) We have a bond with a face value of $10,000 and an annual coupon payment of 5%. Therefore, the payment stream of this bond is $500, $500, $10,500. To calculate the price of the bond, we take the present value using the spot rates given in the yield curve.

$$P = \frac{\$500}{(1.06)} + \frac{\$500}{(1.05)^2} + \frac{\$10{,}500}{(1.04)^3} = \$10.261.91$$

First, let's calculate the Macaulay duration of the bond. Each payment has a Macaulay duration equal to its term (1, 2, and 3). The Macaulay duration of the entire bond is equal to the weighted average of the individual payments where the weights are equal to each payment's contribution to the bond's price.

$$\text{Mac Dur} = \left(\frac{\$471.70}{\$10{,}261.91}\right)(-1) + \left(\frac{\$454.70}{\$10{,}261.91}\right)(-2) + \left(\frac{\$9335.59}{\$10{,}261.91}\right)(-3) = -2.86$$

Now, to get the modified duration, divide the above number by 1 plus the interest rate. Which interest rate do you use? Use an average of the three (5%).

MD = -2.86/1.05 = -2.73

This says the bond's price should drop by 2.73% ($280.15) for every 100 basis point interest rate increase. Therefore a 50 basis point increase lowers the bond's price by $140.08.

Recalculating the bonds price given a 50 basis point increase across the board:

$$P = \frac{\$500}{(1.065)} + \frac{\$500}{(1.055)^2} + \frac{\$10{,}500}{(1.045)^3} = \$10{,}119.82$$

This is a difference of $142.09. The difference has to do with the fact that duration is using a linear approximation of a nonlinear pricing function.

The bonds three key durations are the three components of the above Macaulay duration:

$$D(1) = \left(\frac{\$471.70}{\$10{,}261.91}\right)(-1) = -.046$$

$$D(2) = \left(\frac{\$454.70}{\$10{,}261.91}\right)(-2) = -.089$$

$$D(3) = \left(\frac{\$9335.59}{\$10{,}261.91}\right)(-3) = -2.73$$

First, convert them to Modified durations:

MD(1) = .046/1.05 = .044
MD(2) = .089/1.05 = .084
MD(3) = 2.73/1.05 = 2.6

Now, calculate the percentage change in price:

(-.044)(-1) + (-.089)(0) + (-2.73)(1) = 2.73 - .044 = -2.69% =-$272.22

2) Note that this bond has the same payout as in (1) ($500,$500, $10,500). Given the same yield curve, the bond's initial price is $10,261.91.

   For a callable, bond, if the interest rate drops below 5%, the bond is repaid early. This happens given a 50 basis point drop (the new yield curve is 5.5% 4.5%, 3.5%). Therefore, for a 50 basis point decrease, we have a price of

$$P = \frac{\$500}{(1.055)} + \frac{\$10{,}500}{(1.045)^2} = \$10{,}089.09$$

   Therefore, the effective duration is

   ED = (($10,119.82 - $10,089.09)/$10,261.91)*100 = .3

   Not that this is a positive number rather than a negative number. Due to the call option, the recall triggers a decrease in the bond's value.

3) A 90 Day T-Bill Future with 1 year until expiration is implying the 90 day spot rate 1 year from now – F(360, 90)

   Given the yield curve

Term	**Annual Rate**
1yr	3%
2yr	5%
3yr	6%

We first need to estimate the 1 yr, three month spot rate. The slope of the yield curve between 1 year an 2 years is:

$$slope = \frac{5-3}{360 Days} = .0056\% \text{ per day.}$$

$$90 \text{ Days} * .0056 \text{ per day} = .5\%$$

Therefore, the 1.25 year spot rate (annualized) is 3.5% (1.035). The 1 year spot rate is 3% (1.03) therefore, the F(360, 90) can be found as follows:

Investing $1 at 3.5% for 1.25 years: $(1.035)^{1.25} = 1.044$ = 4.4%

This should give you the same return as investing at the 1 year rate for a year, then investing at the .25 year forward rate:

(1.03)(1 +F) = 1.044

(1+F) = 1.0135 = 1.35% (over the 90 day period) = 5.4% annualized

Therefore, the DY should be 5.4% which gives an IMM of 100 – 5.4 = 94.6

If the current price is 94 (6% annualized yield), then the 90 day return on the futures contract is 1.5% - this is higher than the forward rate implied by the yield curve. Therefore, you should borrow using spot market instruments and then lend out that money in the futures market (go long on futures).

The three year Treasuries from (1) have a modified duration of -2.73. Multiply this by the price to get the dollar duration equal to (.0273)(10,261) = $280.12 per T-Bill. You own 10 of them, so the dollar duration of your portfolio is $2,801.20. Divide this by $2,500 (the dollar duration of the T-Bill future)to get the hedge ratio

$2,801/$2,500 = 1.12 contracts.