Finance 462
Solutions to Practice Midterm #1

1) Recall that the CPI is a *fixed weight index*. Therefore, the price index each year is simply a weighted average of the various individual goods prices. The weights are chosen to match the average household's budget. In this example, the index could be defined as follows:

$$CPI = .40\left(P_{Housing}\right) + .20\left(P_{Trans}\right) + .15\left(P_{Food}\right) + .05\left(P_{Medical}\right) + .05\left(P_{Education}\right) + .05\left(P_{Other}\right)$$

Since the weights are fixed, we can represent the above in terms of inflation rates.

$$\pi_{CPI} = .40\left(\pi_{Housing}\right) + .20\left(\pi_{Trans}\right) + .15\left(\pi_{Food}\right) + .05\left(\pi_{Medical}\right) + .05\left(\pi_{Education}\right) + .05\left(\pi_{Other}\right)$$

Plugging in the respective inflation rates, we get:

$$\pi_{CPI} = .40(2.71) + .20(1.77) + .15(2.44) + .05(3.8) + .05(2.36) + .05(0) = 2.11\%$$

Note that inflation rates vary across different goods as well as across time periods. Most notably, the price of transportation has fluctuated from -2% to 5%. The CPI assumes that even in the face of significant price swings, consumers don't alter their behavior. Foe example, its reasonable to believe that when transportation prices fall, individuals buy more (i.e. you buy a new car when its on sale). Therefore, in the years where transportation was falling in price, the CPI is probably overstating inflation.

Medical care has two key differences from other goods. First, it is pretty much a necessity (in general). Therefore, the substitution effects shouldn't be as big as transportation. However, the other key difference with medical expenses is that most of us have health insurance and therefore don't pay the full cost. Therefore, the rapidly rising costs of healthcare might be overstating the CPI inflation rate.

The GDP Deflator is a *variable weight index*. Its weights are allowed to change as production changes (if automobile sales increase, the weight of automobiles in the deflator adjusts accordingly). Therefore, the deflator avoids the error due to substitution.

2) In any given month, you will find 12 people out of work (10 of them recently lost their job and will find a new one soon while the remaining two have been out of work for a while). The current unemployment rate would be 12/100 = 12%. To calculate duration , first, add up the total number of people unemployed over the course of the year.

10 per month * 12 months per year = 120.

2 additional per year.

Total = 122.

(120/122) = 98% are short term (1 month)

(2/122) = 2% are long term (12 months)

Average duration = (.98)(1) + (.02)(12) = 1.22 months = 4.88 weeks.

In general, the unemployment rate can be misleading because it measures those working as a percentage of *those choosing to be in the labor force*. If employment drops and the labor force shrinks as well (the "discouraged worker effect") the unemployment rate can fall as well.

3) An economy operates according to the following process:

Time t: The current capital stock is fixed. The level of employment determines the economy's ability to produce output:

$Y = F(k, l, A)$

At time t, we must determine how that output is allocated

Y = C + I + G + NX

Y = Total Output (GDP)

I = Investment Expenditures

C = Consumption Expenditures

G = Government Purchases

NX = Exports – Imports

The interest rate determines how output is allocated. For example, higher interest rates inhibit consumption spending which makes more money available for investment spending. It is the job of the financial sector to insure two things:

- Available savings is used to finance capital investment expenditures (S = I + (G-T) + NX
- Given a fixed level of investment, only the most profitable investment projects are undertaken (efficient investment)

For this efficient transfer of resources to take place, the financial system must me able to:

- Create liquidity (i.e. create deep markets)
- Manage Risk (allow for efficient diversification)
- Accurate transmit information

Capital Investment allows the economy to move from period t to period t+1.

k(t+1) = k(t)(1-depreciation) + Investment(t)

That is, today's investment determines tomorrow's capital stock and, hence, tomorrow's production capabilities. If the financial system shuts down, this transfer of resources to investment projects stops and an economy can't grow (which is what happened in Asia).

The problem in Asia was the government exacerbated the information problems that exist in financial markets.

Adverse Selection: A bank manager can't distinguish between the "safe" borrowers and the "risky" borrowers.

Moral Hazard: A bank manager worries that worries that good credit risks will turn into bad risks once the loan has been made.

Ordinarily, a bank has to deal with these issues and, hence, must assess risk and write the loan contracts in way to discourage bad behavior. However, with the government guaranteeing the loans, this was no longer necessary. The result was that Thai banks created a lot of bad loans. When outside investors began observing the underperforming loans, they began selling their Baht position (this caused the Baht to drop in value). With loan portfolios already performing badly, the banking sector couldn't handle the currency devaluation and went bankrupt.

4) All the statistical frameworks are similar in that they are attempts to explain future interest rate movements by characterizing what has happened in the past. Most statistical models can be written in the following form:

$$di = a(i,t)dt + b(i,t)dz$$

Where di is the current change in the interest rate. There are two components:

$a(i,t)dt$: This is the deterministic component. Dt is the change in time. Therefore, at some point in time, the interest rate changes according to some

process described by $a(i,t)$. In general, this is some function of the current value of the interest rate. The t indicates that this function can change over time. This component governs the long run average as well as some of the short term dynamics.

$b(i,t)dz$: This is the random component. Dz is a random error that is mean zero but with a positive variance. This component governs the variance of interest rates. As with the first component, this can be a function of the interest rate and can change over time.

The Vasicek model is the simplest case of the above framework.

$$a(i,t) = \kappa(\theta - i_t)$$

$$b(i,t) = \sigma$$

The contributions of the parameters are as follows:

θ (theta): Theta is the long run value for the interest rate. Note that when the interest rate equals theta , the first term in the interest rate equation is zero. The bigger theta, the higher the long run mean.

κ (kappa): When the interest rate is above/below its long run value, kappa determines how quickly it returns (the bigger kappa is, the faster the interest rate returns to its long run value)

σ (sigma): Sigma determines the magnitude of the impact of the random term dz. Recall that when you multiply a normal distribution by a constant, the variance is increased by the square of that constant. Therefore, if the variance of dz is one, then the variance of the interest rate is σ^2

Now, a more complicated example….suppose that theta and sigma are also random and, hence are described by their own distributions. In this case, the mean of the interest rate is equal to the mean of theta. As in the previous example, the variance is related to the square of whatever is in front of the dz term. On average, sigma is equal to 'b'. Therefore, on average, the variance of the second term is equal to b squared. However, there is now some variance coming from the first term. Remember that the average is no longer constant….it has its own variance (equal to one).

5) First, lets calculate the components that we need:

Year	GDP	Growth	CPI	Inflation	Fed Funds Rate
2004	$10,000	----	$100	----	2%
2005	$10,500	5%	$105	5%	3%
2006	$10,710	2%	$111	6%	4%

(Note that here we have the Fed Funds Rate….not the change in the Fed Funds Rate) Given the forecasting equation, we get the following path for interest rates:

$$\Delta i = -.10 + .1(\%\Delta GDP_t) + .2(Inflation_t) + .2(FF_t) - .5i_t$$

(2004-2005): -.10 + .1(5) + .2(5) + .2(3) - .5(4) = 0%

(2005-2006): -.10 + .1(2) + .2(6) + .2(4) - .5(4) = .10%

Year	Interest Rate	Predicted Change in Interest Rate	Predicted Interest Rate
2004	4%		
2005	---	0%	4%
2006	---	.1%	4.1%

Given the expected path for interest rates, we can calculate a yield curve (the expected average of future interest rates). Note that the exact method is to use geometric averages….a simple arithmetic average is good enough!

1yr: 4%

2 yr: (4% + 4%)/2 = 4%

3 yr: (4% + 4% + 4.1%)/3 = 4.03%

6) Recall that the household condition has two restrictions:

- Any choice for consumption must be affordable (present value of consumption must equal the present value of income). This is given by

$$c_1 + \frac{c_2}{(1+i)} = y_1 + \frac{y_2}{(1+i)}$$

- Further, any choice must be optimal. In general, the optimum is given by

  Marginal Utility of Current = (1+i)* Marginal Utility

  Consumption of Future Consumption

  Where the marginal utility is just the derivative of the utility function. This gives us

$$(1+i)=\left(\frac{1}{\beta}\right)\left(\frac{c_2}{c_1}\right)^{\sigma}$$

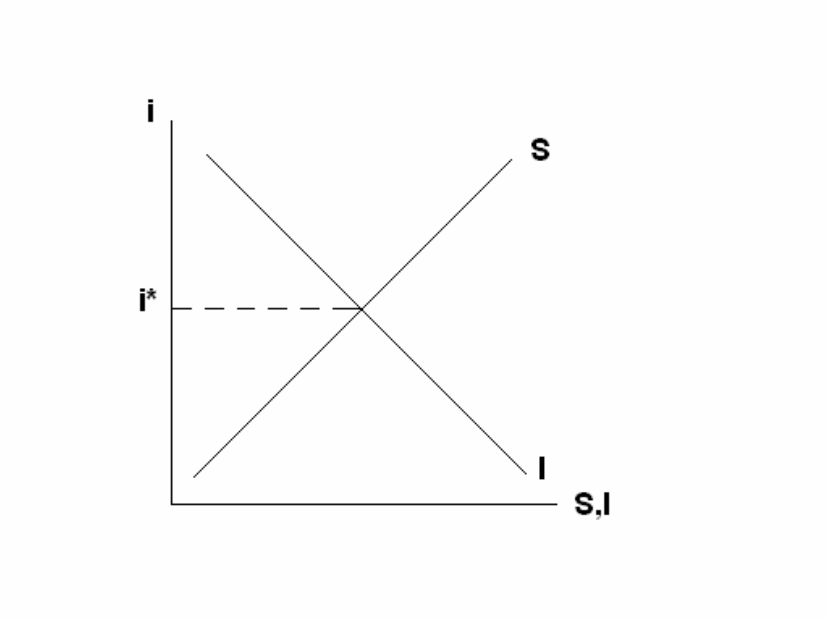
  The firm is choosing investment so that the present value of the lifetime contribution of capital is equal to the current cost:

$$P_k=\frac{MPK}{i}$$

  Where P(k) is the cost of capital, δ is the depreciation rate and MPK (the marginal product is the derivative of the production function with respect to k.

$$i=\alpha\left(\frac{A}{P_k}\right)\left(\frac{k}{l}\right)^{\alpha-1}$$

  In equilibrium, we have S=I (which determines the equilibrium interest rate)



Now, add the tax scheme: To keep things concrete, assume that $100 is taken from corporate profits and given to households in period 2: The household decision is adjusted as follows:

$$c_1 + \frac{c_2}{(1+i)} = y_1 + \frac{y_2 + \$100}{(1+i)}$$

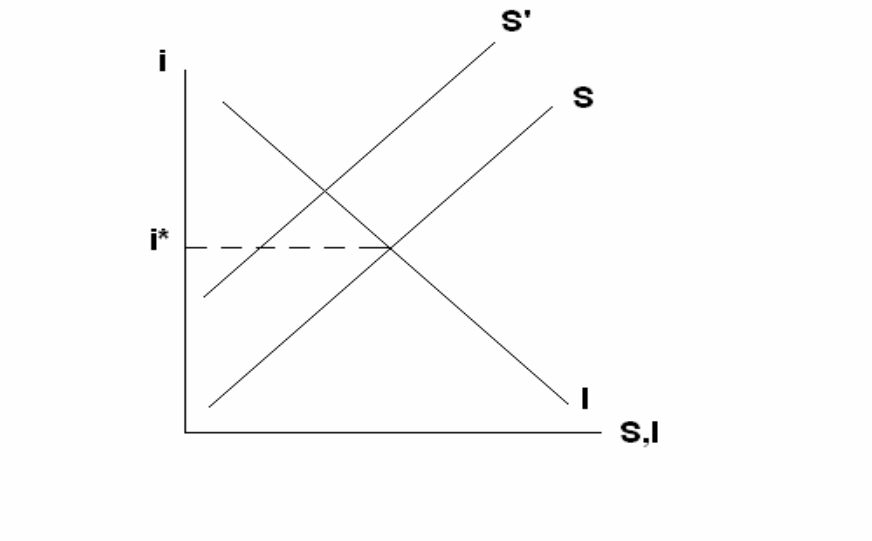
$$(1+i) = \left(\frac{1}{\beta}\right)\left(\frac{c_2}{c_1}\right)^{\sigma}$$

Note that since the interest rate hasn't changed, the household does not change the ration of current /future spending. However, with more income, each is scaled up a bit. Therefore, current income hasn't changed, but with more wealth, current spending rises. Therefore, current savings has to fall.

The firm's condition looks like this:

$$i = \alpha\left(\frac{A}{P_k}\right)\left(\frac{k}{l}\right)^{\alpha-1}$$

The tax is taken lump sum from the firm and, hence, doesn't affect decisions at the margin. (i.e, regardless of what the firm does, profits will be $100 lower).



As Savings drops, the interest rate begins to rise, which lowers the equilibrium amount of investment.