University of Notre Dame
Department of Finance
Economics of the Firm
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Problem Set #2 Solutions

1) Suppose the demand for Bananas is given by

$$P = 10 - .2Q$$

The marginal cost of producing bananas is equal to $2.

a) Show how to set up the problem (i.e. calculate total revenues and marginal revenues, profits, etc).

Quantity	Price	Total Revenue	Marginal Revenue
0	10	0	
1	9.80	9.80	9.80
2	9.60	19.20	9.40

To get price, plug the quantity into the inverse demand curve:

$$P = 10 - .2(0) = 10$$
$$P = 10 - .2(1) = 9.80$$
$$P = 10 - .2(2) = 9.60$$

(Note that each new sale requires a $.20 price drop)

Total Revenue = Price*Quantity

Marginal Revenue is the change in total Revenue

b) Calculate the price and quantity a monopoly would produce

Continuing the chart in excel, we are looking for where marginal revenue equals marginal cost. This happens at a quantity of 20 and a price of $6.

Quantity	Price	Total Revenue	Marginal Revenue
19	6.20	117.8	2.6
20	**6.00**	**120**	**2.2**
21	5.80	121.8	1.8

c) The elasticity of demand at a quantity of 20 (Price equals 6.00).

As quantity increases from 20 to 21…

$$\%\Delta Q = \left(\frac{21-20}{20}\right)*100 = 5\%$$

Price drops from 6 to 5.80…

$$\%\Delta P = \left(\frac{5.80-6}{6}\right)*100 = -3.33\%$$

Elasticity is the ratio of the two…

$$\varepsilon = \frac{\%\Delta Q}{\%\Delta P} = \frac{5}{-3.33} = -1.50$$

Here our markup is

$$\left(\frac{P-MC}{P}\right) = \frac{6-2}{6} = .67 = \frac{1}{1.50}$$

d) Calculate the difference in consumer surplus between a perfectly competitive outcome and a monopolistic outcome

A perfectly competitive firm sets price equal to marginal cost ($2). At this price, we have sales equal to

$$Q = 50 – 5(2) = 40$$

And Consumer surplus = (1/2) (40) (10 – 2) = $160

A monopolist produces Q = 20 and charges a price equal to $6.

Consumer surplus equals (1/2)(20)(10-6) = $40.

Note that some of that goes to profits which equal ($6-$2)(20) = $80 while the remaining $40 is lost.

2) Suppose that you are the manager of an opera house. You have a constant marginal cost of production equal to $50 (i.e. each additional person in the theatre raises your costs by $50 – we will ignore any fixed costs for now.) You have estimated your demand curve for tickets as follows:

$$Q = 150 - P$$

a) Calculate your profit maximizing ticket price.

Here, we simply take the demand curve as given, and solve for price:

$$P = 150 - Q$$

Quantity	Price	Total Revenue	Marginal Revenue
0	150	0	
1	149	149	149
2	148	296	147

As with (1), continuing until MR = MC

Quantity	Price	Total Revenue	Marginal Revenue
49	101	4949	53
50	**100**	**5000**	**51**
51	99	5049	49

Profits = $100(50) - $50(50) = $2500

Now, suppose that you re-estimated your demand curve, but this time, you included different demand curves based on gender:

$$Q = \begin{cases} 175 - P, \text{ if consumer is female} \\ 125 - P, \text{ if consumer is male} \end{cases}$$

b) Now, calculate the prices you would charge if you could distinguish between male and female consumers (i.e. ticket purchasers show up to the box office to buy tickets.) and charge different prices. Why might you be concerned about secondary markets forming for your product?

In This problem, just treat the individual demand curves separately, and solve for the price and quantity separately (follow the same procedure as in (a)).

For Females:

$$Q = 63$$
$$P = 112$$

For Males:

$$Q = 38$$
$$P = 87$$

Profits = \$112 (63) + \$87(38) - \$50(101) = \$5312

3) Continuing with the same example, suppose again, that you are faced with the same demand curve(s)

$$Q = \begin{cases} 175 - P, & \text{(women)} \\ 125 - P, & \text{(men)} \end{cases}$$

a) Suppose that you charged the same price to each consumer (the price calculated in part (b) above. Calculate the consumer surplus for both consumer types.

Given a \$100 price, females buy 75 tickets and generate a consumer surplus of (1/2)(175-100)(75) = \$2,812.50 while men buy 25 tickets and generate a CS of (1/2)(125-100)(25) = \$312.50.

b) Suppose that you were to set a price equal to your marginal cost. Calculate the consumer surplus derived by both consumers.

Given a \$50 price, females buy 125 tickets and generate a consumer surplus of (1/2)(175-50)(125) = \$7812.50 while men buy 75 tickets and generate a CS of (1/2)(125-50)(75) = \$2812.50.

c) If you could distinguish between the types of consumers, how would you set up your prices to maximize profits? (i.e. you could start up an “opera lover’s society” and charge a membership fee)

The ideal situation would be to set up a membership fee for the opera society. Women would pay a membership equal to $7812.50 which would allow them to buy opera tickets for $50 apiece while men would pay a membership price of $2812.50 and would be able to buy tickets at $50. To make sure everybody joins, set a ridiculously high price for non-member ticket prices (say, $1,000 per ticket.)

d) How would your answer to (b) change if you could not distinguish between customer types? (i.e. you could sell different ticket packages.

If we sell a package of 75 tickets for $2,812.50 + 75($50) = $6562.50, then men would buy the package, and get no consumer surplus.

Suppose that a woman bought the 75 ticket package:

We can figure up the total willingness of a women to pay for 75 tickets (using the female demand curve), we get $10,312.50. Subtract off the $6562.50 cost of the 75 ticket package and we find that a women gets a CS of $3750 buying the 75 ticket package. We need to make sure she gets at least a CS of $3750 with the 125 ticket package.

Total Willingness to pay for 125 tickets: $14,062.50:

Minus required surplus: $3750

125 Ticket Package: $10,312.50

4) Suppose that you are George Lucas. You are in the process of packaging the final trilogy (actually the three prequels) of Star Wars for sale to the public. Your marginal costs of production are $2 per movie. Further, you know that there are two types of consumers that you face: children under the age of 10 and everybody else.

Children under 10: Love Jar Jar Binks

Everybody else: Would like to see Jar Jar crushed by a very large truck

Consequently, willingness to pay for each of the three movies is based on how many minutes Jar Jar is on the screen.

Movie	Under 10yrs old	Over 10 yrs Old
Episode 1	$60	$5
Episode 2	$30	$40
Episode 3	$10	$50

a) If you sold these three movies separately, what would your prices be?

Take the first movie:

Movie	Sales	Total Revenues
P=$5	2	$10
P=$60	1	$60

We should charge a price of $60 to maximize revenues. Using similar logic, we should charge a price of $30 for the second and $50 for the third. Calculate profits:

Profits = ($60)(1) + ($30)(2) + ($50)(1) - $2(4) = $162

b) If you only sold these movies as a box set, what should you charge?

As a box set, we can add up each consumers' willingness to pay over the three movies:

Movie	Under 10	Over 10
Box Set	$100	$95

Setting a box set price equal to $95 get us two sales (6 total movies sold)

Profits = 2($95) - $2(6) = $178.

5) Suppose that you have two manufacturers: one company specializes in the production of left shoes (They have a store called "The Left Shoe Emporium"). Another company specializes in right shoes ("Right Shoes 'R' Us"). Consumers have a demand for shoes given by:

$$Q = 150 - P$$

Where P is the price of a pair of shoes: $P = P_L + P_R$. For simplicity, assume that marginal costs for each firm constant and equal to zero.

a) Suppose that "Right Shoes are US" choose to set a 30 price for a right shoe. Write down the inverse demand curve faced by "The Left Shoe Emporium"

We can plug in a right shoe price of $30.

$$Q = 150 - P = 150 - (P_l + P_r) = 150 - (30 + P_l) = 120 - P_l$$

b) Set up the maximization faced by "The Left Shoe Emporium" (i.e. show how total revenue and marginal revenue are calculated).

Quantity	Price	Total Revenue	Marginal Revenue
0	120	0	
1	119	119	119
2	118	236	117

Note: to get the price associated with each quantity, we have

$$P_l = 120 - Q$$

c) Calculate the "Left Shoe Emporiums" profit maximizing price.

Once again, looking for where MR = MC

Quantity	Price	Total Revenue	Marginal Revenue
59	61	3599	
60	**60**	**3600**	**1**
61	59	3599	-1

d) Given your answer to (c), calculate "Right Shoes 'R' Us' profit maximizing price.

We can plug in a left shoe price of $60 from part c.

$$Q = 150 - P = 150 - (P_l + P_r) = 150 - (60 + P_l) = 90 - P_l$$

Now, we can set up the maximization…

Quantity	Price	Total Revenue	Marginal Revenue
0	90	0	
1	89	89	89
2	88	176	87

Note: to get the price associated with each quantity, we have

$$P_l = 90 - Q$$

Once again, looking for where MR = MC

Quantity	Price	Total Revenue	Marginal Revenue
44	46	2024	
45	**45**	**2025**	**1**
46	44	2024	-1

We get a price of $45.

e) Show that if $50 per shoe is an optimal price for both companies (i.e. if one company charges $50, the other company's best choice of price is $50.)

We can plug in a left shoe price of $50

$$Q = 150 - P = 150 - (P_l + P_r) = 150 - (50 + P_l) = 100 - P_l$$

Now, we can set up the maximization…

Quantity	Price	Total Revenue	Marginal Revenue
0	100	0	
1	99	99	99
2	98	196	97

Note: to get the price associated with each quantity, we have

$$P_l = 100 - Q$$

Once again, looking for where MR = MC

Quantity	Price	Total Revenue	Marginal Revenue
49	51	2499	
50	**50**	**2500**	**1**
51	49	2499	-1

We get a price of $50. Therefore, if one firm charges $50, the other firm should also charge $50. The cost of a pair of shoes is $100.

f) Now, suppose that these two companies merged. What would happen to the price of a pair of shoes?

Now, we have a demand curve for a pair of shoes given by

$$Q = 150 - P$$

Repeating the same process…

Quantity	Price	Total Revenue	Marginal Revenue
74	76	5624	
75	**75**	**5625**	**1**
76	74	5624	-1

Now a pair of shoes costs $75.