Parity Nonconservation in Cesium: Is the Standard Model in Trouble?

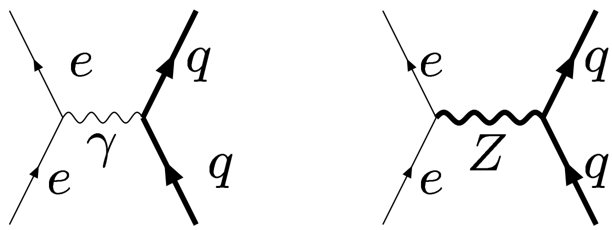
Walter Johnson
Department of Physics
Notre Dame University
`http://www.nd.edu/~johnson`

May 10, 2001

Abstract

This is a brief review of the current status of PNC in cesium including a discussion of the reported $2.3\,\sigma$ disagreement between experiment and the Standard Model

Overview



$$H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}}\, Q_W\, \gamma_5\; \rho(r)$$

where the *conserved* weak charge is

$$Q_W = -N + Z\left(1 - 4\sin^2\theta_W\right)$$

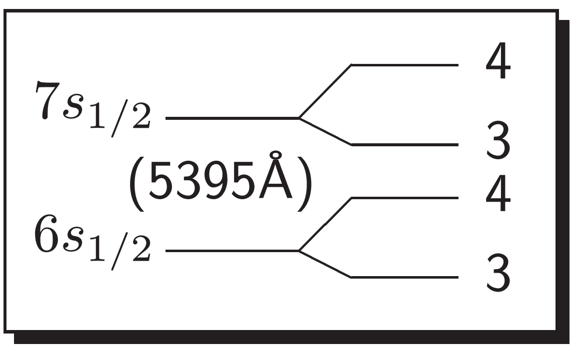
For states v and w that have the same parity
$\langle w|ez|v\rangle = Q_W \times$ "Structure factor"

Measure: $E_{\text{PNC}} = \langle w|ez|v\rangle$
Calculate: "Structure factor"
Ratio gives Q_W

Some Properties of Cesium

- Configuration: [Xe] 6s 55 electrons
- A=133 (100%) N=78 Z=55
- I=7/2 $g_{7/2}$ valence proton
- $\mu = 2.5826\ \mu_N$
- $Q = -0.0037$ b

Levels of Interest



$\nu_{43}(6s) = 9,192,631,770$ Hz defines the second!

Z-e Coupling from Standard Model

$$\mathcal{H}^{(1)} = \frac{G}{\sqrt{2}}\left(\bar{\psi}_e\gamma_\mu\gamma_5\psi_e\right)\sum_i\left[c_{1p}\left(\bar{\psi}_{pi}\gamma^\mu\psi_{pi}\right) + c_{1n}\left(\bar{\psi}_{ni}\gamma^\mu\psi_{ni}\right)\right]$$

$$\mathcal{H}^{(2)} = \frac{G}{\sqrt{2}}\left(\bar{\psi}_e\gamma_\mu\psi_e\right)\sum_i\left[c_{2p}\left(\bar{\psi}_{pi}\gamma^\mu\gamma_5\psi_{pi}\right) + c_{2n}\left(\bar{\psi}_{ni}\gamma^\mu\gamma_5\psi_{ni}\right)\right]$$

where the standard-model coupling constants are

$$\begin{aligned} c_{1p} &= \tfrac{1}{2}\left(1 - 4\sin^2\theta_W\right) \approx 0.038, \\ c_{1n} &= -\tfrac{1}{2}, \\ c_{2p} &= \tfrac{1}{2}\,g_A\left(1 - 4\sin^2\theta_W\right) \approx 0.047, \\ c_{2n} &= -\tfrac{1}{2}\,g_A\left(1 - 4\sin^2\theta_W\right) \approx -0.047. \end{aligned}$$

In the above, $g_A \approx 1.25$ is a scale factor for the partially conserved axial current A_N.

Nonrelativistic Nucleons

$\mathcal{H}^{(1)}$:

$$\left(\bar{\psi}_p \gamma^\mu \psi_p\right) \rightarrow \phi_p^\dagger \phi_p \, \delta_{\mu 0} \qquad \left(\bar{\psi}_n \gamma^\mu \psi_n\right) \rightarrow \phi_n^\dagger \phi_n \, \delta_{\mu 0}$$

where ϕ_p and ϕ_n are nonrelativistic field operators. From this we extract an "effective" Hamiltonian to be used in the electron sector; namely,

$$H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 \left[2\, Z\, c_{1p}\, \rho_p(r) + 2\, N\, c_{1n}\, \rho_n(r) \right].$$

Here, $\rho_p(r)$ and $\rho_n(r)$ proton and neutron density functions normalized to 1. Assuming $\rho_p(r) = \rho_n(r) = \rho(r)$, we may rewrite the effective Hamiltonian as

$$\boxed{H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 \, Q_W \, \rho(r)}$$

where

$$Q_W = \left[2\, Z\, c_{1p} + 2\, N\, c_{1n} \right] = -N + Z \left(1 - 4 \sin^2 \theta_W\right)$$

Axial Nuclear Current

$\mathcal{H}^{(2)}$:

$$\left(\bar{\psi}_p \gamma^\mu \gamma_5 \psi_p\right) \rightarrow \phi_p^\dagger \sigma_i \phi_p \, \delta_{\mu i} \quad \left(\bar{\psi}_n \gamma^\mu \gamma_5 \psi_n\right) \rightarrow \phi_n^\dagger \sigma_i \phi_n \, \delta_{\mu i}$$

The corresponding effective Hamiltonian in the electron sector is obtained from

$$H_{\text{eff}}^{(2)} = -\frac{G}{\sqrt{2}} \alpha \cdot \left[c_{2p} \left\langle \phi_p^\dagger \, \sigma \phi_p \right\rangle + c_{2n} \left\langle \phi_n^\dagger \, \sigma \phi_n \right\rangle \right]$$

Only unpaired valence nucleons (with polarization corrections) contribute, so the size of $H^{(2)}$ is smaller than that from $H^{(1)}$ by a factor of $\approx 1/A$.

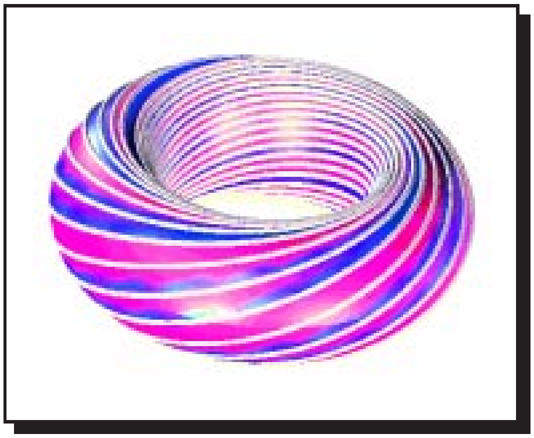
For a single valence proton, this reduces to:

$$\boxed{H_{\text{eff}}^{(2)} = -\frac{G}{\sqrt{2}} c_{2p} \frac{\kappa - 1/2}{I(I+1)} \alpha \cdot \mathbf{I} \, \rho_p(r)}$$

where $\kappa = \mp(I + 1/2)$ for $I = L \pm 1/2$.

Anapole Moment

PNC in nucleus ⇒ nuclear anapole:



The anapole is a toroidal electromagnetic current localized to the nucleus.

$$H^{(a)}_{\rm eff} = \frac{G}{\sqrt{2}} K_a \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I} \, \rho_p(r)$$

Combining the two spin-dependent interactions:

$$H^{(a)}_{\rm eff} + H^{(2)}_{\rm eff} = \frac{G}{\sqrt{2}} K \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I} \, \rho_p(r)$$

with

$$K = K_a - \frac{\kappa - 1/2}{\kappa} c_{2p}$$

Another Spin-Dependent Term

The action of $\left[H_{\text{hyperfine}} \times H_{\text{eff}}^{(1)}\right]$ gives yet another nuclear spin-dependent correction

$$H_{\text{eff}}^{(Q_W)} = \frac{G}{\sqrt{2}} K_{Q_W} \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I} \, \rho_p(r)$$

with[1]

$$K_{Q_W}(^{133}\text{Cs}) \approx 0.0307$$

[1] C. Bouchiat and C. A. Piketty, Z. Phys. C 49, 91 (1991); Phys. Lett. B 269, 195 (1991).

Atomic Structure

For the $6s \to 7s$ transition in atomic cesium:

$$\langle 7s|ez|6s\rangle = \sum_n \left\{ \frac{\langle 7s|ez|np_{1/2}\rangle\langle np_{1/2}|H^{(1)}|6s\rangle}{E_{np} - E_{6s}} + \frac{\langle 7s|H^{(1)}|np_{1/2}\rangle\langle np_{1/2}|ez|6s\rangle}{E_{np} - E_{7s}} \right\}$$

1. $\sum_{n=6}^{9}$ with SD wave functions & energies (90%)
2. $\sum_{n=10}^{\infty}$ "weak RPA" level (10%)
3. Breit interaction at "weak HF" level (0.2%)
4. Nucleon structure correction $\rho_N(r)$ (<0.1%)
5. $H^{(2')}$ contribution at "weak RPA" level

Singles-Doubles Equations

$$\Psi_v = \Psi_{\mathrm{DHF}} + \delta\Psi$$

$$\delta\Psi = \left\{ \sum_{am} \rho_{ma}\, a_m^\dagger a_a + \frac{1}{2} \sum_{abmn} \rho_{mnab}\, a_m^\dagger a_n^\dagger a_b a_a + \sum_{m\neq v} \rho_{mv}\, a_m^\dagger a_v + \sum_{bmn} \rho_{mnvb}\, a_m^\dagger a_n^\dagger a_b a_v \right\} \Psi_{\mathrm{DHF}}$$

$$E_C = E_C^{\mathrm{DHF}} + \delta E_C$$

$$E_v = E_v^{\mathrm{DHF}} + \delta E_v$$

(we also include limited triples)

Core Excitation Equations

$$(\epsilon_a - \epsilon_m)\rho_{ma} = \sum_{bn} \tilde{v}_{mban}\rho_{nb}$$

$$+ \sum_{bnr} v_{mbnr}\tilde{\rho}_{nrab} - \sum_{bcn} v_{bcan}\tilde{\rho}_{mnbc}$$

$$(\epsilon_a + \epsilon_b - \epsilon_m - \epsilon_n)\rho_{mnab} = v_{mnab}$$

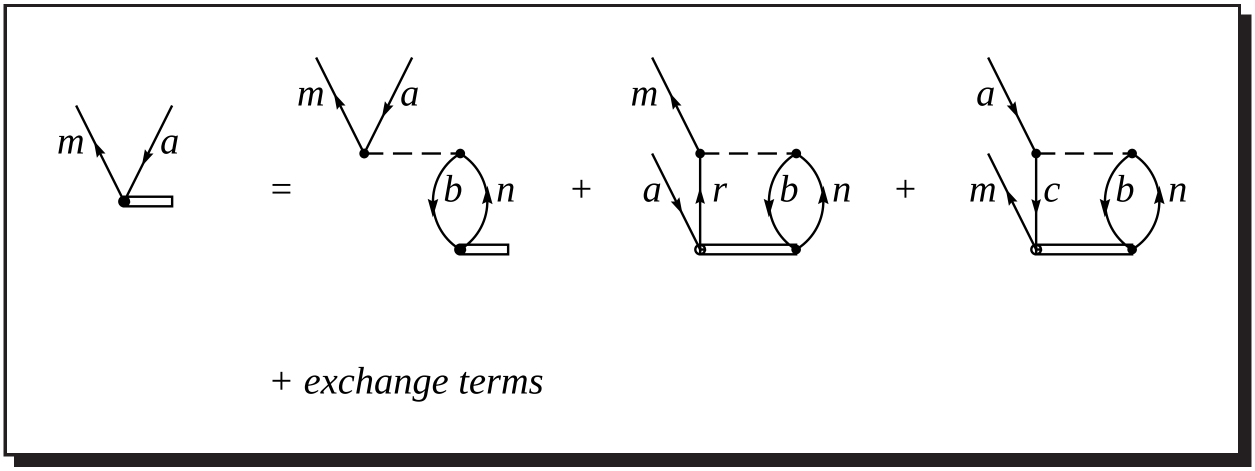
$$+ \sum_{cd} v_{cdab}\rho_{mncd} + \sum_{rs} v_{mnrs}\rho_{rsab}$$

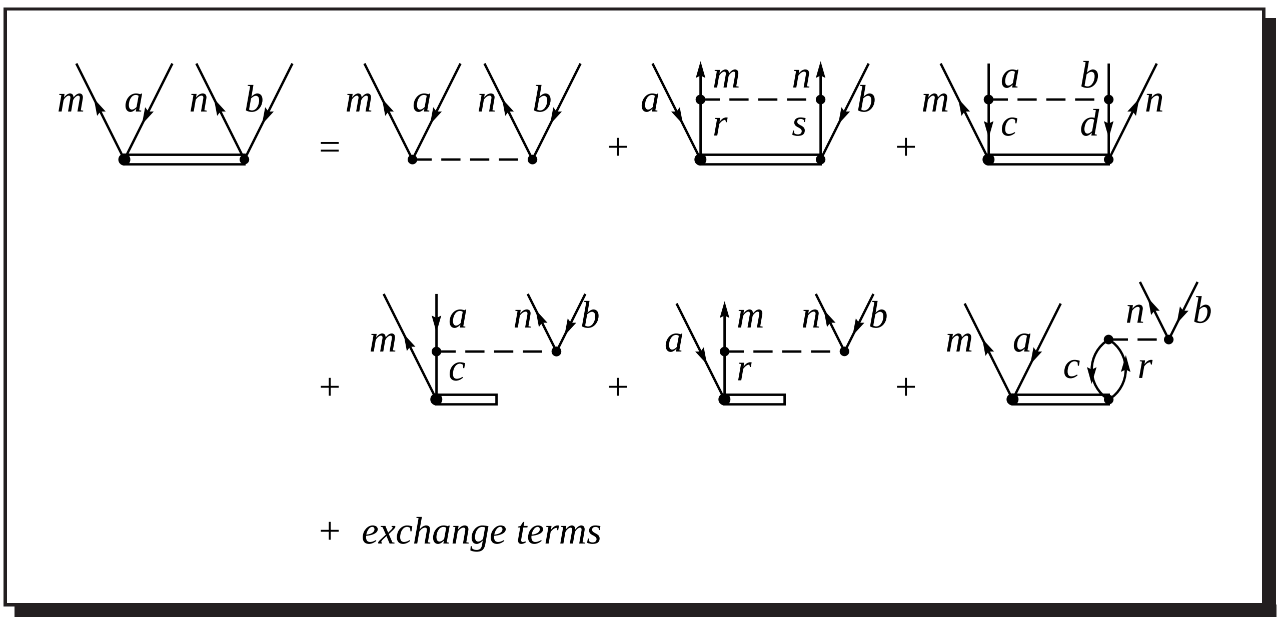
$$+ \left[\sum_{r} v_{mnrb}\rho_{ra} - \sum_{c} v_{cnab}\rho_{mc} + \sum_{rc} \tilde{v}_{cnrb}\tilde{\rho}_{mrac}\right]$$

$$+ \left[\ a \leftrightarrow b \quad m \leftrightarrow n\ \right]$$

$$\delta E_C = \frac{1}{2}\sum_{abmn} v_{abmn}\tilde{\rho}_{mnab}$$

$\approx$ 15,000,000 ρ_{mnab} coefficients for Cs ($\ell = 6$).





Brueckner-Goldstone Diagrams for the core SD equations.

Valence Equations

$$(\epsilon_v - \epsilon_m + \delta E_v)\rho_{mv} = \sum_{bn} \tilde{v}_{mbvn}\rho_{nb}$$

$$+ \sum_{bnr} v_{mbnr}\tilde{\rho}_{nrvb} - \sum_{bcn} v_{bcvn}\tilde{\rho}_{mnbc}$$

$$(\epsilon_v + \epsilon_b - \epsilon_m - \epsilon_n + \delta E_v)\rho_{mnvb} = v_{mnvb}$$

$$+ \sum_{cd} v_{cdvb}\rho_{mncd} + \sum_{rs} v_{mnrs}\rho_{rsvb}$$

$$+ \left[\sum_{r} v_{mnrb}\rho_{rv} - \sum_{c} v_{cnvb}\rho_{mc} + \sum_{rc} \tilde{v}_{cnrb}\tilde{\rho}_{mrvc}\right]$$

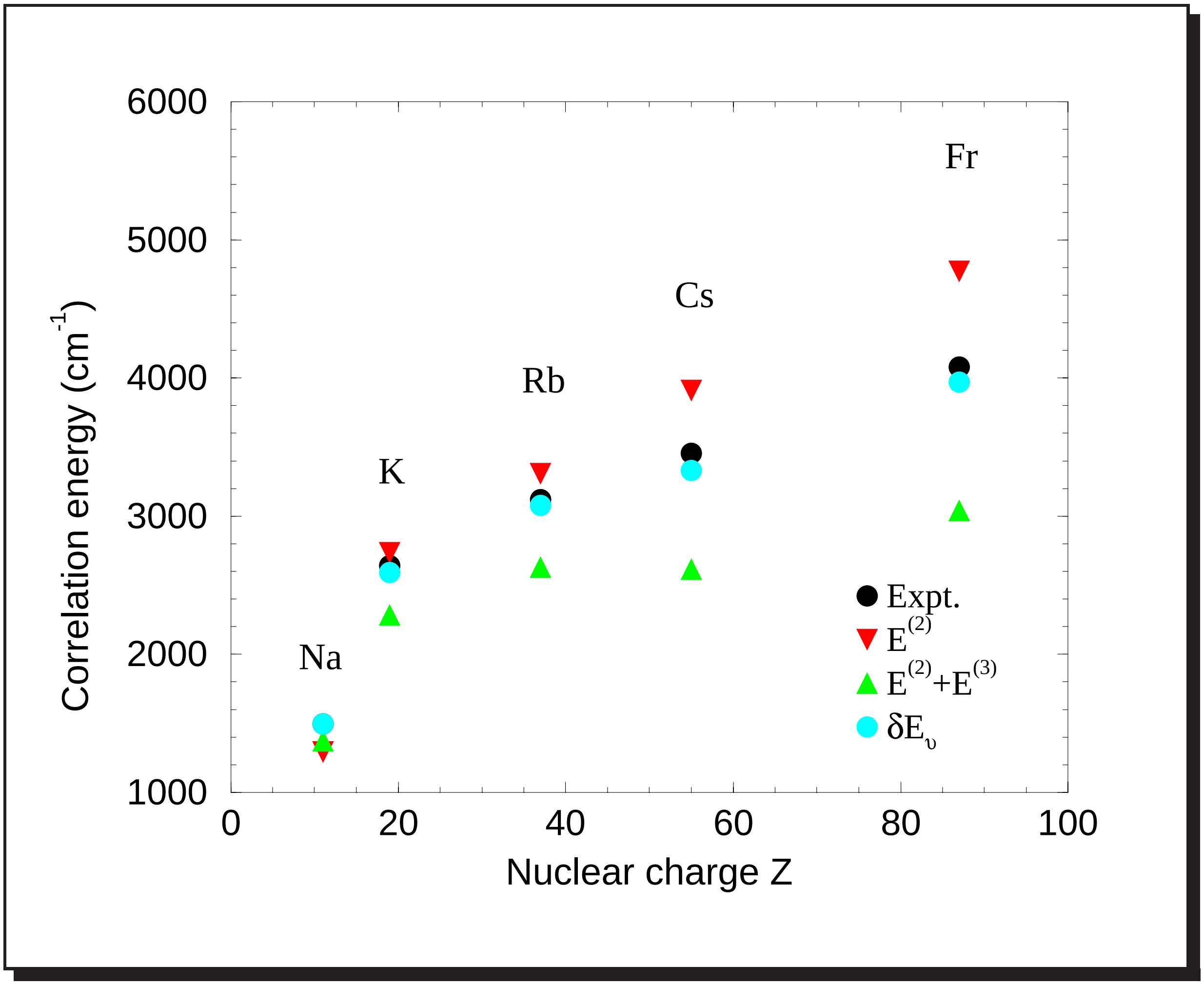
$$+ \left[\ v \leftrightarrow b \quad m \leftrightarrow n\ \right]$$

$$\delta E_v = \sum_{ma} \tilde{v}_{vavm}\rho_{ma} + \sum_{mab} v_{abvm}\tilde{\rho}_{mvab}$$

$$+ \sum_{mna} v_{vbmn}\tilde{\rho}_{mnvb}$$

$\approx$ 1,000,000 ρ_{mnvb} coefficients for each state (Cs)

SD Correlation Energy



Ground-state correlation energies for alkali-metal atoms

Calculations of cesium $6s \rightarrow 7s$ PNC Amplitude

Group	E_{PNC}	Breit
Novosibirsk	0.908 $\pm$ 0.010	-
Notre Dame	0.905 $\pm$ 0.010	HF-level

units: $i\, e\, a_0 \times 10^{-11} \, \frac{Q_W}{-N}$

Status of PNC Experiments

(a) Optical rotation: $n_+ \neq n_-$ $\quad \phi = E_{\rm PNC}/M_1$

$6p_{1/2} - 6p_{3/2}$ transition

Element	Group	$10^8 \times \phi$
Thallium	Oxford	-15.7(5)
Thallium	Seattle	-14.7(2)
Lead	Oxford	-9.8(1)
Lead	Seattle	-9.9(1)
Bismuth	Oxford	-10.1(20)

(b) Stark interference: Add $E(t) = A \cos \omega t$ and detect the hetrodyne signal R $= E_{\rm PNC}/\beta$

$6s_{1/2} - 7s_{1/2}$ (mV/cm)

Element	Group	R_{4-3}	R_{3-4}
Cesium	Paris (1984)	-1.5(2)	-1.5(2)
Cesium	Boulder (1988)	-1.64(5)	-1.51(5)
Cesium	Boulder (1997)	-1.635(8)	-1.558(8)

Bennett & Wieman[2]

- Measured $\beta = 27.024\,(43)_{\rm expt}(67)_{\rm theor}a_0^3$
- Updated theory error estimates!

Expt	Tests	Diff $\times 10^3$		$\sigma_{\rm expt}$
		Novo	ND	
Stark($6s$-$7s$)	$\langle 7p\|ez\|ns\rangle$	-3.4	-0.7	1.0
$\tau_{6p_{1/2}}$	$\langle 6s\|ez\|6p\rangle$	-4.4	4.3	1.0
$\tau_{6p_{3/2}}$	$\langle 6s\|ez\|6p\rangle$	-2.6	7.9	2.3
α	$\langle vs\|ez\|np\rangle$	-	-1.4	3.2
β	$\langle vs\|ez\|np\rangle$	-	-0.8	3.0
A_{6s}	$\Psi_{6s}(0)$	1.8	-3.1	-
A_{7s}	$\Psi_{7s}(0)$	-6.0	-3.4	0.2
$A_{6p_{1/2}}$	$\langle 1/r^3\rangle_{6p}$	-6.1	2.6	0.2
$A_{7p_{1/2}}$	$\langle 1/r^3\rangle_{7p}$	-7.1	-1.5	0.5

[2] S. C. Bennett & C. E. Wieman, Phys. Rev. Letts. **82**, 2484 (1999).

Cesium: Theory vs. Experiment

$\beta = 27.024\ (43)_{\rm exp}\,(67)_{\rm th}\, a_0^3$ (1999)

(eliminating axial vector + anapole contribution)

$\Im(E_{\rm PNC}) = -0.8379\ (37)_{\rm exp}\,(21)_{\rm th} \times 10^{-11}|e|a_0$

(dividing by theoretical matrix element)

$$Q_W = -72.06\ (29)_{\rm exp}\,(34)_{\rm th}$$

Marciano & Rosner (with radiative corrections)

$$Q_W^{SM} = -73.09\ (03)_{\rm rad.\ corr.}$$

Expt. - Theory = 2.3 σ

This difference has been cited as evidence for "new physics" beyond the Standard Model!

Result for Anapole Moment

Difference $R_{3-4} - R_{4-3}$ leads to:

$$\begin{aligned} K &= 0.441\,(63) \\ -(\kappa - 1/2)/\kappa \; K^{(2)} &= 0.055 \\ K^{(Q_W)} &= 0.031 \\ K^{(a)} &= 0.355\;(63) \end{aligned}$$

Theoretical estimates[3]

$$K^{(a)} = 0.25 - 0.75$$

[3]W. C. Haxton and C. E. Wieman, arXiv:nucl-th/0104026

Table 10.4: (continued)

Quantity	Value	Standard Model	Pull
m_t [GeV]	174.3 ± 5.1	172.9 ± 4.6	0.3
M_W [GeV]	80.448 ± 0.062	80.378 ± 0.020	1.1
	80.350 ± 0.056		-0.5
M_Z [GeV]	91.1872 ± 0.0021	91.1870 ± 0.0021	0.1
Γ_Z [GeV]	2.4944 ± 0.0024	2.4956 ± 0.0016	-0.5
$\Gamma(\text{had})$ [GeV]	1.7439 ± 0.0020	1.7422 ± 0.0015	—
$\Gamma(\text{inv})$ [MeV]	498.8 ± 1.5	501.65 ± 0.15	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.96 ± 0.09	84.00 ± 0.03	—
σ_{had} [nb]	41.544 ± 0.037	41.480 ± 0.014	1.7
R_e	20.803 ± 0.049	20.740 ± 0.018	1.3
R_μ	20.786 ± 0.033	20.741 ± 0.018	1.4
R_τ	20.764 ± 0.045	20.786 ± 0.018	-0.5
R_b	0.21642 ± 0.00073	0.2158 ± 0.0002	0.9
R_c	0.1674 ± 0.0038	0.1723 ± 0.0001	-1.3
$A_{FB}^{(0,e)}$	0.0145 ± 0.0024	0.0163 ± 0.0003	-0.8
$A_{FB}^{(0,\mu)}$	0.0167 ± 0.0013		0.3
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5
$A_{FB}^{(0,b)}$	0.0988 ± 0.0020	0.1034 ± 0.0009	-2.3
$A_{FB}^{(0,c)}$	0.0692 ± 0.0037	0.0739 ± 0.0007	-1.3
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1035 ± 0.0009	-0.5
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2321 ± 0.0010	0.2315 ± 0.0002	0.6

Table 10.4: (continued)

Quantity	Value	Standard Model	Pull
A_e	0.15108 ± 0.00218	0.1475 ± 0.0013	1.7
	0.1558 ± 0.0064		1.3
	0.1483 ± 0.0051		0.2
A_μ	0.137 ± 0.016		-0.7
A_τ	0.142 ± 0.016		-0.3
	0.1425 ± 0.0044		-1.1
A_b	0.911 ± 0.025	0.9348 ± 0.0001	-1.0
A_c	0.630 ± 0.026	0.6679 ± 0.0006	-1.5
A_s	0.85 ± 0.09	0.9357 ± 0.0001	-1.0
R^-	$0.2277 \pm 0.0021 \pm 0.0007$	0.2299 ± 0.0002	-1.0
κ^ν	$0.5820 \pm 0.0027 \pm 0.0031$	0.5831 ± 0.0004	-0.3
R^ν	$0.3096 \pm 0.0033 \pm 0.0028$	0.3091 ± 0.0002	0.1
	$0.3021 \pm 0.0031 \pm 0.0026$		-1.7
$g_V^{\nu e}$	-0.035 ± 0.017	-0.0397 ± 0.0003	—
	-0.041 ± 0.015		-0.1
$g_A^{\nu e}$	-0.503 ± 0.017	-0.5064 ± 0.0001	—
	-0.507 ± 0.014		0.0
$Q_W(\mathrm{Cs})$	$-72.06 \pm 0.28 \pm 0.34$	-73.09 ± 0.03	2.3
$Q_W(\mathrm{Tl})$	$-114.8 \pm 1.2 \pm 3.4$	-116.7 ± 0.1	0.5
$\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to ce\nu)}$	$3.26^{+0.75}_{-0.68} \times 10^{-3}$	$3.15^{+0.21}_{-0.20} \times 10^{-3}$	0.1

Possible Explanation of $2.3\,\sigma$

The previous result suggested the possible existence of a Z' particle to several authors:

1. R. Casalbuoni, S. De Curtis, D. Dominici, and R. Gatto, Phys. Lett. **B460**, 135 (1999).
2. J. L. Rosner, Phys. Rev. **D61**, 016006 (2000).
3. J. Erler and P. Langacker, Phys. Rev. Lett. **84**, 212 (2000).

Breit Revisited

Weak HF level:

$$\left(h_0 + V^{\rm HF} - \epsilon_v^{\rm HF}\right) \tilde{\psi}_v^{\rm HF} = -h_{\rm PNC}\, \psi_v^{\rm HF}$$

$$E_{\rm PNC} = \langle \psi_{7s}^{\rm HF} | ez | \tilde{\psi}_{6s}^{\rm HF} \rangle + \langle \tilde{\psi}_{7s}^{\rm HF} | ez | \psi_{6s}^{\rm HF} \rangle$$

Type	$\langle 7s \| ez \| \widetilde{6s} \rangle$	$\langle \widetilde{7s} \| ez \| 6s \rangle$	$E_{\rm PNC}$
Coul	0.27492	-1.01439	-0.73947
+Breit	0.27411	-1.01134	-0.73722
Δ%	-0.29%	-0.30%	-0.30%

This correction was included in ND calculation

$$E_{\rm PNC} = -0.907 + 0.002 = -0.905$$

but not in Novosibirsk calculation.

Derevianko's observation

Brueckner level:

$$\left(h_0 + V^{\mathrm{HF}} + \hat{\Sigma} - \epsilon_v^{\mathrm{Br}}\right) \tilde{\psi}_v^{\mathrm{Br}} = -h_{\mathrm{PNC}}\, \psi_v^{\mathrm{Br}} \\ -\delta V_{\mathrm{PNC}}^{\mathrm{HF}}\, \psi_v^{\mathrm{Br}}$$

$$E_{\mathrm{PNC}} = \langle \psi_{7s}^{\mathrm{Br}} | ez + \delta_{\mathrm{RPA}}(ez) | \tilde{\psi}_{6s}^{\mathrm{Br}} \rangle \\ + \langle \tilde{\psi}_{7s}^{\mathrm{Br}} | ez + \delta_{\mathrm{RPA}}(ez) | \psi_{6s}^{\mathrm{Br}} \rangle$$

Type	$\langle 7s \vert ez \vert \widetilde{6s} \rangle$	$\langle \widetilde{7s} \vert ez \vert 6s \rangle$	E_{PNC}
Coul	0.43942	-1.33397	-0.89456
+ Breit	0.43680	-1.32609	-0.88929
Δ%	-0.60%	-0.59%	-0.59%

Using this result for the Breit correction, the final theoretical PNC amplitudes become

Group	Coul	Breit	E_{PNC}
Novosibirsk	0.908	-0.005	0.903
Notre Dame	0.907	-0.005	0.902

Summary

With Breit corrections:

$$E_{\mathrm{PNC}}^{\mathrm{theor}} = -0.902\,(4) \quad \text{or} \quad (10)?$$

and the deviation of the Q_W from the standard model is reduced to $1.5\,\sigma$ if 0.4% theoretical accuracy is still assumed. However, if a more realistic 1% theoretical uncertainty is assumed, the corresponding value of the weak-charge becomes

$$Q_W(^{133}\mathrm{Cs}) = -72.42\,(0.28)_{\mathrm{expt}}(0.74)_{\mathrm{theor}}$$

and shows NO significant deviation from the standard model.[4]

HELP! – accurate calculations needed – HELP!

[4] A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000); V. A. Dzuba *et al.*, Phys. Rev. A **63**, 044103 (2001); M. G. Kozlov *et al.*, arXiv:physics (0101053).