Finance 462
Solutions to Problem Set #4

1) Given the Vacicek model, we can calculate the path of interest rates given a starting value and some error terms.

$$\Delta i_t = .2(6 - i_t) + 2\varepsilon_t$$
$$\varepsilon_t \in N(0,1)$$

Suppose we have the following errors {-.1, .4, -.3, -.2}. The current 1 year TBill yield (from the *WSJ*) is (approximately) 2.8%. Therefore,

F(0,1) = 2.8%	**F(1,1) = 3.24%**	**F(2,1) = 4.59%**	**F(3,1) = 4.27%**	**F(4,1) = 4.22%**
.2(6-2.8) = .64	.2(6-3.24) = .55	.2(6-4.59) = .28	.2(6-4.27) = .35	
2*(-.1) = -.2	2*(.4) = .8	2*(-.3) = -.6	2*(-.2) = -.4	
Change = .44	Change = 1.35	Change = -.32	Change = -.05	

With the path for interest rates, we can price the bond:

$$P = \frac{\$60}{(1.028)} + \frac{\$60}{(1.028)(1.0324)} + \frac{\$60}{(1.028)(1.0324)(1.0459)} + \frac{\$60}{(1.028)(1.0324)(1.0459)(1.0427)}$$
$$+ \frac{\$1{,}060}{(1.028)(1.0324)(1.0459)(1.0427)(1.0422)} = \$1099.54$$

The current yield (from the *WSJ*) is

Term	**Yield**
1yr	2.8%
2yr	3.21%
3yr	3.36%
4yr	3.47%
5yr	3.58%

Using the spot rate pricing method:

$$P = \frac{\$60}{(1.028)} + \frac{\$60}{(1.0321)^2} + \frac{\$60}{(1.0336)^3} + \frac{\$60}{(1.0347)^4} + \frac{\$60}{(1.0358)^5} = \$1{,}110.43$$

2) The formula for an N period STRIP is

$$P = \frac{FV}{(1+i)^N}$$

Assuming a face value of $100 (STRIPS are priced per $100 of face value) we get a price of:

$$P = \frac{\$100}{(1.055)^{10}} = \$58.54$$

Recall that the Macaulay duration is equal to the maturity (10 in this case). To get the modified duration, divide by the interest rate.

$$MD = \frac{10}{1.055} = 9.48$$

To get the dollar duration, multiply by the bond's price (and divide by 100)

$$DD = (9.48)(\$58.44) = \$553.93/100 = \$5.54$$

Therefore, a drop in interest rates from 5.5% to 4.5% (a 100 basis point drop) will raise this Bond's price by $5.54 to $64.08.

If we actually revalue the bond at an interest rate of 4.5%, we get

$$P = \frac{\$100}{(1.045)^{10}} = \$64.39$$

The difference between the actual price change and price change predicted by the dollar duration is due to the curvature of the pricing function.

3) First, the value of the bond (at a 5% YTM) is as follows:

$$P = \frac{\$400}{(1.05)} + \frac{\$400}{(1.05)^2} + \frac{\$10{,}400}{(1.05)^3}$$

$$P = \$380.95 + \$362.81 + \$8983.91 = \$9727.67$$

First, let's calculate the Macaulay duration. This is equal to the weighted average of each individual payment's Macaulay duration (equal to the term) where the weights are each payment's contribution to the price.

$$\text{Mac. Dur.} = \left(\frac{\$380.95}{\$9727.67}\right)(-1) + \left(\frac{\$362.81}{\$9727.67}\right)(-2) + \left(\frac{\$8983.91}{\$9727.67}\right)(-3) = 2.87$$

To get modified duration, divide by the yield.

MD = 2.87/1.05 = 2.74

A 50 basis point rise in the interest rate would lower this bond's price by (.5)(2.74) = 1.37 percent ($13.31).
The key durations are just the individual pieces from the above calculation: The key Macaulay durations are:

Mac(1) = -.04
Mac(2) = -.074
Mac(3) = -2.76

Therefore, the key modified durations are:

MD(1) = -.038
MD(2) = -.070
MD(3) = -2.63

Note that the sum of the key durations equals 2.74

4) For asset A, the three possible values are:

P(1) = $100/1.05 = $95.24
P(2) = $100/1.07 = $93.46
P(3) = $100/1.03 = $97.08

Taking the average, we get $95.26

Repeating for asset B

P(1) = $100/1.05 = $95.24
P(2) = $110/1.07 = $102.80
P(3) = $105/1.03 = $101.94

Taking the average, we get $99.99

5) The spread between a 2 year TIP and a 2 year Treasury (i.e. both with expiration in 2007) is 3.23 - .619 = 2.611. This is an approximation of the market's expectation of inflation (remember, there could be a variance term at play as well!).