Finance 462
Solutions to Problem Set #3

1) This question is just repeated use of the equation governing inflation expectations. In this case, we are assuming expectations are *adaptive*. Individuals base their current inflation expectation partly on their past forecast (people are stubborn) and the most recently observed inflation rate. Assume inflation has been zero long enough that nobody expects inflation. Suddenly, we have a 5% inflation rate. Using the formula, we can calculate the path of expectations:

   Inflation Expectations: $\pi_t^e = .5\pi_{t-1} + .5\pi_{t-1}^e$

   Time 1: 0
   Time 2: .5(5%) + .5(0) = 2.5%
   Time 3: .5(5%) + .5(2.5%) = 3.75%
   Time 4: .5(5%) + .5(3.75%) = 4.375%
   Time 5: .5(5%) + .5(4.375%) = 4.68% (you get the idea…it takes a while to learn)

   The nominal interest rate should equal the real rate (assumed constant at 5%) plus the expected inflation rate:

   Nominal Interest Rate = Real Interest Rate + Expected Inflation

   Time 1: 5% + 0% = 5%
   Time 2: 5% + 2.5% = 7.5%
   Time 3: 5% + 3.75% = 8.75%
   Time 4: 5% + 4.375% = 9.375%
   Time 5: 5% + 4.68% = 9.68% (eventually the nominal rate will reach 10%)

   The realized real rate of return is the nominal interest rate minus actual inflation:

   Real Interest Rate = Nominal Interest Rate – Actual Inflation

   Time 1: 5% - 5% = 0%
   Time 2: 7.5% - 5% = 2.5%
   Time 3: 8.75% - 5% = 3.75%
   Time 4: 9.375% -5% = 4.375%
   Time 5: 9.68% - 5% = 4.68%% (eventually the real rate will return to5%)

2) This is just a numerical version of what we talked about in class. If we have a description of savings and investment, then the market real interest rate should adjust to equate the two.

   S = I

   Now, insert the equations…

.2Y + 300r = 4,000 -200r

Solve for the interest rate….

r = (4,000/500) – (.2/500)Y = 8 - .0004Y

Y = 10,000. Therefore, the equilibrium interest rate is equal to 4%. Equilibrium savings = equilibrium Investment = 3200.

3) Our economic model assumes that households maximize utility subject to lifetime budget constraints. The resulting decisions are savings and consumption as functions of income (or, more accurately, wealth) and the interest rate. In this example, utility is given by

$$U(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \beta\left(\frac{c_2^{1-\sigma}}{1-\sigma}\right)$$

Note that is satisfies the properties that we assume for utility:

- If $\tilde{c}_1 \geq c_1, \tilde{c}_2 \geq c_2$, then $U(\tilde{c}_1, \tilde{c}_2) > U(c_1, c_2)$ (Utility is increasing in consumption)
- If $\tilde{c}_1 \succ c_1$, then $U_1(\tilde{c}_1, c_2) \leq U_1(c_1, c_2)$ (Diminishing marginal utility)

Households are constrained by the fact that the present value of lifetime must equal the present value of lifetime income (Wealth). Note that if we combine the two given constraints by solving for savings, we get:

$$c_1 + \frac{c_2}{(1+r)} = y_1 + \frac{y_2}{(1+r)} = W$$

Therefore our maximization problem can be formally written as

Choose c1 and c2 to maximize $\left\{\frac{c_1^{1-\sigma}}{1-\sigma} + \beta\left(\frac{c_2^{1-\sigma}}{1-\sigma}\right)\right\}$

Subject to: $c_1 + \frac{c_2}{(1+r)} = y_1 + \frac{y_2}{(1+r)}$

The Efficiency condition for this problem states that at the margin, you should be indifferent between consuming more now or more later (i.e. to maximize a function, you set the derivative equal to zero. For this problem, we have

$$U_1(c_1, c_2) = (1+r)U_2(c_1, c_2)$$

The left hand side is the marginal utility of current consumption. It represents the loss in utility of a small decrease in current spending. The right hand represents the marginal gain of deferring a little current consumption. For each dollar you give up today, you get (1+r) dollars tomorrow. $U_2(c_1, c_2)$ converts those dollars into utility.

Remember that the marginal utilities are the partial derivatives of utility with respect to current and future consumption respectively. This gives us:

$$c_1^{-\sigma} = \beta(1+r)c_2^{-\sigma}$$

This condition, along with the restriction that the present value of consumption equals the present value of wealth gives us two equations in two unknowns. As a result, we have:

$$c_1 = \left(\frac{1}{1+\beta[\beta(1+r)]^{\frac{1}{1-\sigma}}}\right)\left(y_1 + \frac{y_2}{(1+r)}\right) = \Theta(r)W$$

$$S = y_1 - c_1$$

Consumption is a fraction of wealth. The fraction is a function of the interest rate.

In equilibrium, savings equals zero and, hence, consumption each period equals income. Putting this into the efficiency condition and we get:

$$(1+r) = \left(\frac{1}{\beta}\right)\left(\frac{y_2}{y_1}\right)^{\sigma}$$

Beta will govern the long run level of the interest rate (as was the case in class) while sigma governs the volatility of interest rates due to changes in economic growth.

In the long run, GDP grows at 3%. Assuming sigma equals 1.5 (as is the case empirically) and beta equals .98, the long run real interest rate should equal

$$(1+r) = \left(\frac{1}{.98}\right)(1.03)^{1.5} = 1.066 = 6.6\%$$

4) We have the following yield curve:

Term	**Yield**
1yr	5%
2yr	5.5%
3yr	6%

Given the current term structure, we can calculate the implied forward rates…

F(0,1) = 5%
F(1,1) = (1.055)(1.055)/(1.05) = 1.06= 6%
F(2,1) = (1.06)(1.06)(1.06)/(1.055)(1.055) = 1.07 = 7%

Our structural model yields (pardon the pun!) a relationship between interest rates and (expected) economic growth.

$$(1+r) = \left(\frac{1}{.98}\right)\left(\frac{y_2}{y_1}\right)^{1.5}$$

Solving for economic growth, we get

$$\left((1+r)(.98)\right)^{\frac{1}{1.5}} = \left(\frac{y_2}{y_1}\right)$$

Therefore, future rates of economic growth are expected to be:

1yr: 1.019 (1.9%)
2yr: 1.025 (2.5%)
3yr: 1.032 (3.2%)

5) The answer is that no lender would ever *knowingly* accept a negative real return (they would be paid back with less purchasing power than they originally loaned out. Therefore, the must not *expect* to earn a negative real return. Remember, loans are contracted based on expectations of inflation. If inflation is underestimated, then it is possible for real rates to be negative.