University of Notre Dame
Department of Finance
Economics of the Firm
Fall 2009

Project #2: Due in class, Saturday, October 31st.

1) In the class notes, I attempted to describe the production function faced by Notre Dame as well as the managerial decision facing the college. In this question I would like you to attempt the same description for your business.
    a) What is your company/business?
    b) Describe the production function for your company/business. Specifically, what is your output(s)? For your output, do we need to worry about dimensions other than simply quantity (for example, quality)? What are your inputs? Over what time horizons would you consider some inputs fixed?
    c) Would you consider your input as substitutes, compliments, or both? Explain.
    d) Would you say the primary decision for your business is "minimize costs for a fixed production target" or "maximize production given a fixed budget" or both? Explain.
    e) Would you say that your production exhibits increasing marginal returns, decreasing marginal returns, or both?
    f) Does your production function exhibit increasing, decreasing, or constant returns to scale?

2) Consider the following production function:

$$Q = 3L^2K^2 - .1L^3K^3$$

Where L indicates labor input and K indicates capital input.

    a) Assume that the capital input is fixed in the short run at 3. Labor can be adjusted in increments of .1. Plot out your production function.
    b) Calculate the marginal product of labor, average product of labor and elasticity of production with respect to labor.
    c) Over what range does production exhibit increasing marginal returns to labor? Over what range does production exhibit decreasing marginal returns to labor?

d) At what point is average product of labor maximized? What is the elasticity of production at this point?
e) At what point is the marginal product of labor maximized?
f) Does this production function exhibit increasing returns to scale, constant returns to scale, or decreasing returns to scale (i.e. what happens to quantity when all inputs are increased by the same proportion)?

3) Continuing with the same example:

$$Q = 3L^2K^2 - .1L^3K^3$$

Where L indicates labor input and K indicates capital input. Capital is currently fixed at 3 units.

Now, suppose that you face a wage rate of $20 per hour. Capital costs $100 per unit and you can sell your output for $3 per unit.

a) Suppose that your production constraint is 269 units. How much labor will you need? Calculate your profits at this point.
b) Given your answers to (2), calculate marginal costs, average costs, and average variable costs for all values of labor. At what point is average variable cost minimized? At what point is marginal cost minimized?
c) Calculate profits (total revenues – total costs) for every choice of labor. What is the minimal level of production associated with positive profits? What is the maximum level of production associates with positive profits?
d) Find the level of production associated with maximum profits.

4) Consider the following production function:

$$Q = L^{\frac{2}{3}}K^{\frac{1}{3}}$$

Both capital and labor can be adjusted in increments of 1, but capital is fixed at 2 units in the short term. Wages are equal to $10/hr, the price of capital is $20 per unit, and output sells for $30.

a) Calculate total production and the marginal product of labor given a fixed capital stock of 2. Calculate marginal costs.
b) Identify the profit maximizing level of production.
c) Calculate profits to verify your answer to (b).
d) Calculate your production function for all values of capital and labor (this is a bit tricky…try setting up a grid with capital along one row and labor in one column and then calculate the output value for each square in the grid).
e) Now, assume that your production target is still fixed at your answer to (b). We want to find the long term cost minimizing combination of capital and labor that hits this production target. See if you can locate the isoquant (i.e. all capital and labor choices that hit (or at least come close) to your production target.
f) Given your answer to (e), calculate total costs for each point on the isoquant. What is the cost minimizing choice for capital and labor (note: There could be more than one combination that minimizes costs).