Finance 462
Solutions to Problem Set #5

1) With the given lattice structure, there are two possible movements for interest rates each period:

- $i_{t+1} = i_t(1.2)$ (With probability of 50%)
- $i_{t+1} = \frac{i_t}{(1.2)}$ (With probability of 50%)

Therefore, we have the following tree:

Time 0	Time 1	Time 2
6%	7.2%	8.64%
	5.0%	6%
		6%
		4.16%

There are 4 possible interest rate paths (up, up; up, down; down, up; down, down)

What is the expected return over two years? Remember, the expected value is the weighted average of all possible events where the weights are the probabilities of each event.

(.5)(1.06)(1.072) + (.5)(1.06)(1.05) = 1.125 (compounded over two years)

Now, convert the above rate into an annual return:

$\sqrt{(1.125)}$ = 1.061 = 6.1%

Similarly, for the three year return…

(.25)(1.06)(1.072)(1.0864) + (.25)(1.06)(1.072)(1.06) + (.25)(1.06)(1.05)(1.06) + (.25)(1.06)(1.05)(1.042) = 1.194 (compounded over three years)

$\sqrt[3]{1.194} = 1.061 = 6.1\%$

2) Given a three year Treasury with a $2,000 face value and a 5% annual coupon, we have the following set of cash flows:

1 year: $100
2 years: $100
3 years: $2,100
The price of the bond is the present value of payments using the yield curve:

$$P = \frac{\$100}{(1.06)} + \frac{\$100}{(1.061)^2} + \frac{\$2{,}100}{(1.061)^3}$$

$$P = \$94.34 + \$88.83 + \$1{,}758.22 = \$1{,}941.39$$

3) First, let's calculate the Macaulay duration. This is equal to the weighted average of each individual payment's Macaulay duration (equal to the term) where the weights are each payment's contribution to the price.

$$\text{Mac. Dur.} = \left(\frac{\$94.34}{\$1{,}941.39}\right)(-1) + \left(\frac{\$88.83}{\$1{,}941.39}\right)(-2) + \left(\frac{\$1{,}758.22}{\$1{,}941.39}\right)(-3) = 2.86$$

To get modified duration, divide by the yield.

MD = 2.86/1.06 = 2.70

4) To calculate the value of the bond, we find the value over each path and then average the paths (note that the bond is never called).

$$P_1 = \frac{\$100}{1.06} + \frac{\$100}{(1.06)(1.072)} + \frac{\$2{,}100}{(1.06)(1.072)(1.0864)} = \$1{,}883.44$$

$$P_2 = \frac{\$100}{1.06} + \frac{\$100}{(1.06)(1.072)} + \frac{\$2{,}100}{(1.06)(1.072)(1.06)} = \$1{,}925.80$$

$$P_3 = \frac{\$100}{1.06} + \frac{\$100}{(1.06)(1.05)} + \frac{\$2{,}100}{(1.06)(1.05)(1.06)} = \$1{,}964.18$$

$$P_4 = \frac{\$100}{1.06} + \frac{\$100}{(1.06)(1.05)} + \frac{\$2{,}100}{(1.06)(1.05)(1.0416)} = \$1{,}995.62$$

Averaging over the paths we get **$1,942.26.**

To get the duration, raise every interest rate by 50 basis points and repeat part (a). Again, note that the bond is never recalled early.

$$P_1 = \frac{\$100}{1.065} + \frac{\$100}{(1.065)(1.077)} + \frac{\$2{,}100}{(1.065)(1.077)(1.0914)} = \$1{,}858.61$$

$$P_2 = \frac{\$100}{1.065} + \frac{\$100}{(1.065)(1.077)} + \frac{\$2{,}100}{(1.065)(1.077)(1.065)} = \$1{,}900.19$$

$$P_3 = \frac{\$100}{1.065} + \frac{\$100}{(1.065)(1.055)} + \frac{\$2{,}100}{(1.065)(1.055)(1.065)} = \$1{,}937.86$$

$$P_4 = \frac{\$100}{1.065} + \frac{\$100}{(1.065)(1.055)} + \frac{\$2{,}100}{(1.065)(1.055)(1.0466)} = \$1{,}968.71$$

Averaging, we get **$1,916.34**

Now, lower every price by 50 basis points and note that along the lower paths, the bond is called in the second period.

$$P_1 = \frac{\$100}{1.055} + \frac{\$100}{(1.055)(1.067)} + \frac{\$2{,}100}{(1.055)(1.067)(1.0814)} = \$1{,}908.72$$

$$P_2 = \frac{\$100}{1.055} + \frac{\$100}{(1.055)(1.067)} + \frac{\$2{,}100}{(1.055)(1.067)(1.055)} = \$1{,}951.90$$

$$P_3 = \frac{\$100}{1.055} + \frac{\$2100}{(1.055)(1.045)} = \$1{,}999.59$$

$$P_4 = \frac{\$100}{1.055} + \frac{\$2100}{(1.055)(1.045)} = \$1{,}999.59$$

The average across paths is **$1,964.95**

To calculate the duration, take the percentage change:

ED = ($1,916.34 - $1,964.95)/ $1,942.26 *100 = 2.5%

Note that the bond's modified duration would be the same as in (3) – 2.7!