

## Proof of the “Monty Hall Problem”:

- 1) The probability that the prize is behind door 1, 2, or 3 is

$$P(1) = \frac{1}{3} \quad P(2) = \frac{1}{3} \quad P(3) = \frac{1}{3}$$

**Suppose that the contestant chooses door number 1:**

- 2) Given that the contestant has chosen door number 1, what’s the probability of the host opening door number 3 conditional on where the prize is located?

$P(III / 1) = \frac{1}{2}$  (The probability of the host opening door III given that the prize is behind door number 1. Since the contestant has chosen door number 1, the host can open either door 2 or door 3 with equal probability)

$$P(III / 2) = 1$$

$$P(III / 3) = 0$$

- 3) Now, what’s the probability of the prize being behind door 2, given that door 3 was opened? This is an application of Bayes Law:

$$P(A_k / B) = \frac{P(B / A_k)P(A_k)}{\sum_i P(B / A_i)P(A_i)}$$

In this case, we are interested in:

$$P(2 / III) = \frac{P(III / 2)P(2)}{P(III / 1)P(1) + P(III / 2)P(2) + P(III / 3)P(3)}$$

$$= \frac{(1)\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + (1)\left(\frac{1}{3}\right) + (0)\left(\frac{1}{3}\right)} = \frac{2}{3}$$

Now, what's the probability that the prize is behind door number 1, given that door number 3 was opened?

$$P(1/III) = \frac{P(III/1)P(1)}{P(III/1)P(1) + P(III/2)P(2) + P(III/3)P(3)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + (1)\left(\frac{1}{3}\right) + (0)\left(\frac{1}{3}\right)} = \frac{1}{3}$$

**Therefore, you are better off switching!**