Finance 462
Solutions to Problem Set #9

1) With no fees, we have the following demand for loans:

$$Q = 125 - 624r_l - 90.4UR$$

First, to simplify, set the unemployment rate to 5% (.05)

$$Q = 125 - 624r_l - 90.4(.05) = 120.48 - 624r_l$$

To calculate the elasticity, we first need the derivative with respect to the interest rate.

$$\frac{dQ}{dr_l} = -624$$

Next, divide by the quantity of loans and multiply by the interest rate to get the elasticity.

$$\frac{dQ}{dr_l}\frac{r_l}{Q} = -624\left(\frac{r_l}{Q}\right)$$

To get total revenues as a function of L, first solve the demand curve for the interest rate.

$$r_l = \left(\frac{120.48}{624}\right) - \left(\frac{1}{624}\right)Q = .193 - .0016Q$$

Monthly revenues equal the interest rate charged (divided by twelve) times the quantity of loans issued times $100,000.

$$TR = \$100{,}000(Q)\frac{r_l}{12} = 1608Q - 13Q^2$$

Marginal revenue is the derivative with respect to L

$$MR = 1608 - 2(13)Q = 1608 - 26Q$$

Now, take the cost function

$$TC = 10{,}000 + \left(\frac{.05}{12}\right)(100{,}000)Q = 10{,}000 + 416Q$$

Marginal cost is the derivative with respect to Q

$$MC = 416$$

Now, to get the optimal amount of loans, set MR=MC and solve for Q

$$1608 - 26Q = 416$$
$$Q = 46$$

Now, given L, the interest rate can be found using the demand curve.

$$r_l = .193 - .0016Q = .193 - .0016(46) = .1194 = 11.94\%$$

Therefore,

$$TR = \$100{,}000\left(\frac{.1194}{12}\right)46 = \$45{,}770$$

$$TC = 10{,}000 + 416(46) = \$29{,}136$$

Profits = $16,634

At the profit maximizing point, elasticity of demand is

$$\frac{dQ}{dr_l}\frac{r_l}{Q} = -624\left(\frac{.1194}{46}\right) = 1.61$$

If we add the fees, the procedure is the same, by the demand curve becomes:

$$Q = 125 - 624r_l - .026(1200) - 90.4(.05) = 89 - 624r_l$$

Solving for the interest rate, we get

$$r_l = \left(\frac{89}{624}\right) - \left(\frac{1}{624}\right)Q = .1431 - .0016Q$$

Total Revenues now include interest income and fee income:

$$TR = \$100{,}000(Q)\frac{r_l}{12} + \left(\frac{\$1200}{12}\right)Q = 1292Q - 13Q^2$$

Total Costs are unchanged at

$$TC = 10{,}000 + \left(\frac{.05}{12}\right)(100{,}000)Q = 10{,}000 + 416Q$$

Set marginal revenue equal to marginal cost as in part (a) and the optimum is 33 loans and an interest rate of 9%. Profits are equal to$4322.

In part (d), things get interesting. The demand curve doesn't change, but now, Total revenues become:

$$TR = \$100{,}000(Q)\frac{r_l}{12} + (\$1200)Q = 2392Q - 13Q^2$$

Because all the fees are being paid up front. The optimal interest rate becomes 2.5%, 74 loans are created, and profits are $63,000!

2) Perfectly competitive firms are restricted to charge a price (or, more accurately, a spread) equal to marginal cost. Therefore, in a perfectly competitive market, the entire increase in marginal cost will be reflected in interest rates. A monopolist, however, charges a markup above marginal cost. This markup is chosen optimally given that the bank will lose some demand if it raises its price. Therefore, to save some sales, the bank will increase price (here, the loan rate) by an amount less than the marginal cost increase.

3) Optimal decisions are made at the margin for both competitive firms and monopolists. Therefore, a change in a fixed cost (a cost that is independent of sales) will have no effect on price – only on profits.

4) Given the information on Assets/Liabilities, your balance sheet is as follows (interest paid/received in parentheses):

Assets	**Liabilities**
Cash: $15,000	Checking: $200,000
Short Term Loans: $160,000 (7%)	Savings: $50,000 (2%)
Government Securities: $100,000 (3%)	
	Equity Capital: $25,000

a) The reserve requirement states that the bank must hold 5% of total deposits (5% of $250,000 is $12,500) in the form of either cash or deposits at the Federal Reserve. Therefore, your current cash holdings of $15,000 in cash can be broken up into $12,500 in required reserves and $2,500 in excess

reserves. Your equity is defined as total assets minus total liabilities. In this case, $275,000 (Assets) - $250,000 (Liabilities) results in $25,000 in equity. This as a percentage of non-cash assets is 9.6% - well above the requirement of 4%.

b) Your profit is defined as your revenues from interest on loans/securities, minus your interest paid on deposits. In this case, your profits are (.07)*(160,000) + (.03)*(100,000) – (.02)*(50,000) = $11,200+$3,000 - $1,000 = $13,200. Return on Assets expresses this profit as a percentage of total assets, or (13,200/$275,000)*100 = 4.8%. Return on equity expresses profits as a percentage of equity, or (13,200/25,000)*100 = 52.8%.

c) A $10,000 withdrawal would result is a deduction of $10,000 from the cash category of assets and an equal deduction of $10,000 deduction from checking under liabilities (balance sheets must always balance!). Note that this withdrawal reduces our cash position to $5,000 or 2% of deposits. Therefore, we must raise cash to get back above the reserve requirement. This can be done by liquidating some assets (selling securities or recalling loans) or by taking out a short-term loan in the fed funds market.

d) A $30,000 default is a much bigger problem. This deducts $30,000 in loans from the asset side and $30,000 from equity capital on the liability side. Now, our equity is -$5,000 (obviously below the requirement). This could only be solved by bringing in addition capital (i.e., investing more capital in the bank).

5) Given the information on Assets/Liabilities, your balance sheets shoud look like the following (duration of assets/liabilities in parentheses):

Assets	**Liabilities**
Cash: $18,000 (0)	Checking: $100,000 (0)
1 yr. Loans: $90,000 (1)	1yr. CDs: $50,000 (1)
2 yr. Loans: $80,000 (2)	2yr. CDs: $20,000 (2)
	Equity Capital: $18,000

a) A $10,000 withdrawal would result is a deduction of $10,000 from “Cash” and “Checking” with no impact on equity. This would, however, lower cash reserves below the requirement and will need to be rectified) most likely through a fed funds loan or a repo).

b) A $20,000 load default would deduct $20,000 from “2 yr. Loans” and “equity capital”. With equity capital at -$2,000, more would need to invest in the bank for it to stay in business.

c) The duration of liabilities (the weighted average of the duration of each individual liability where the weights are equal to each individual liability’s percentage of total liabilities). In this case (I dropped the zeroes for

simplicity): (18/188)*0 + (90/188)*1 + (80/188)*2 = 1.34. Similarly, the duration of assets is (100/170)*0 + (50/170)*1 + (20/170)*2 = .53. Lastly the duration gap is the difference between the duration of assets and the liability to asset ratio weighted duration of liabilities. In this case, 1.34 – (170/188)*.53 = .86. Duration represents the sensitivity to interest rates. In this example, a 3% rise in the interest rate will cause a 3*(.86) = 2.58% drop is equity as a percentage of assets (in this example, the duration of assets is bigger than the duration of liabilities. Therefore, when interest rates rise, the value of assets falls by more than the value of liabilities, resulting in a decrease in net worth.