

**Finance 30220**  
**Solutions to Problem Set #2**

1) Consider the following data from the US economy.

Year	Real GDP	Real Capital Stock	Employment
1980	5,830	7,446	90,800
1990	7,646	8,564	109,151

Assume that production can be represented by the following production function:

$$Y = AK^{\frac{1}{4}}L^{\frac{3}{4}}$$

Calculate the average annual rate of productivity growth during the 1980's.

First, we need the growth accounting expression derived from the production function:

$$\% \Delta Y = \varepsilon_A (\% \Delta A) + \varepsilon_K (\% \Delta K) + \varepsilon_L (\% \Delta L)$$

Where

$$\varepsilon_A = \left( \frac{dY}{dA} \right) \frac{A}{Y} = K^{\frac{1}{4}} L^{\frac{3}{4}} \left( \frac{A}{AK^{\frac{1}{4}}L^{\frac{3}{4}}} \right) = 1$$

$$\varepsilon_K = \left( \frac{dY}{dK} \right) \frac{K}{Y} = \left( \frac{1}{4} \right) AK^{-\frac{3}{4}} L^{\frac{3}{4}} \left( \frac{K}{AK^{\frac{1}{4}}L^{\frac{3}{4}}} \right) = \frac{1}{4}$$

$$\varepsilon_L = \left( \frac{dY}{dL} \right) \frac{L}{Y} = \left( \frac{3}{4} \right) K^{\frac{1}{4}} L^{-\frac{1}{4}} \left( \frac{L}{AK^{\frac{1}{4}}L^{\frac{3}{4}}} \right) = \frac{3}{4}$$

Or,

$$\% \Delta Y = (\% \Delta A) + \frac{1}{4} (\% \Delta K) + \frac{3}{4} (\% \Delta L)$$

Now, solve for productivity growth

$$\% \Delta A = (\% \Delta Y) - \frac{1}{4}(\% \Delta K) - \frac{3}{4}(\% \Delta L)$$

Now, calculate average output, labor, and capital growth

$$\% \Delta Y = \left[ \frac{\ln(7,646) - \ln(5,830)}{10} \right] * 100 = 2.71$$

$$\% \Delta K = \left[ \frac{\ln(8,564) - \ln(7,446)}{10} \right] * 100 = 1.40$$

$$\% \Delta L = \left[ \frac{\ln(109,151) - \ln(90,800)}{10} \right] * 100 = 1.84$$

Now, find productivity growth:

$$\% \Delta A = (2.71) - \frac{1}{4}(1.40) - \frac{3}{4}(1.84) = .98\%$$

2) Consider the following economy:

$$Y = AK^{\frac{1}{3}}L^{\frac{2}{3}}$$

Parameter	Value
$\theta$	5%
$\delta$	10%
$g_L$	4%
$g_A$	2%
A	8
$t$	0

Currently, the economy has a labor force of 2,000 and a capital stock equal to 4,000.

a) Calculate this country's current rate of economic growth.

First, convert the production function to per capita terms:

$$\frac{Y}{L} = \frac{AK^{\frac{1}{3}}L^{\frac{2}{3}}}{L} = A\left(\frac{K}{L}\right)^{\frac{1}{3}}$$

Or,

$$y = Ak^{\frac{1}{3}}$$

Now, calculate per capita capital:  $k = \left(\frac{K}{L}\right) = \frac{4,000}{2,000} = 2$

Now, calculate per capita output:

$$y = Ak^{\frac{1}{3}} = 8(2)^{\frac{1}{3}} = 10.1$$

Now, calculate per capita savings:

$$s = \theta(y - t) = .05(10.1 - 0) = .505$$

Now, calculate the new capital stock:

$$k' = \frac{(1 - \delta)k + i}{1 + g_l} = \frac{(1 - .1)2 + .505}{1.04} = 2.21$$

Now, calculate the new level of output per capita:

$$y' = 8(2.21)^{\frac{1}{3}} = 10.41$$

Finally, calculate the growth rate:

$$\% \Delta y = [\ln(10.41) - \ln(10.1)] * 100 = 3.02\%$$

$$\% \Delta Y = 3.02 + \% \Delta L = 3.02 + 4 = 7.02\%$$

- b) Assuming productivity at its current level of 8, calculate the steady state.

The expression for the steady state is given by:

$$k = \left( \frac{\theta A}{\delta + g_L} \right)^{\frac{3}{2}}$$

Plugging in values:

$$k = \left( \frac{\theta A}{\delta + g_L} \right)^{\frac{3}{2}} = \left( \frac{.05 * 8}{.10 + .04} \right)^{\frac{3}{2}} = 4.83$$

$$y = Ak^{\frac{1}{3}} = 8(4.83)^{\frac{1}{3}} = 13.5$$

$$s = i = \theta y = .05 * 13.5 = .68$$

$$c = y - s = 12.82$$

- c) How fast will this country grow annually once its steady state is reached?

This country will grow (in per capita terms at the rate of productivity growth – equal to 2% per year. Total output growth at productivity growth plus labor growth – equal to 6%.

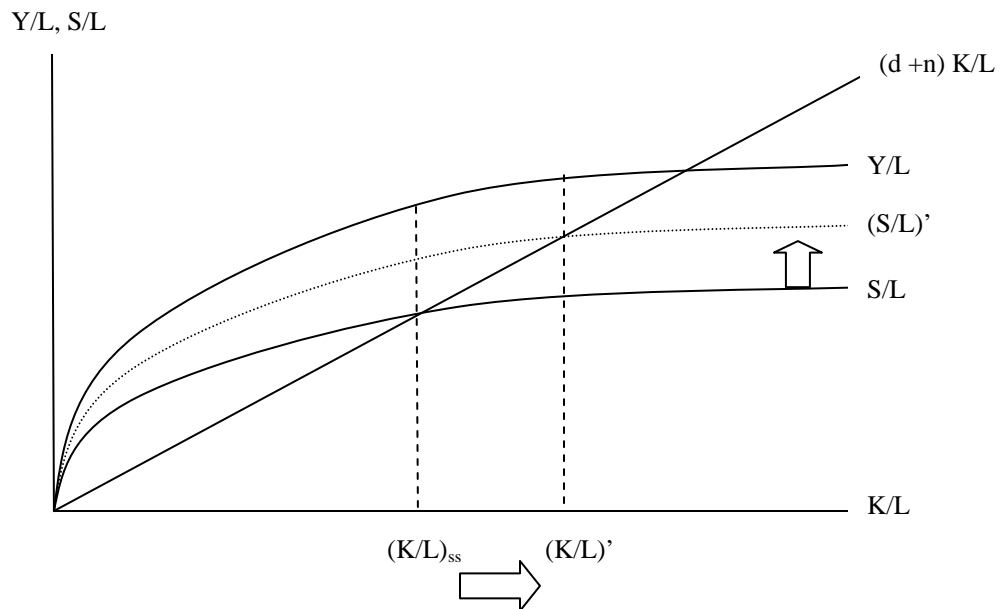
- d) Calculate the steady state level of per capita output that maximizes steady state consumption per capita.

$$k^* = \left( \frac{A}{3(\delta + g_L)} \right)^{\frac{3}{2}} = \left( \frac{8}{3(.10 + .04)} \right)^{\frac{3}{2}} = 83.1$$

$$y = Ak^{\frac{1}{3}} = 8(83.1)^{\frac{1}{3}} = 35$$

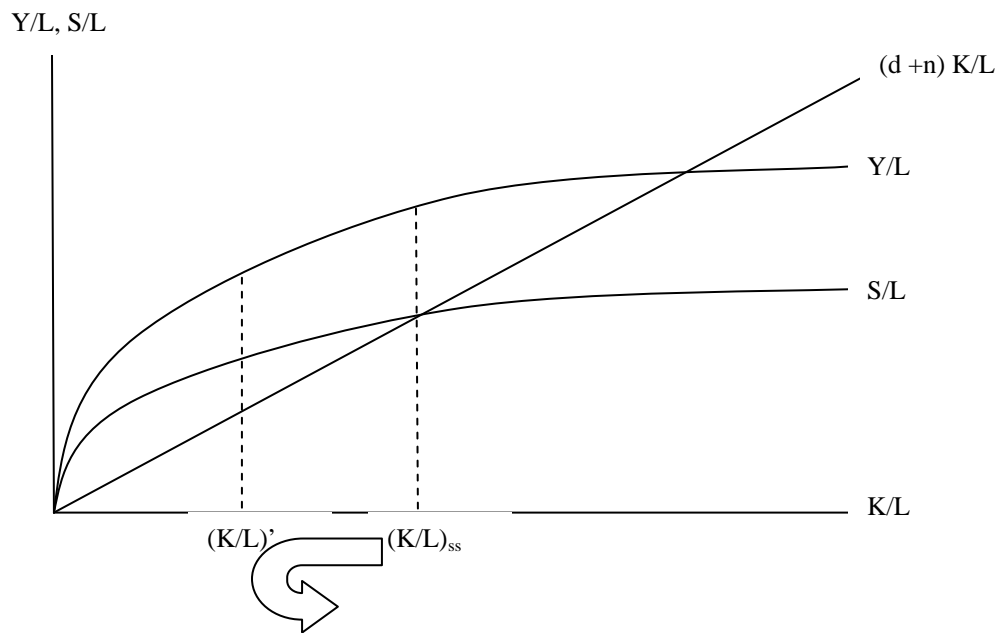
- 3) Suppose that the privatization of the social security system raises the savings rate in the U.S. Explain the long run dynamics of output per capita in the U.S.

The higher rate of savings lowers the interest rate and thus increases investment levels. Higher investment rates can support a larger steady state capital per capital. GDP per capita in the US will grow (i.e. GDP will grow faster than population growth until the new steady state is reached). Then GDP per capita will be constant.



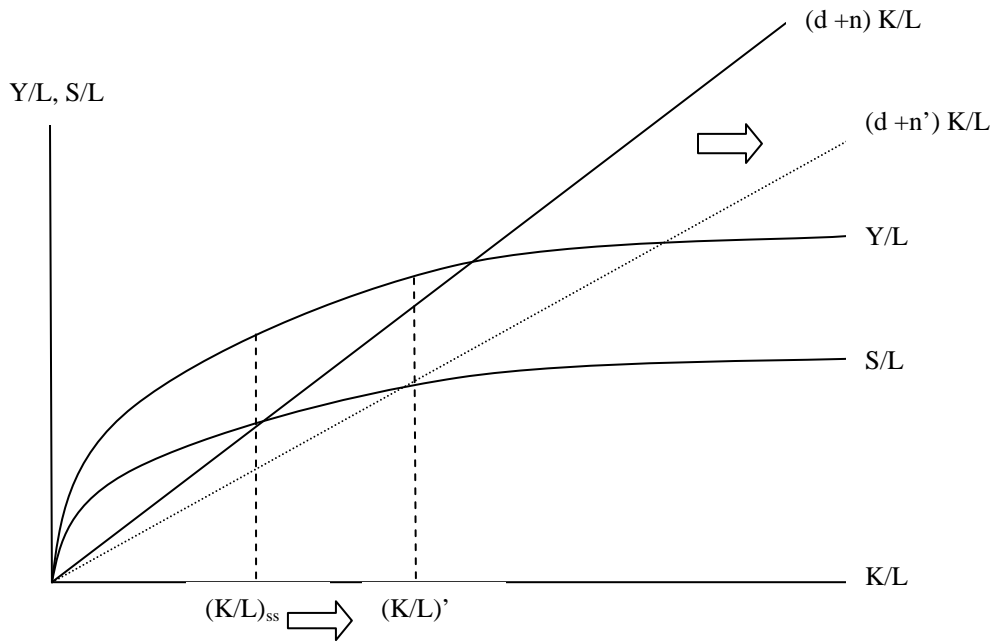
- 4) During World War II, many countries (most notably Germany) lost substantial portions of its capital stock while the U.S. emerged relatively unharmed. Explain the long run impact of the war on Germany's level and growth rate of per capita output.

Assuming that both the US and Germany were initially at their steady states, Germany's loss of capital dramatically lowers their capital to labor ratio. Since Germany is now below its steady state, GDP per capita will fall, but will have to grow to return to its steady state. (GDP grows faster than population growth).



- 5) The data shows that population growth in developed countries is lower than that of developing countries. Explain the impact of a decline in population growth on GDP growth.

A decline in population growth lowers the investment rate required to maintain a higher capital to labor ratio. The steady state increases and, therefore, GDP per capita grows towards the new, higher steady state. Once the new steady state is reached, GDP per capita stays constant.



- 6) The US government is continually growing relative to the overall US economy. Assuming that government spending is completely wasteful, explain the impact on economic growth of an expanding government.

An increase in government requires higher levels of taxes to maintain a balanced budget. With lower levels of disposable income, savings falls which, in turn lowers investment rates. The initial steady state level of capital to labor can't be maintained and so the steady state falls. GDP per capita will shrink (i.e. GDP grows at a slower rate than population) until the new lower steady state is reached.

