Finance 462
Outline for Midterm #2

I: Assessing Interest Rate Risk

Every financial asset involves payouts over time. Therefore, every asset can be priced by calculating the *present value* of the expected payments. Recall that as interest rates rise, present values fall. Further, as payments are moved further out into the future, their present value becomes more sensitive to interest rate changes. Duration is an attempt to measure this sensitivity.

Non-contingent payments (US Treasuries, Municipal Bonds)

Consider a bond with one fixed payment (X) to be paid 'N' years from now. The price of this bond can be written as a function of the interest rate (i):

$$P(i) = \frac{X}{(1+i)^N}$$

Dollar duration (the change in price due to a 100 basis point increase in the interest rate) is the derivative with respect to the interest rate

$$\text{DD} = P'(i) = \frac{-NX}{(1+i)^{N-1}}$$

Modified duration (the percentage change in price due to a 100 basis point increase in the interest rate) is dollar duration divided by price

$$MD = \frac{P'(i)}{P} = \frac{\dfrac{-NX}{(1+i)^{N-1}}}{\dfrac{X}{(1+i)^N}} = \frac{-N}{(1+i)}$$

Macaulay duration (the percentage change in price due to a 1% increase in the 1+i) is modified duration multiplied by (1+i)

$$Mac.D = MD(1+i) = \frac{-N}{(1+i)}(1+i) = -N$$

For assets with multiple payments, duration of the asset equals the weighted average of individual payment durations where the weights are equal to each payment's contribution to the asset's price)

$$Mac.D = \sum_{j=1}^{N} w_j Mac.D(j)$$

$$w_j = \frac{\left(\frac{X_j}{(1+i)^j} \right)}{P}$$

Weaknesses of duration:

1) The pricing function P(i) is non-linear. Therefore, duration is only an acceptable measure for very small interest rate changes.
2) Duration only measures the response of an asset price to a parallel shift in the yield curve – this rarely happens. Key durations would be a better measure for changes in the shape of the yield curve.

Contingent Payments (Corporate debt, Asset Backed/Mortgage Backed securities, stocks, callable bonds, etc)

Assessing interest rate risk is difficult when the promised payments are variable, but the process is the same. We need to measure the impact of interest rate changes on price. The usual measure is called effective duration:

$$ED = \frac{P_{+50} - P_{-50}}{P}$$

II: Hedging Risk: Interest Rate Futures

Futures

Futures are easy to work with because they are symmetric contracts. The buyer (long position) agrees to buy a commodity at a pre-specified price while the seller (short position) agrees to sell at a pre-specified price. Upon expiration, profits are equal to

Profit (long position) = S-F (S = Spot price, F = Futures price)

If the spot price rises above the futures price, the long position benefits. If the spot price falls, the short position benefits.

Pricing futures involves constructing an alternative investment strategy with the same payout. If two investments have the same payout, then they should have the same price. The result is the following formula:

Treasury Bill Futures are the most straightforward of the interest rate futures because the commodity is well defined – it is a T-Bill with 90 Days left until maturity and $1M of face value.

Pricing Treasury bill Futures:

Treasury bill futures are priced through a zero arbitrage condition. Consider a Treasury bill future that expires in 1 month (30 days). The price of the Treasury bill is written in terms of an IMM (international monetary market) index:

IMM = 100 – Discount Yield (DY)

Where

$$DY = \left(\frac{\text{Face Value - Price}}{\text{Face Value}} \right) * \left(\frac{360}{\text{Days to Maturity}} \right) * 100$$

Consider a Treasury bill future that expires in 30 days with an IMM of 96. This implies an annualized DY of 4% (1% yield over the 90 day period). The timeline for this transaction is:

Now: Enter into Contract

30 Days from now: Purchase $1M in 90 Day T-Bills for $99 (per $100) of Face Value - $990,000.

120 Days from Now: Receive $1M.

Therefore, the futures price implies a forward rate. IN this case, the 90 Day spot rate 30 days forward. F(30,90). If this rate is inconsistent with the forward rate implied by the current yield curve, there exists a profit opportunity.

Hedging with T-Bill Futures:

If you are concerned about the value of your portfolio dropping when interest rates rise, you will want to go *short* in T-Bill futures (as interest rates rise, T-Bill prices fall – the short position benefits from this. To figure out the hedge ratio (# of contracts to buy)

Hedge Ratio = Dollar Duration of Portfolio/Dollar Duration of T-Bill Future

= Dollar Duration of Portfolio/$2,500