Finance 462
Solutions to Problem Set #6

1) The long position in a futures contract is an agreement to purchase the underlying commodity at the futures price. Therefore, the long position only benefits if the price of the underlying commodity rises. In this case, the long position loses because the price of cocoa falls.

Loss = ($1,803 - $1,790)(10) = $130.

2) First, remember that the price of a T-Bill future is expressed an IMM (international monetary market) index. IMM = 1 – (Annualized) discount yield.

a) A one point increase in the IMM index is caused by a 1 point drop in the annualized discount yield.

b) The discount yield for a 90 day T-Bill with $100 Face Value is given by:

$$DY = \left(\frac{\$100 - P}{\$100}\right)\left(\frac{360}{90}\right)*100$$

Solving for price:

$$P = 100 - \frac{DY}{4}$$

For every 1 point drop in the DY, the price increases by $.25 per $100 of face value

c) Multiply $.25 by 10,000 to scale up to $1M worth of face value to get $2,500.

3) The IMM of 93 implies an annualized yield of 7%. This yield is the forward rate between days 180 and 270 – F(180, 90). The 90 day return is 7%/4 = 1.75%.

We can also find the forward rate implied by the yield curve. First, we need the 270 day spot rate – S(270). This is halfway between, 180 days and 1 year, so linear extrapolation will give us 3.5% (annualized).

3.5% *(3/4) = 2.625% (270 day return)
3%*(1/2) = 1.5% (180 day return)

(1.015)(1+ F(180,90)) = (1.0265)

(1+F(180,90)) = 1.0265/1.015 = 1.0113 = 1.13% - the implied yield off the yield curve.

The futures rate is too high! Use the spot market to borrow at 1.13% and the use the futures market to lend (i.e. go long in futures).

4) A treasury note/bond future requires delivery of a 6% coupon bond. However, because there might not be a 6% coupon bond available, the short position has the *cheapest to deliver* option. However, the bond with the lowest coupon rate will always be the cheapest to deliver. Conversion factors try to correct for this.

5) First, calculate the price of the bond:

$$P = \frac{\$400{,}000}{(1.08)} + \frac{\$400{,}000}{(1.08)^2} + \frac{\$5{,}400{,}000}{(1.08)^3} = \$5{,}000{,}000$$

$$P = \$370{,}370 + \$342{,}935 + \$4{,}286{,}694 = \$5{,}000{,}000$$

Now, calculate the duration:

$$MAc.Dur = \left(\frac{\$370{,}370}{\$5M}\right)(1) + \left(\frac{\$342{,}935}{\$5M}\right)(2) + \left(\frac{\$4{,}286{,}694}{\$5M}\right)(3) = 2.78$$

Modified Duration = 2.78/1.08 = 2.57

Dollar Duration = (.0257)($5M) = $128,500

The dollar duration of the TBill future is $2,500. Therefore, you would need

$128,500/$2,500 = 51.4 = 50 contracts.

If the volatility of short term interest rates was lower, you would need to increase the number of contracts.