

**University of Notre Dame
Department of Finance
Economics of the Firm
Fall 2011**

Problem Set #3 Solutions

- 1) Consider the following version of the prisoners dilemma game (Player one's payoffs are in bold):

		Player Two	
		Cooperate	Cheat
Player One	Cooperate	\$10 \$10	\$0 \$12
	Cheat	\$12 \$0	\$5 \$5

- a) What is each player's dominant strategy? Explain the Nash equilibrium of the game. Start with player one:
- If player two chooses cooperate, player one should choose cheat (\$12 versus \$10)
 - If player two chooses cheat, player one should also cheat (\$0 versus \$5).

Therefore, the optimal strategy is to always cheat (for both players) this means that (cheat, cheat) is the only nash equilibrium.

- b) Suppose that this game were played three times in a row. Is it possible for the cooperative equilibrium to occur? Explain.

If this game is played multiple times, then we start at the end (the third playing of the game). At the last stage, this is like a one shot game (there is no future). Therefore, on the last day, the dominant strategy is for both players to cheat. However, if both parties know that the other will cheat on day three, then there is no reason to cooperate on day 2. However, if both cheat on day two, then there is no incentive to cooperate on day one.

- 2) You and your sister have just inherited \$3M that needs to be split between the two of you. You get to make an initial offer, your sister then gets to respond by either accepting your initial offer or making a counter offer. Finally, you can respond by either accepting your sister's offer or making a final offer. Assume that each period, \$1M is removed from the total (each round of negotiation costs \$1M in lawyer's fees). Further, assume that both you and your sister value future payments just as much as current payments (i.e. no discounting future payments). Calculate the Nash equilibrium for this game.

Stage 3: Note that if your sister rejects your offer at this stage, the money disappears. Therefore, your sister will accept anything positive.

You offer: \$1M to you, \$0 to your sister

Stage 2: Now, your sister must make an offer that you will accept (if the game gets to stage three, she gets nothing). If it gets to stage three, you get \$1M.

Your sister offers: \$1M to you, \$1M to her.

Stage 3: Now, you must make an offer that your sister will accept (and is preferable to you – if this is not possible, then you will make an offer she rejects and the game goes to stage 2). Your sister gets \$1M if the game reaches stage two.

You offer: \$2M to you, \$1M to your sister

- 3) Consider a variation of the previous problem: Same rules as in (3), However, this time, you learn something about your sister: You discover that your sister has always hated you. All she cares about with regards to splitting the \$3M is that she gets more than you do (i.e. an allocation of \$500,000 for you and \$1M for her is preferred by her to an allocation of \$1.5M apiece!). Calculate the new Nash equilibrium of the game. (Note: Your incentives are the same as in (4), you want to get as much as possible.) Again, assume that you are making the first offer.

Stage 3: Note that now, your sister's happiness is based on relative earnings (earnings relative to you). You must come up with an offer she will accept or you both get nothing.

You offer: \$500K to you, \$500K to your sister

Stage 2: Now, your sister must make an offer that you will accept (if the game gets to stage three, she gets \$500K). If it gets to stage three, you get \$500K.

Your sister offers: \$500K to you, \$1.5M to her (three to one ratio)

Stage 3: Now, you must make an offer that your sister will accept (and is preferable to you – if this is not possible, then you will make an offer she rejects and the game goes to stage 2). Your sister gets \$1.5M if the game reaches stage two.

You offer: \$750,000 to you, \$2.25M to your sister (three to one ratio)

- 4) Consider the game of chicken. Two players drive their cars down the center of the road directly at each other. Each player chooses SWERVE or STAY. Staying wins you the admiration of your peers (a big payoff) only if the other player swerves. Swerving loses face if the other player stays. However, clearly, the worst output is for both players to stay! Specifically, consider the following payouts. (Player one's payoffs are in bold):

		Player Two	
		Stay	Swerve
Player One	Stay	-6 -6	2 -2
	Swerve	-2 2	1 1

- a) Does either player have a dominant strategy? Explain.

In this case, neither player has a dominant strategy. Suppose player two chooses to stay. Then player one's best response is to swerve (-6 vs. -2). However, if player two swerves, then player one should stay (2 vs. 1).

- b) Suppose that Player B has adopted the strategy of Staying 1/5 of the time and swerving 4/5 of the time. Show that Player A is indifferent between swerving and staying.

We need to show that if player B follows the strategy (stay = 1/5, swerve = 4/5) then player A is indifferent between swerving and staying. If we calculate the expected reward to player A from staying/swerving, we get

$$E(\text{stay}) = (1/5)(-6) + (4/5)(2) = 2/5$$

$$E(\text{swerve}) = (1/5)(-2) + (4/5)(1) = 2/5$$

They are in fact equal.

- c) If both player A and Player B use this probability mix, what is the chance that they crash?

Both players are staying 1/5 of the time. Therefore, the probability that the crash (stay, stay) is $(1/5)(1/5) = 1/25 = 4\%$.

- 5) Suppose that the (inverse) market demand for fax paper is given by

$$P = 400 - 2Q$$

Where Q is total industry output. There are two firms that produce fax paper. Each firm has a constant marginal cost of production equal to \$40 and they are competing in quantities. That is, they each choose production levels simultaneously.

- a) Suppose that Firm 2 chooses a production level equal to 40. Complete the following chart:

Firm 1 Output	Firm 2 Output	Total Output	Price	Total Revenue	Marginal Revenue
1	40	41	318	318	318
2	40	42	316	632	314
3	40	43	314	942	310
4	40	44	312	1248	306

- b) Calculate firm 1's best response to firm 2's choice of output equal to 40 (this will be easier in excel).

Continuing the chart above and looking for where $MR = MC$

Firm 1 Output	Firm 2 Output	Total Output	Price	Total Revenue	Marginal Revenue
68	40	108	184	12512	50
69	40	109	182	12558	46
70	40	110	180	12600	42
71	40	111	178	12638	38

- c) How would your answer to (b) change if firm 2's production increased to 80? Explain.

As firm 2 expands production, it decreases demand for firm 1 (with higher production by firm 2, the market price will be lower for any choice by firm 1). Firm 1's best choice is to reduce production. We can calculate the change by firm 1 by repeating the process above:

Firm 1 Output	Firm 2 Output	Total Output	Price	Total Revenue	Marginal Revenue
48	80	128	144	6912	50
48	80	129	142	6958	46
50	80	130	140	7000	42
51	80	131	138	7038	38

- d) Show that each firm setting a production level equal to 60 is a Nash equilibrium (i.e. neither firm has an incentive to change).

As with part (c), if firm 2 selects 60,

Firm 1 Output	Firm 2 Output	Total Output	Price	Total Revenue	Marginal Revenue
58	60	118	164	9512	50
59	60	119	162	9558	46
60	60	120	160	9600	42
61	60	121	158	9638	38

Firm 1's best response is 60.

- 6) Bertrand competition is a very severe form of competition. In fact, with competition in prices, it only takes two firms in the marketplace to drive price down to marginal cost and profits to zero.
- a) How do capacity constraints influence the equilibrium in Bertrand competition?
 - Restrict capacity: by restricting capacity, they create excess demand for their products. This will drive the price up (see the example in the notes about the movie theatre).
 - b) How does product variety influence the equilibrium in Bertrand competition?
 - Variety: By offering different varieties, they can create market niches given consumer preferences. See the spatial competition section of the notes for a complete answer.