

# Relativistic many-body calculations of magnetic dipole transitions in Be-like ions

U.I. Safronova, W.R. Johnson, A. Derevianko

*Department of Physics, University of Notre Dame, Notre Dame, IN 46556*

Reduced matrix elements and transition rates are calculated for all magnetic dipole ( $M1$ ) transitions within  $2l2l'$  configurations and for some  $2l3l' - 2l2l'$  transitions in Be-like ions with nuclear charges ranging from  $Z = 4$  to 100. Many-body perturbation theory (MBPT), including the Breit interaction, is used to evaluate retarded  $M1$  matrix elements. The calculations start with a  $(1s)^2$  Dirac-Fock potential and include all possible  $n = 2$  configurations, leading to 4 odd-parity and 6 even-parity states, and some  $n = 3$  configurations. First-order perturbation theory is used to obtain intermediate coupling coefficients. Second-order MBPT is used to determine the matrix elements, which are evaluated for all 11  $M1$  transitions within  $2l2l'$  configurations and for 35  $M1$  transitions between  $2l3l'$  and  $2l2l'$  states. The transition energies used in the calculation of oscillator strengths and transition rates are obtained from second-order MBPT. The importance of negative-energy contributions to  $M1$  transition amplitudes is discussed.

32.70.Cs, 31.15.Md, 31.25.-v, 31.30.Jv

## I. INTRODUCTION

Studies of  $M1$  transitions are much less complete than studies of  $E1$  transitions, as testified to by the small number of theoretical papers that have been devoted to  $M1$  transitions [1–7] during the past thirty years. This lack of interest is associated in part with the fact that the probability of  $M1$  transitions is several orders magnitude smaller than the probability of  $E1$  transitions. In spite of the small probability,  $1s^2 \ ^1S_0 - 1s2s \ ^3S_1$   $M1$  transitions have been observed in the solar corona and  $M1$  transitions between states within  $1s^2 2s^2 2p^3$  configurations in OII have been observed in spectra of gaseous nebulae. Recently [8], the lifetime of the  $1s^2 2s 2p \ ^3P_2$  level of Ar XV was measured using metastable  $\text{Ar}^{14+}$  ions produced by an electron cyclotron resonance ion source. It was found [8] that the rate derived from the measured lifetime differed significantly from both nonrelativistic [9] and relativistic [7] calculations of  $M1$  transition rates. It should be noted that, for  $M1$  transitions within a single configuration, differences between relativistic and nonrelativistic calculations are insignificant.

The purpose of this paper is to calculate the forbidden  $2s^2 - 2s3s$ ,  $2s2p - 2p3s$ , and  $2s2p - 2s3p$   $M1$  transitions in berylliumlike ions. We also calculate  $M1$  transitions within  $2s2p$  and  $2p^2$  configurations, and confirm that our results for allowed  $2l - 2l$  transitions are in good agreement with other theoretical results.

In the present paper, we use relativistic many-body perturbation theory (MBPT) to determine reduced matrix elements and transition rates for all lines in  $2l2l'$  configurations and for some  $2l3l' - 2l2l'$  lines in Be-like ions with nuclear charges ranging from  $Z = 4$  to 100. Our calculations start from a  $(1s)^2$  Dirac-Fock potential, and include all possible  $n = 2$  configurations, leading to 4 odd-parity and 6 even-parity states; they also include some  $n = 3$  configurations. Second-order MBPT is used to determine the matrix elements, which are evaluated for the 11 transitions inside  $2l2l'$  configurations and for 35 transitions between states of  $2l3l'$  and  $2l2l'$  configurations. The transition energies used in the calculation of transition rates are evaluated using second-order MBPT. Negative-energy contributions to the transition amplitudes are included. The present calculations provide a theoretical benchmark for future experiments and calculations.

## II. METHOD

In this section, we write down and discuss the relativistic MBPT formulas for first- and second-order transition matrix elements in atomic systems with two valence electrons. It should be noted that the primary difference in the theoretical expressions for  $E1$  and  $M1$  transition matrix elements occurs in the first-order. Details of second-order MBPT calculations for  $E1$  matrix elements were presented in [10]; here we give the expression for the first-order  $M1$  matrix element only and refer the reader to Ref. [10] for a discussion of second-order diagrams. The present results are given in the Coulomb gauge. It should be noted that gauge transformations of the type considered in [10] have no effect on  $M1$  matrix elements.

## A. Basic Formulas

The first-order reduced dipole matrix element  $M^{(1)}$  for the transition between two states  $vw(J) - v'w'(J')$  is [11]

$$M^{(1)}[v_1w_1(J) - v_2w_2(J')] = \sqrt{[J][J']} \sum_{vw} \sum_{v'w'} S^J(v_1w_1, vw) S^{J'}(v_2w_2, v'w') \\ \times (-1)^{1+j_w+j_{v'}} \left\{ \begin{matrix} J & J' & 1 \\ j_{v'} & j_w & j_v \end{matrix} \right\} M_{v'w} \delta_{vw'}, \quad (2.1)$$

where  $[J] = 2J + 1$ . The quantity  $S^J(v_1w_1, vw)$  is a symmetry coefficient defined by

$$S^J(v_1w_1, vw) = \eta_{v_1w_1} [\delta_{v_1v} \delta_{w_1w} + (-1)^{j_v+j_w+J+1} \delta_{v_1w} \delta_{w_1v}], \quad (2.2)$$

where  $\eta_{vw}$  is a normalization factor given by

$$\eta_{vw} = \begin{cases} 1 & \text{for } w \neq v \\ 1/\sqrt{2} & \text{for } w = v. \end{cases}$$

The magnetic dipole matrix element  $M_{vw}$ , which includes retardation, is given by

$$M_{vw} = \langle -\kappa_v \| C_1 \| \kappa_w \rangle \frac{6}{\alpha k} \int_0^\infty dr \frac{(\kappa_v + \kappa_w)}{2} j_1(kr) [G_v(r)F_w(r) + F_v(r)G_w(r)]. \quad (2.3)$$

Here  $\kappa_v$  is the angular momentum quantum number [ $\kappa_v = \mp(j_v + \frac{1}{2})$  for  $j_v = (l_v \pm \frac{1}{2})$ ], and  $k = \omega\alpha$ , where  $\omega = \varepsilon_w - \varepsilon_v$  is the photon energy and  $\alpha$  is the fine-structure constant. The quantity  $C_{1q}(\hat{r})$  is a normalized spherical harmonic. The functions  $G_a(r)$  and  $F_a(r)$  are large- and small-component radial Dirac wave functions, respectively.

The nonrelativistic limit is different for transitions inside of one configurations,  $n_v = n_w$ , and transitions between different configurations,  $n_v \neq n_w$ . For the case  $n_v = n_w$ , we obtain

$$M_{vw}^{\text{NR}} = -\langle -\kappa_v \| C_1 \| \kappa_w \rangle (\kappa_v + \kappa_w - 1) \frac{(\kappa_v + \kappa_w)}{2} \int_0^\infty dr P_v(r) P_w(r), \quad (2.4)$$

where  $P_v(r)$  is a nonrelativistic limit the large-component radial wave function  $G_v(r)$ . The case  $n_v \neq n_w$  was considered in [11–16] for  $1s^2 \ ^1S_0$ - $1s2s \ ^3S_1$  transitions. The nonrelativistic limit in this case is proportional to  $Z^2$ , and for transitions with  $l_v = l_w = 0$  the matrix element is given by

$$M_{vw}^{\text{NR}} = -(\alpha Z)^2 \sqrt{\frac{2}{3}} \left[ \frac{3}{Z} \int_0^\infty \frac{1}{r} dr P_v(r) P_w(r) + \frac{\omega^2}{2Z^2} \int_0^\infty r^2 dr P_v(r) P_w(r) \right]. \quad (2.5)$$

Using hydrogenic wave functions, we find

$$M_{1s2s}^{\text{NR}} = -(\alpha Z)^2 \frac{16}{27\sqrt{3}}, \quad M_{2s3s}^{\text{NR}} = -(\alpha Z)^2 \frac{2^4 2^3}{5^5}. \quad (2.6)$$

Second-order contributions to  $E1$  reduced matrix elements are written out in [10]. It is only necessary to replace the quantities  $Z_{vw}$  in Ref. [10] by  $M_{vw}$  to obtain the corresponding second-order contributions to  $M1$  reduced matrix elements. The reduced matrix element for the derivative term is given by

$$M^{(\text{deriv})}[vw(J) - v'w'(J')] = \alpha (E_{vw}^{(1)} - E_{v'w'}^{(1)}) P^{(\text{deriv})}[vw(J) - v'w'(J')], \quad (2.7)$$

where  $E_{vw}^{(1)}$  is the first-order correction to the energy given by Eqs.(2.8-2.10) of Ref. [21]. The quantity  $P^{(\text{deriv})}$  introduced above is given by

$$P^{(\text{deriv})}[v_1w_1(J) - v_2w_2(J')] = \sqrt{[J][J']} \sum_{vw} \sum_{v'w'} S^J(v_1w_1, vw) S^{J'}(v_2w_2, v'w') \\ \times (-1)^{1+j_w+j_{v'}} \left\{ \begin{matrix} J & J' & 1 \\ j_{v'} & j_w & j_v \end{matrix} \right\} M_{v'w}^{(\text{deriv})} \delta_{vw'}, \quad (2.8)$$

where

$$M_{vw}^{(\text{deriv})} = \langle \kappa_v \| C_1 \| \kappa_w \rangle \frac{6}{\alpha k} \int_0^\infty dr \frac{(\kappa_v + \kappa_w)}{2} [j_1(kr) - (kr) j_2(kr)] [G_v(r)F_w(r) + F_v(r)G_w(r)]. \quad (2.9)$$

All second-order correlation corrections above are from the residual Coulomb interaction. To include correlation corrections from the Breit interaction, the Coulomb matrix element  $X_k(abcd)$  must be modified according to the rule:

$$X_k(abcd) \rightarrow X_k(abcd) + M_k(abcd) + N_k(abcd). \quad (2.10)$$

The magnetic radial integrals  $M_k$  and  $N_k$  are defined by Eqs.(A4,A5) in Ref. [17].

## B. Uncoupled Matrix Elements

In Table I, we list values of  $M^{(1)}$ ,  $M^{(\text{HF})}$ ,  $M^{(\text{RPA})}$ ,  $M^{(\text{corr})}$ , and  $P^{(\text{deriv})}$  for transitions in Be-like iron,  $Z=26$ . Two types of transitions are presented: those with no change of principal quantum number (2-2 transitions) and those with a unit change of principal quantum number (2-3 transitions). It should be noted that the contribution from the HF diagram,  $M^{(\text{HF})}$ , vanishes in the present calculation since we use HF basis functions. In the table, we use the label  $B$  to denote the Breit contributions and we tabulate  $1000 \times B^{(\text{HF})}$ ,  $1000 \times B^{(\text{RPA})}$ ,  $1000 \times B^{(\text{corr})}$ , together with the sum,  $1000 \times B^{(2)}$ . It can be seen from Table I that the largest difference (a factor of 1000) between two types of transitions occurs in the values of the first-order matrix element  $M^{(1)}$ . However, there are no large differences for the second-order diagram contributions ( $M^{(\text{RPA})}$ ,  $M^{(\text{corr})}$ ) and Breit contributions ( $B^{(\text{HF})}$ ,  $B^{(\text{RPA})}$ ,  $B^{(\text{corr})}$ ). To interpret this observation we consider the  $Z$ -dependence of a typical second-order diagram

$$M^{(\text{corr})} \sim \sum_{iw \notin P'} \frac{M_{iv} g_{v'w'} w_i}{\varepsilon_i + \varepsilon_w - \varepsilon_{v'} - \varepsilon_{w'}}. \quad (2.11)$$

Here  $P'$  denotes the model space of  $w'v'$ . The intermediate state energy  $\varepsilon_i$  takes both positive,  $\varepsilon_i > mc^2$ , and negative,  $\varepsilon_i < -mc^2$ , values. To begin, we focus on the positive-energy contribution to such diagrams. The magnetic-dipole matrix element  $M_{iv}$  involving an intermediate state can be either of order of 1, provided  $|i\rangle = |v\rangle$  is allowed by the condition  $vw \notin P'$ , or of order of  $(\alpha Z)^2$ , otherwise. The leading term of Coulomb matrix element  $X_k(abcd)$  is proportional to  $Z$ . The Breit interaction is proportional to  $\alpha^2 Z^3$  and  $\varepsilon_i$  is proportional to  $Z^2$ . As a result, we obtain two different  $Z$ -dependences for  $M^{(\text{corr})}$  and  $B^{(\text{corr})}$ :

$$M^{(\text{corr})} \propto \begin{cases} 1/Z & \text{for } n_i = n_v \\ \alpha^2 Z & \text{for } n_i \neq n_v \end{cases}, \quad B^{(\text{corr})} \propto \begin{cases} \alpha^2 Z & \text{for } n_i = n_v \\ \alpha^4 Z^3 & \text{for } n_i \neq n_v. \end{cases}$$

For transitions inside the same model space ( $2l_1 2l_2 - 2l_3 2l_4$ ), the upper case is never realized, since the condition  $vw \notin P'$  is not satisfied. The upper case is realized for transitions such as  $2s 2p_{3/2} - 2s 3p_{3/2}$ . For such transitions, the lowest-order matrix element is suppressed by  $(\alpha Z)^2$ ; thus, the leading contribution to uncoupled matrix elements arises from the second-order diagrams.

## C. Contribution from the negative-energy states

*Ab initio* relativistic calculations require a careful treatment of negative-energy states (virtual electron-positron pairs), which, if included improperly, lead to the *continuum dissolution* problem discussed by Sucher [18]. A common approach to relativistic calculations is to use the *no-pair* Hamiltonian, which excludes the negative-energy states (NES) [18,19]. However, the NES contribution appears in the evaluation of transition amplitudes. In the second-order expressions (2.11), such contribution explicitly arises from the terms in the sum over states  $i$  and  $n$  for which  $\varepsilon_i < -mc^2$ . The effect of the NES contributions to MI-amplitudes has been studied recently for heliumlike ions in [15,16,20]. In Ref. [20], it was demonstrated that for nonrelativistically forbidden magnetic-dipole transitions the relative contribution of NES grows for low  $Z$  as  $1/Z$  in calculations using a model-potential basis.

To understand the relative importance of NES for Be-like ions, we consider the scaling of leading terms in an  $\alpha Z$  expansion of various diagrams for uncoupled matrix elements. As discussed in Section B, the leading regular *no-pair* contribution to the total value of uncoupled matrix element for  $2l_1 2l_2 - 2l_3 2l_4$  transitions scales either as  $(\alpha Z)^2$  or as  $1/Z$ , depending on the selection rules intermediate states. The leading term for  $2l_1 2l_2 - 2l_3 2l_4$  transitions is of order of 1 and arises from the non-relativistically allowed lowest order. The analysis of NES contribution is similar to that given in Ref. [20]. First, we notice that the energy denominator can be replaced by  $-2mc^2$  with an accuracy

of  $(\alpha Z)^2$ . Further, we take into account that the meaning of “large” and “small” components for a negative-energy state is reversed, i.e., the large component is  $\alpha Z$  times smaller than the small component. The magnetic-dipole matrix element  $M_{iv}$  couples the large component of the positive-energy state  $|v\rangle$  and the small component of the negative-energy state  $|i\rangle$ . Therefore, as can be seen from Eq.(2.3),  $M_{iv} \propto 1/\alpha Z$ . Applying the same argument to the Coulomb and Breit matrix elements, we arrive at the conclusion that the NES contribution scales as  $\alpha^2 Z$  for both Breit and Coulomb contributions.

$$M_-^{(\text{corr})} \sim B_-^{(\text{corr})} \sim \alpha^2 Z.$$

We see that the relative contribution of NES to uncoupled magnetic-dipole amplitude scales as  $\alpha^2 Z$  for  $2l_1 2l_2 - 2l_3 2l_4$  transitions, and as  $(\alpha Z)^2$  or  $1/Z$  for  $2l_1 2l_2 - 2l_3 3l_4$  transitions, depending on the leading order of the regular *no-pair* contribution.

#### D. Coupled Matrix Elements

Physical two-particle states are linear combinations of uncoupled two-particle states  $(vw)$  in the model space that have fixed values of angular momentum and parity; consequently, transition amplitudes between physical states are linear combinations of the uncoupled transition matrix elements given in Table I. Expansion coefficients and energies are obtained by diagonalizing the effective Hamiltonian as discussed in [21]. The first-order expansion coefficient  $C_1^\lambda(vw)$  is the  $\lambda$ -th eigenvector of the first-order effective Hamiltonian, and  $E_1^\lambda$  is the corresponding eigenvalue. In the present calculation, both Coulomb and Breit interactions are included in the first-order effective Hamiltonian. The coupled transition matrix element between the  $I$ -th, initial eigenstate which has angular momentum  $J$  and the  $F$ -th final state which has angular momentum  $J'$  is given by

$$\begin{aligned} Q^{(1+2)}(I-F) &= \frac{1}{E_1^I - E_1^F} \sum_{vw} \sum_{v'w'} C_1^I(vw) C_1^F(v'w') \\ &\times \left\{ [\epsilon_{vw} - \epsilon_{v'w'}] \left[ M^{(1+2)}[vw(J) - v'w'(J')] + B^{(2)}[vw(J) - v'w'(J')] \right] \right. \\ &\left. + [E_1^I - E_1^F - \epsilon_{vw} + \epsilon_{v'w'}] P^{(\text{deriv})}[vw(J) - v'w'(J')] \right\}. \end{aligned} \quad (2.12)$$

Here,  $\epsilon_{vw} = \epsilon_v + \epsilon_w$  and  $M^{(1+2)} = M^{(1)} + M^{(\text{RPA})} + M^{(\text{corr})}$ . Using these formulas and the results for uncoupled reduced matrix elements given in Table I, we transform from uncoupled reduced matrix elements to intermediate coupled reduced matrix elements between physical states.

To cite a specific example, in Fig. 1 we present the mixing coefficients as functions of  $Z$  for the  $2s3p^3P_1$  level. It should be noted that the odd parity complex with  $J=1$  includes 7 two-particle states  $2s3p_{1/2}$ ,  $2s3p_{3/2}$ ,  $2p_{1/2}3s$ ,  $2p_{3/2}3s$ ,  $2p_{1/2}3d_{3/2}$ ,  $2p_{3/2}3d_{3/2}$ , and  $2p_{3/2}3d_{5/2}$ . After diagonalization, the 7 eigenvectors are labeled  $2s3p^1P_1$ ,  $2s3p^3P_1$ ,  $2p_{3s}^3P_1$ ,  $2p_{3s}^1P_1$ ,  $2p_{3d}^3D_1$ ,  $2p_{3d}^3P_1$ , and  $2p_{3d}^1P_1$ . These states form the complex of final states, labeled by “F” in Eq.(2.12),

$$\begin{aligned} \Psi(F) &= C^F(2s3p_{1/2})\Psi(2s3p_{1/2}) + C^F(2s3p_{3/2})\Psi(2s3p_{3/2}) + C^F(2p_{1/2}3s)\Psi(2p_{1/2}3s) \\ &+ C^F(2p_{3/2}3s)\Psi(2p_{3/2}3s) + C^F(2p_{1/2}3d_{3/2})\Psi(2p_{1/2}3d_{3/2}) \\ &+ C^F(2p_{3/2}3d_{3/2})\Psi(2p_{3/2}3d_{3/2}) + C^F(2p_{3/2}3d_{5/2})\Psi(2p_{3/2}3d_{5/2}). \end{aligned} \quad (2.13)$$

In Fig. 1, we show the  $Z$ -dependence of the 7 coefficients  $C^F(vw)$  in the above expansion. We can see from this figure that the largest contribution to the  $2s3p^3P_1$  level in the range  $Z=6-12$  arises from the  $2s3p_{1/2}$  state, for  $Z=13-50$  is from the  $2s3p_{3/2}$  state, but for  $Z=51-100$  the contribution of the  $2p_{1/2}3s$  state becomes dominant. We observe similar complex behavior of mixing coefficients for other levels.

The importance of these mixing coefficients is demonstrated in Fig. 2. We can see a series deep minima in Fig. 2, where the  $M1$  coupled matrix elements are shown as functions of  $Z$  with the single initial state  $2s2p^3P_0$  and seven final states from the odd-parity complex with  $J=1$ . Most of these minima are at the mixing coefficient crossings shown in Fig. 1. We investigated the  $Z$ -dependence of the uncoupled matrix elements  $M^{(1+2)} = M^{(1)} + M^{(\text{RPA})} + M^{(\text{corr})}$  and  $B^{(2)}$  for the seven uncoupled matrix elements with  $(vw)=2s2p_{1/2}[0]$  and  $(v'w')=2s3p_{1/2}[1]$ ,  $2s3p_{3/2}[1]$ ,  $2p_{1/2}3s[1]$ ,  $2p_{3/2}3s[1]$ ,  $2p_{1/2}3d_{3/2}[1]$ ,  $2p_{3/2}3d_{3/2}[1]$ , and  $2p_{3/2}3d_{5/2}[1]$ . There are no peaks for these matrix elements. There are minima in two curves for  $B^{(2)}$ , but the contributions of these matrix elements are 100 times smaller than the contribution of the other five matrix elements for  $B^{(2)}$ . Moreover, they do not effect the coupled matrix elements

shown in Fig. 2. We conclude that the mixing coefficients alone are responsible for the minima in the coupled matrix elements.

Let us return to the discussion of negative-energy contributions. In Table II, we list the values of the coupled reduced matrix elements calculated with and without NES contribution for  $\text{Fe}^{+22}$ . We can see that the relative contribution of NES is  $\sim 0.03\%$  for transitions inside the  $2l2l'$  configuration space and  $\sim 3\%$  for  $2l_12l_2 - 2l_33l_4$  transitions. Since the scaling of NES contribution, discussed in Section B, does not depend on a transition, we devise the qualitative rule: the negative-energy contribution is most important for the weakest transitions in a given transition array. Predictions from this rule can be easily verified by analyzing values in Table II.

Figure 3 illustrates the relative size of contributions of NES to several transitions inside the  $2l2l'$  configuration space. We observe very small relative NES contributions for these transitions. These contributions grow with increasing nuclear charge, in agreement with the expected  $Z$ -dependence, and are bound by  $0.3\%$  at  $Z = 100$ . The relatively small NES effect is explained by the non-relativistically allowed character of the transitions inside the  $2l2l'$  configuration space, the relative contribution of the second-order diagrams being comparably small. The transitions between  $2l_12l_2$  and  $2l_33l_4$  configuration spaces are non-relativistically forbidden; therefore, the NES effect on the transition amplitudes is amplified. To illustrate this assertion, in Fig. 4 we present the  $Z$ -dependence of relative NES contribution to the line strength (or rate) for several  $2l_12l_2 - 2l_33l_4$  transitions. We see that the ratio  $(S_{n+p} - S_p)/S_{n+p}$  for these transitions can be as large as  $25\%$  and the NES contribution is the most important for low- $Z$  calculations, in accordance with the expected  $Z$  dependence. On this plot, we can see several irregularities which are the reflections of minima in the total values of  $M1$  coupled matrix elements. It is worth emphasizing that the contribution from the negative-energy states can be comparable to the regular *no-pair* contribution for transitions involving different configuration spaces.

### III. RESULTS AND DISCUSSION

We calculate line strengths and transition probabilities for the 11  $M1$  transitions inside of  $2l2l'$  configurations and the 35  $M1$  transitions between  $2l3l'$  and  $2l2l'$  states for all ions up to  $Z=100$ . The theoretical energies used to evaluate these transition probabilities are calculated by using second-order MBPT [21].

In Table III, we list the values of line strengths  $S$ , calculated for all of the above mentioned  $M1$  transitions. These values are given for  $Z= 10, 20, 40, 60, 80, \text{ and } 100$ . We use both  $LS$ - and  $jj$ -coupling designations to label the levels. We can see from this table, that some  $LS$  forbidden transitions with very small S-values for  $Z= 10, 20$ , for example  $2s2p\ ^3P_J - 2s2p\ ^1P_1$  and  $2s2p\ ^3P_0 - 2s3p\ ^1P_1$ , become allowed in  $jj$ -coupling with substantially larger S-values for high nuclear charges. On the other hand, some  $LS$  allowed transitions, for example  $2p^2\ ^3P_0 - 2p3p\ ^3P_1$ , become forbidden in the  $jj$ -coupling. It should be noted that even though the S-values for transitions inside the same configuration space are much larger than the S-values for transitions between different configurations, the A-values for transition probabilities are of the same order. This is explained by small energy differences for transitions inside the same configuration space. The A-values for the 2-2 transitions are proportional to  $Z^{12}$ , since the leading term of S is constant and the energy difference is proportional to  $Z^4$ . The magnetic-dipole A-values for the 2-3 transitions are proportional to  $Z^{10}$ , since the leading term of S is proportional to  $Z^4$  and the energy difference is proportional to  $Z^2$ .

The  $Z$  dependence of transition probabilities for the five transitions between levels of  $2s2p$  configurations are shown in Fig. 5. The A-values for six transitions with the initial level  $2s^2\ ^1S_0$  and the 6 even parity  $2l3l'$  levels with  $J=1$  are shown in Fig. 6. We see from these plots that the A-values for these transitions increase from  $10^{-5}\ \text{s}^{-1}$  to  $10^{11}\ \text{s}^{-1}$  in the interval of  $Z=6 - 100$ . The  $Z$  dependence for the A-value of transitions inside the same configuration space, shown in Fig. 5, is rather smooth, compared to that of the 2-3 transitions, presented in Fig. 6. The  $Z$  dependence for the 2-3 transitions is more complex reflecting the tangled behavior of the mixing coefficients.

In Tables IV and V, we present wavelengths  $\lambda$  ( $\text{\AA}$ ), line strengths  $S$  (a.u.), and transition probabilities  $A$  ( $\text{s}^{-1}$ ) for five transitions  $^3P_0 - ^3P_1, ^3P_1 - ^3P_2, ^3P_J - ^1P_1$  within the  $2s2p$  configuration, five transitions  $^3P_0 - ^3P_1, ^3P_1 - ^3P_2, ^3P_1 - ^1S_0, ^3P_1 - ^1D_2, ^3P_2 - ^1D_2$  within the  $2p^2$  configuration, and for the transition between  $2s^2\ ^1S_0$  and  $2p^2\ ^3P_1$  levels. It is worth noting that the transition  $2s^2\ ^1S_0 - 2p^2\ ^3P_1$  is nonrelativistically allowed only due to configuration mixing. (The model space for the  $2s^2\ ^1S_0$  level includes three two-particle states:  $2s2s[0] + 2p_{1/2}2p_{1/2}[0] + 2p_{3/2}2p_{3/2}[0]$ .) In the Tables IV and V, we compare our data for  $Z = 6 - 26$  with the other theoretical results [5–7]. All theoretical values agree for LS-allowed transitions:  $^3P_0 - ^3P_1, ^3P_1 - ^3P_2$  transitions in  $2s2p$  and  $2p^2$  configurations. Such agreement can be expected, since correlation and relativistic effects do not affect these transitions. For these four transitions, the line strengths are almost independent of  $Z$ ;  $S \simeq 2$  for  $^3P_0 - ^3P_1$  transitions and  $S \simeq 5/2$  for  $^3P_1 - ^3P_2$  transitions. The corresponding A-value can be accurately determined from the transition energies, which can be obtained from precise spectroscopic data. The experimental A-value of  $74(4)\ \text{s}^{-1}$  for the LS-allowed  $2s2p\ ^3P_1 - 2s2p\ ^3P_2$  transition in  $\text{Ar}^{+14}$  has been recently reported in Ref. [8]. This value disagrees with previous theoretical results,  $63.3\ \text{s}^{-1}$  [6] and  $63.0$

$s^{-1}$  [7]; it also disagrees with our predictions  $64.0 s^{-1}$ , obtained with the MBPT energies and  $63.8 s^{-1}$  obtained with experimental energies.

In contrast to the LS-allowed transitions, we disagree with the previous theoretical values for transitions with  $\Delta S = 1$ . The  $A$ -values for transitions with  $\Delta S = 1$  are determined by the mixing coefficients, which are very sensitive to the theoretical approximation employed. Our coefficients were obtained as the result of diagonalizing the first-order effective relativistic Hamiltonian. On the other hand, the LS-coupling scheme was employed in papers [5,6] and the Breit-Pauli approximation was used for relativistic corrections. Owing to the common approach used in [5] and [6], the results of these papers are in a much better agreement with each other than with our results, especially for small  $Z$ . Further analysis is necessary to assess the accuracy of two approaches.

#### IV. CONCLUSION

We have presented a systematic second-order relativistic MBPT study of reduced matrix elements, line strengths, and transition rates for two types of magnetic dipole transitions: with and without change of principle quantum number. The calculations involving a change of principal quantum number are completely new for Be-like ions. In order to confirm the correctness of our method, we also investigated the well-known transitions inside one configuration and we obtained good agreement with other calculations for LS-allowed transitions where relativistic corrections are not very important. We believe that our results for intercombination transitions are more accurate than previous theoretical results since we treat relativistic contributions more carefully. Contributions from negative-energy states were also included in the second-order matrix elements. It was shown that these contributions are very important for 2-3 transition probabilities. Matrix elements from the present calculations provide basic theoretical input for calculations of reduced matrix elements, oscillator strengths, and transition rates in three-electron boronlike ions.

#### ACKNOWLEDGMENTS

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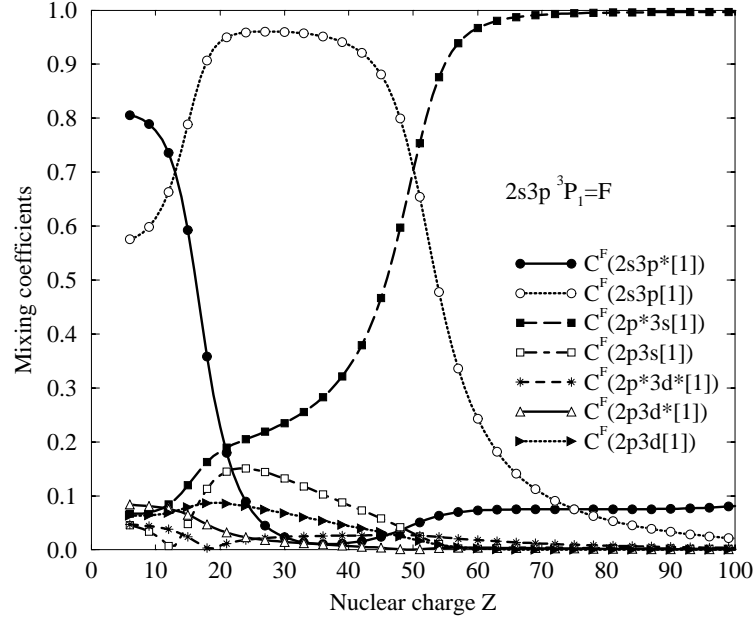


FIG. 1.  $Z$ -dependence of the mixing coefficients for  $2s3p\ ^3P_1$  level

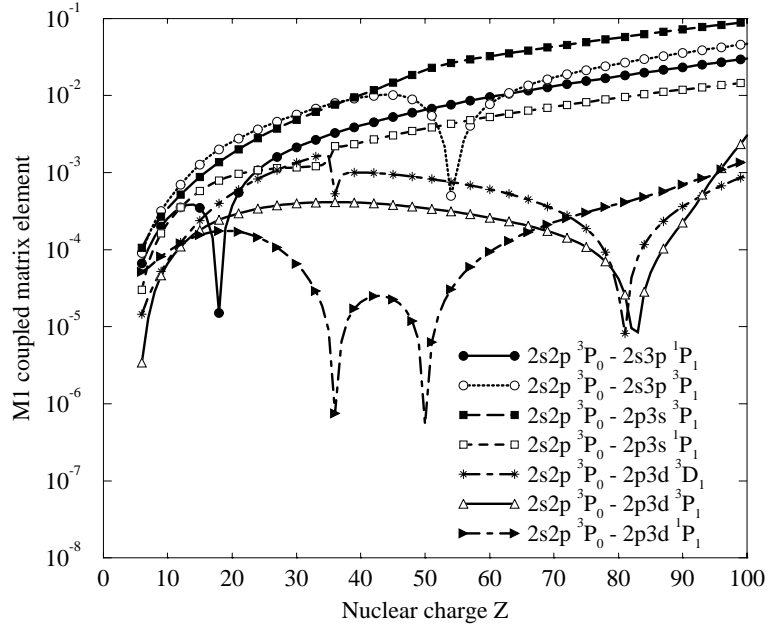


FIG. 2.  $Z$ -dependence of the  $M1$  coupled matrix elements.

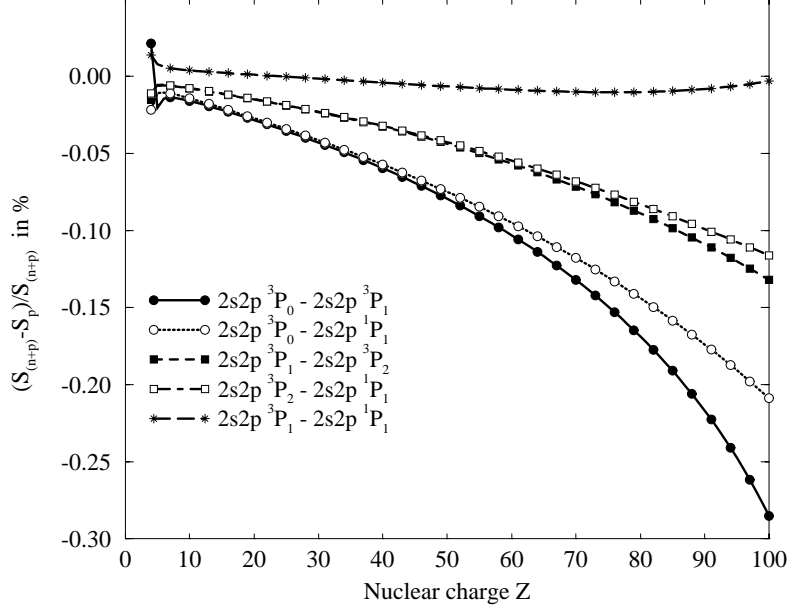


FIG. 3.  $Z$ -dependence of the ratio  $(S_{n+p} - S_p)/S_{n+p}$  in % for transitions into  $2s2p$  configuration

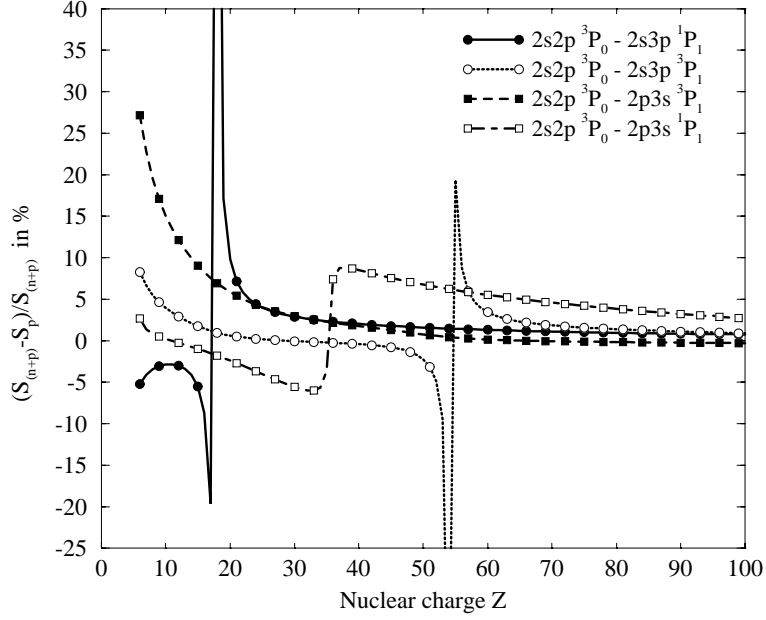


FIG. 4.  $Z$ -dependence of the ratio  $(S_{n+p} - S_p)/S_{n+p}$  in % for 2-3 transitions

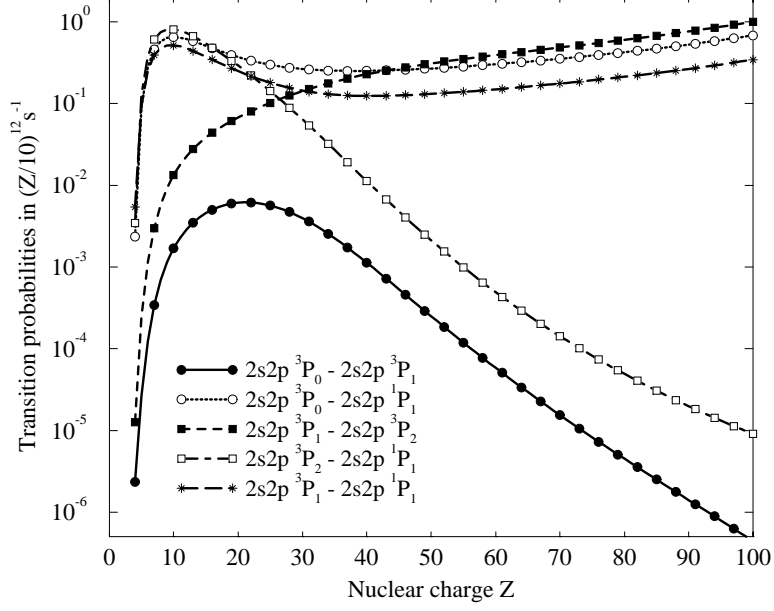


FIG. 5.  $Z$ -dependence of transition probabilities  $A$  into  $2s2p$  configuration.

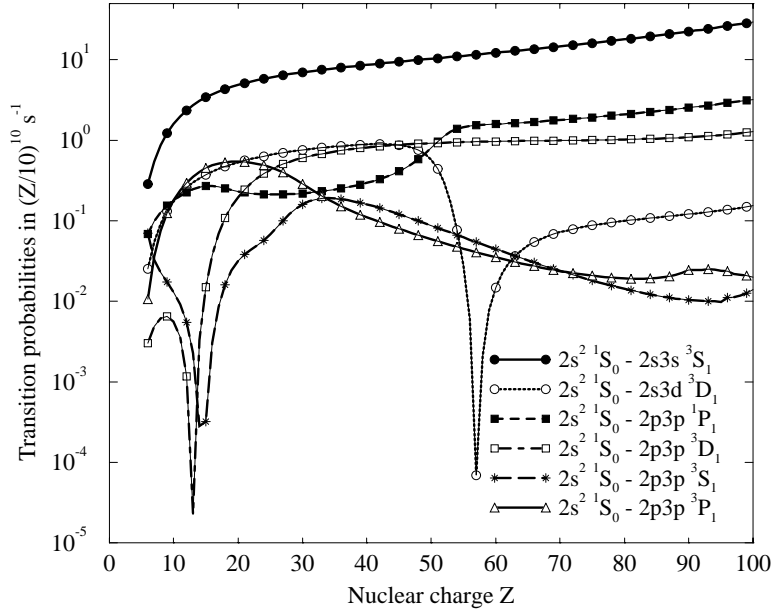


FIG. 6.  $Z$ -dependence of transition probabilities  $A$  between  $2s^2 \ ^1S_0$  levels and 6 even parity  $2l3l'$  levels with  $J=1$

TABLE I. Uncoupled  $M1$  reduced matrix for  $\text{Fe}^{+22}$ . Notation:  $np = np_{3/2}$ ,  $np^* = np_{1/2}$ ,  $3d = 3d_{5/2}$ ,  $3d^* = 3d_{3/2}$ 

(a) Coulomb Interaction:					
$vw[J]$	$v'w'[J']$	$M^{(1)}$	$P^{(\text{derv})}$	$M^{(\text{RPA})}$	$M^{(\text{corr})}$
$2s2p^*[0]$	$2s2p^*[1]$	1.154620	1.154620	-0.000023	0.000027
$2s2p^*[0]$	$2s2p[1]$	0.814777	0.814777	0.000065	-0.000016
$2s2p[1]$	$2s2p[1]$	2.848083	2.848083	0.000112	0.000137
$2p^*2p[2]$	$2p^*2p[2]$	6.365538	6.365538	0.000006	0.000800
$2p^*2p[2]$	$2p2p[2]$	-1.288276	-1.288275	-0.000103	-0.000059
$2p2p[2]$	$2p^*2p[2]$	-1.288276	-1.288275	-0.000103	-0.000059
$2p2p[2]$	$2p2p[2]$	7.279830	7.279830	0.000110	0.000598
$2s2s[0]$	$2s3s[1]$	0.003755	0.000018	0.000053	-0.000243
$2s2s[0]$	$2s3d^*[1]$	-0.001221	0.000694	0.000002	0.000131
$2p2p[0]$	$2p3p[1]$	0.005110	0.001239	-0.000024	-0.000325
$2s2p^*[1]$	$2s3p^*[0]$	-0.001347	-0.001787	-0.000023	-0.000102
$2s2p[1]$	$2p3d^*[0]$	-0.000611	0.000347	0.000001	-0.000051
(b) Breit Interaction with factor $10^3$					
$vw[J]$	$v'w'[J']$	$B^{(\text{HF})}$	$B^{(\text{RPA})}$	$B^{(\text{corr})}$	$B^{(2)}$
$2s2p^*[0]$	$2s2p^*[1]$	-0.214620	0.153805	-0.024369	-0.085184
$2s2p^*[0]$	$2s2p[1]$	-0.069154	0.015593	-0.039520	-0.093081
$2s2p[1]$	$2s2p[1]$	0.012768	0.102957	-0.095533	0.020192
$2p^*2p[2]$	$2p^*2p[2]$	0.395632	-0.210116	-0.398355	-0.212839
$2p^*2p[2]$	$2p2p[2]$	0.109343	-0.024654	-0.064385	0.020303
$2p2p[2]$	$2p^*2p[2]$	0.109343	-0.024654	-0.064385	0.020303
$2p2p[2]$	$2p2p[2]$	0.042419	0.131645	-0.155396	0.018668
$2s2s[0]$	$2s3s[1]$	-0.012301	0.031090	0.039747	0.058535
$2s2s[0]$	$2s3d^*[1]$	0.004822	-0.014542	-0.069244	-0.078964
$2p2p[0]$	$2p3p[1]$	-0.012286	-0.031559	0.093758	0.049912
$2s2p^*[1]$	$2s3p^*[0]$	0.111038	-0.090110	-0.101976	-0.081049
$2s2p[1]$	$2p3d^*[0]$	0.002411	-0.007271	0.013019	0.008159

TABLE II. Coupled reduced matrix elements  $M1$  for  $\text{Fe}^{+22}$ . Comparison  $M1$  - calculated with (a) and without (b) negative-energy component. Notation:  $np = np_{3/2}$ ,  $np^* = np_{1/2}$ ,  $3d = 3d_{5/2}$ ,  $3d^* = 3d_{3/2}$

$l_1 l_2 LSJ$	$l'_1 l'_2 L' S' J'$	$M1^a$	$M1^b$	$vw[J]$	$v'w'[J']$
$2s2p^3P_0$	$2s2p^3P_1$	1.396484	1.396742	$2s2p^*[0]$	$2s2p^*[1]$
$2s2p^3P_0$	$2s2p^1P_1$	0.215871	0.215909	$2s2p^*[0]$	$2s2p[1]$
$2s2p^3P_2$	$2s2p^3P_1$	1.559213	1.559367	$2s2p[2]$	$2s2p^*[1]$
$2s2p^3P_2$	$2s2p^1P_1$	0.242869	0.242893	$2s2p[2]$	$2s2p[1]$
$2s^2^1S_0$	$2s3s^3S_1$	0.003715	0.003701	$2s2s[0]$	$2s3s[1]$
$2s^2^1S_0$	$2s3d^3D_1$	0.001171	0.001161	$2s2s[0]$	$2s3d^*[1]$
$2p^2^3P_0$	$2s3s^3S_1$	0.001237	0.001271	$2p^*2p^*[0]$	$2s3s[1]$
$2p^2^3P_0$	$2p3p^3D_1$	0.006483	0.006535	$2p^*2p^*[0]$	$2p^*3p[1]$
$2p^2^3P_0$	$2p3p^3S_1$	0.001821	0.001835	$2p^*2p^*[0]$	$2p3p^*[1]$
$2p^2^3P_0$	$2p3p^3P_1$	0.002964	0.002915	$2p^*2p^*[0]$	$2p3p[1]$
$2p^2^1S_0$	$2p3p^1P_1$	0.000964	0.000997	$2p2p[0]$	$2p^*3p^*[1]$
$2p^2^1S_0$	$2p3p^3D_1$	0.001608	0.001629	$2p2p[0]$	$2p^*3p[1]$
$2p^2^1S_0$	$2p3p^3S_1$	0.001354	0.001321	$2p2p[0]$	$2p3p^*[1]$
$2s2p^3P_1$	$2s3p^3P_0$	0.001906	0.001932	$2s2p^*[1]$	$2s3p^*[0]$
$2s2p^3P_1$	$2p3s^3P_0$	0.002133	0.002062	$2s2p^*[1]$	$2p^*3s[0]$
$2s2p^3P_0$	$2s3p^1P_1$	0.001422	0.001396	$2s2p^*[0]$	$2s3p^*[1]$
$2s2p^3P_0$	$2p3s^3P_1$	0.003406	0.003340	$2s2p^*[0]$	$2p^*3s[1]$
$2s2p^3P_0$	$2p3s^1P_1$	0.001119	0.001143	$2s2p^*[0]$	$2p3s[1]$

TABLE III. Line strengths (S) as function of Z.

[LS designations]		Z = 10	Z = 20	Z = 40	Z = 60	Z = 80	Z = 100	[jj designations]	
$2s2p^3P_0$	$2s2p^3P_1$	2.00(0)	1.99(0)	1.72(0)	1.47(0)	1.39(0)	1.37(0)	$2s2p^*[0]$	$2s2p^*[1]$
$2s2p^3P_0$	$2s2p^1P_1$	6.73(-5)	9.67(-3)	2.77(-1)	5.10(-1)	5.67(-1)	5.50(-1)	$2s2p^*[0]$	$2s2p[1]$
$2s2p^3P_2$	$2s2p^3P_1$	2.50(0)	2.48(0)	2.12(0)	1.77(0)	1.62(0)	1.47(0)	$2s2p[2]$	$2s2p^*[1]$
$2s2p^3P_2$	$2s2p^1P_1$	8.76(-5)	1.23(-2)	3.49(-1)	6.45(-1)	7.26(-1)	7.24(-1)	$2s2p[2]$	$2s2p[1]$
$2s2p^3P_1$	$2s2p^1P_1$	5.42(-5)	7.42(-3)	1.81(-1)	2.84(-1)	2.98(-1)	2.83(-1)	$2s2p^*[1]$	$2s2p[1]$
$2p^2^3P_1$	$2p^2^3P_2$	2.50(0)	2.36(0)	1.11(0)	9.21(-1)	9.03(-1)	9.49(-1)	$2p^*2p[1]$	$2p^*2p[2]$
$2p^2^3P_1$	$2p^2^1D_2$	8.23(-4)	1.36(-1)	1.38(0)	1.55(0)	1.53(0)	1.43(0)	$2p^*2p[1]$	$2p2p[2]$
$2p^2^3P_1$	$2p^2^3P_0$	2.00(0)	1.97(0)	1.55(0)	1.35(0)	1.26(0)	1.15(0)	$2p^*2p[1]$	$2p^*2p^*[0]$
$2p^2^3P_1$	$2p^2^1S_0$	2.26(-4)	2.92(-2)	4.21(-1)	5.80(-1)	5.98(-1)	5.65(-1)	$2p^*2p[1]$	$2p2p[0]$
$2s^2^1S_0$	$2p^2^3P_1$	2.21(-6)	2.45(-4)	6.36(-3)	1.30(-2)	1.28(-2)	1.08(-2)	$2s2s[0]$	$2p^*2p[1]$
$2s^2^1S_0$	$2s3s^3S_1$	1.90(-7)	4.46(-6)	8.58(-5)	4.91(-4)	1.77(-3)	5.09(-3)	$2s2s[0]$	$2s3s[1]$
$2s^2^1S_0$	$2s3d^3D_1$	1.65(-8)	4.34(-7)	8.14(-6)	5.52(-7)	9.26(-6)	2.54(-5)	$2s2s[0]$	$2s3d^*[1]$
$2s^2^1S_0$	$2p3p^1P_1$	1.31(-8)	1.68(-7)	2.61(-6)	5.70(-5)	1.76(-4)	4.31(-4)	$2s2s[0]$	$2p^*3p^*[1]$
$2s^2^1S_0$	$2p3p^3D_1$	40.4(-9)	1.41(-7)	6.95(-6)	3.31(-5)	8.22(-5)	1.67(-4)	$2s2s[0]$	$2p^*3p[1]$
$2s^2^1S_0$	$2p3p^3S_1$	92.1(-9)	2.24(-8)	1.27(-6)	1.29(-6)	8.49(-7)	1.12(-6)	$2s2s[0]$	$2p3p^*[1]$
$2s^2^1S_0$	$2p3p^3P_1$	1.24(-8)	3.77(-7)	8.60(-7)	9.57(-7)	1.02(-6)	2.84(-7)	$2s2s[0]$	$2p3p[1]$
$2p^2^3P_0$	$2s3s^3S_1$	2.74(-8)	5.22(-7)	7.50(-6)	2.57(-5)	5.34(-5)	1.00(-4)	$2p^*2p^*[0]$	$2s3s[1]$
$2p^2^3P_0$	$2s3d^3D_1$	91.7(-9)	3.99(-9)	3.56(-7)	1.21(-4)	4.62(-4)	1.34(-3)	$2p^*2p^*[0]$	$2s3d^*[1]$
$2p^2^3P_0$	$2p3p^1P_1$	6.97(-8)	1.08(-6)	2.15(-5)	7.14(-6)	2.71(-6)	3.52(-6)	$2p^*2p^*[0]$	$2p^*3p^*[1]$
$2p^2^3P_0$	$2p3p^3D_1$	2.26(-8)	7.26(-6)	3.07(-4)	1.74(-3)	5.99(-3)	1.59(-2)	$2p^*2p^*[0]$	$2p^*3p[1]$
$2p^2^3P_0$	$2p3p^3S_1$	7.64(-7)	9.07(-6)	1.60(-6)	2.00(-6)	2.57(-6)	5.73(-6)	$2p^*2p^*[0]$	$2p3p^*[1]$
$2p^2^3P_0$	$2p3p^3P_1$	4.54(-9)	1.80(-6)	6.89(-6)	2.38(-6)	1.02(-6)	8.05(-9)	$2p^*2p^*[0]$	$2p3p[1]$
$2p^2^1S_0$	$2s3s^3S_1$	9.70(-9)	4.28(-8)	1320(-9)	8.94(-9)	6.26(-9)	3.87(-7)	$2p2p[0]$	$2s3s[1]$
$2p^2^1S_0$	$2s3d^3D_1$	3.41(-9)	4.52(-8)	4.27(-7)	4.69(-7)	1.92(-7)	9.40(-8)	$2p2p[0]$	$2s3d^*[1]$
$2p^2^1S_0$	$2p3p^1P_1$	1.18(-7)	8.32(-7)	5.45(-7)	2.89(-7)	5.82(-7)	1.10(-6)	$2p2p[0]$	$2p^*3p^*[1]$
$2p^2^1S_0$	$2p3p^3D_1$	656.(-9)	1.19(-6)	1.84(-6)	2.11(-7)	3.23(-8)	5.25(-7)	$2p2p[0]$	$2p^*3p[1]$
$2p^2^1S_0$	$2p3p^3S_1$	1.15(-7)	2.17(-7)	2.54(-4)	1.89(-3)	7.68(-3)	2.56(-2)	$2p2p[0]$	$2p3p^*[1]$
$2p^2^1S_0$	$2p3p^3P_1$	5.30(-7)	1.88(-5)	2.17(-4)	9.36(-4)	2.93(-3)	7.24(-3)	$2p2p[0]$	$2p3p[1]$
$2s2p^3P_1$	$2s3p^3P_0$	8.13(-8)	1.86(-6)	7.82(-7)	2.06(-5)	1.62(-4)	6.24(-4)	$2s2p^*[1]$	$2s3p^*[0]$
$2s2p^3P_1$	$2p3s^3P_0$	3.26(-8)	1.19(-6)	4.01(-5)	2.55(-4)	9.10(-4)	2.54(-3)	$2s2p^*[1]$	$2p^*3s[0]$
$2s2p^3P_1$	$2p3d^3P_0$	1.45(-9)	2.56(-8)	2.39(-7)	1.95(-7)	4.44(-8)	6.20(-7)	$2s2p^*[1]$	$2p3d^*[0]$
$2s2p^1P_1$	$2s3p^3P_0$	6.01(-7)	1.46(-5)	3.34(-4)	2.01(-3)	7.85(-3)	2.57(-2)	$2s2p[1]$	$2s3p^*[0]$
$2s2p^1P_1$	$2p3s^3P_0$	8.64(-8)	1.36(-6)	8.76(-6)	1.39(-5)	1.36(-5)	1.46(-6)	$2s2p[1]$	$2p^*3s[0]$
$2s2p^1P_1$	$2p3d^3P_0$	44.2(-9)	1.24(-8)	4.11(-7)	8.08(-6)	3.77(-5)	7.33(-5)	$2s2p[1]$	$2p3d^*[0]$
$2s2p^3P_0$	$2s3p^1P_1$	6.37(-8)	1.37(-7)	1.67(-5)	9.05(-5)	3.14(-4)	9.06(-4)	$2s2p^*[0]$	$2s3p^*[1]$
$2s2p^3P_0$	$2s3p^3P_1$	1.80(-7)	6.30(-6)	8.89(-5)	5.94(-5)	6.68(-4)	2.19(-3)	$2s2p^*[0]$	$2s3p[1]$
$2s2p^3P_0$	$2p3s^3P_1$	1.16(-7)	3.14(-6)	1.03(-4)	1.04(-3)	3.17(-3)	8.13(-3)	$2s2p^*[0]$	$2p^*3s[1]$
$2s2p^3P_0$	$2p3s^1P_1$	4.82(-8)	8.27(-7)	6.00(-6)	2.79(-5)	8.76(-5)	2.20(-4)	$2s2p^*[0]$	$2p3s[1]$
$2s2p^3P_0$	$2p3d^3D_1$	5.13(-9)	2.82(-7)	1.01(-6)	3.67(-7)	67.6(-9)	9.00(-7)	$2s2p^*[0]$	$2p^*3d^*[1]$
$2s2p^3P_0$	$2p3d^3P_1$	4.33(-9)	7.80(-8)	1.65(-7)	6.77(-8)	1.75(-9)	3.31(-6)	$2s2p^*[0]$	$2p3d^*[1]$
$2s2p^3P_0$	$2p3d^1P_1$	8.80(-9)	3.04(-8)	43.8(-9)	8.70(-9)	1.55(-7)	1.30(-8)	$2s2p^*[0]$	$2p3d[1]$

TABLE IV. Wavelengths  $\lambda$  ( $\text{\AA}$ ), line strengths  $S$  (a.u.), and transition probabilities  $A$  ( $\text{s}^{-1}$ ) for  $M1$  transition in  $1s^2 2s 2p$  configuration as function of  $Z$ . Comparison present calculations (a) with other theoretical results (b)- Glass (1983), (c)- Idrees *et al* (1989), (d)-Oboladze *et al* (1980).

$Z$	$\lambda^a$	$S^a$	$A^a$	$A^b$	$A^c$
Transition ${}^3P_0-{}^3P_1$					
6	4100000	2.00(0)	2.68(-7)	2.16(-7)	2.09(-7)
7	1560000	2.00(0)	4.74(-6)	4.28(-6)	4.17(-6)
8	725000	2.00(0)	4.75(-5)	4.50(-5)	4.37(-5)
9	381000	2.00(0)	3.25(-4)	3.10(-4)	3.08(-4)
10	220000	2.00(0)	1.69(-3)	1.63(-3)	1.63(-3)
11	135900	2.00(0)	7.15(-3)		6.97(-3)
12	88746	2.00(0)	2.57(-2)	2.52(-2)	2.52(-2)
13	60499	2.00(0)	8.12(-2)		
14	42764	2.00(0)	2.30(-1)	2.27(-1)	.
15	31160	2.00(0)	5.94(-1)		
16	23306	2.00(0)	1.42(+0)		
17	17832	2.00(0)	3.16(+0)		3 17(+0)
18	13920	1.99(0)	6.65(+0)	6.61(+0)	6.70(+0)
19	11062	1.99(0)	1.32(+1)		1 33(+1)
20	8934.2	1.99(0)	2.51(+1)	2.49(+1)	2.54(+1)
21	7321.4	1.99(0)	4.55(+1)		
22	6080.5	1.98(0)	7.92(+1)		8.07(+1)
23	5112.5	1.97(0)	1.33(+2)		
24	4347.8	1.97(0)	2.15(+2)		
25	3736.8	1.96(0)	3.38(+2)		
26	3243.5	1.95(0)	5.14(+2)	5.06(+2)	5.30(+2)
$Z$	$WL^a$	$S^a$	$A^a$	$A^b$	$A^c$
Transition ${}^3P_1-{}^3P_2$					
6	1754400	2.50(0)	2.50(-6)	2.14(-6)	2.23(-6)
7	689650	2.50(0)	4.11(-5)	3.76(-5)	3.82(-5)
8	325100	2.50(0)	3.92(-4)	3.71(-4)	3.72(-4)
9	173040	2.50(0)	2.60(-3)	2.46(-3)	2.50(-3)
10	100420	2.50(0)	1.33(-2)	1.28(-2)	1.29(-2)
11	62212	2.50(0)	5.60(-2)		5.45(-2)
12	40548	2.50(0)	2.02(-1)	1.96(-1)	1.97(-1)
13	27524	2.50(0)	6.46(-1)		
14	19314	2.50(0)	1.87(+0)	1.83(+0)	.
15	13934	2.50(0)	4.98(+0)		
16	10290	2.49(0)	1.24(+1)		
17	7751.8	2.49(0)	2.89(+1)		2 84(+1)
18	5941.5	2.49(0)	6.40(+1)	6.33(+1)	6.30(+1)
19	4623.0	2.49(0)	1.36(+2)		1 34(+2)
20	3645.0	2.48(0)	2.77(+2)	2.73(+2)	2.72(+2)
21	2907.9	2.48(0)	5.44(+2)		
22	2344.2	2.47(0)	1.04(+3)		1.02(+3)
23	1907.7	2.46(0)	1.91(+3)		
24	1565.8	2.45(0)	3.45(+3)		
25	1295.3	2.44(0)	6.07(+3)		
26	1079.1	2.43(0)	1.04(+4)	1.05(+4)	1.03(+4)
$Z$	$WL^a$	$S^a$	$A^a$	$A^b$	$A^c$
Transition ${}^3P_2-{}^1P_1$					
6	2095.9	8.48(-7)	8.28(-4)	1.98(-3)	2.75(-3)
7	1628.0	4.04(-6)	8.42(-3)	1.63(-2)	1.98(-2)
8	1336.6	1.37(-5)	5.14(-2)	8.77(-2)	9.59(-2)
9	1136.5	3.73(-5)	2.28(-1)	3.41(-1)	3.81(-1)
10	990.07	8.76(-5)	8.11(-1)	1.15(+0)	1.26(+0)
11	878.24	1.84(-4)	2.45(+0)		3.56(+0)
12	790.03	3.56(-4)	6.49(+0)	8.53(+0)	8.97(+0)
13	718.75	6.42(-4)	1.56(+1)		2.07(+1)
14	660.03	1.10(-3)	3.43(+1)	4.29(+1)	4.38(+1)

15	610.89	1.79(-3)	7.04(+1)		8.74(+1)
16	569.25	2.79(-3)	1.36(+2)		1.65(+2)
17	533.58	4.22(-3)	2.50(+2)		2.97(+2)
18	502.76	6.19(-3)	4.38(+2)	5.12(+2)	5.10(+2)
19	475.92	8.83(-3)	7.37(+2)		8.43(+2)
20	452.39	1.23(-2)	1.19(+3)	1.36(+3)	1.34(+3)
21	431.63	1.67(-2)	1.87(+3)		2.07(+3)
22	413.21	2.23(-2)	2.84(+3)		3.10(+3)
23	396.79	2.92(-2)	4.20(+3)		4.53(+3)
24	382.07	3.75(-2)	6.05(+3)		6.42(+3)
25	368.81	4.74(-2)	8.50(+3)		8.90(+3)
26	356.81	5.90(-2)	1.17(+4)	1.24(+4)	1.21(+4)
Z	$WL^a$	$S^a$	$A^a$	$A^b$	$A^d$
Transition ${}^3P_0^{-1}P_1$					
6	2092.3	6.23(-7)	6.12(-4)	1.59(-3)	2.21(-3)
7	1622.4	3.02(-6)	6.36(-3)	1.32(-2)	1.60(-2)
8	1328.7	1.04(-5)	3.97(-2)	7.13(-2)	7.82(-2)
9	1125.7	2.85(-5)	1.79(-1)	2.80(-1)	3.14(-1)
10	976.05	6.73(-5)	6.51(-1)	9.57(-1)	1.06(+0)
11	860.53	1.42(-4)	2.01(+0)		3.03(+0)
12	768.23	2.76(-4)	5.47(+0)	7.40(+0)	7.81(+0)
13	692.44	4.99(-4)	1.35(+1)		1.85(+1)
14	628.83	8.55(-4)	3.09(+1)	3.95(+1)	4.06(+1)
15	574.44	1.39(-3)	6.62(+1)		8.43(+1)
16	527.20	2.19(-3)	1.34(+2)		1.66(+2)
17	485.63	3.31(-3)	2.60(+2)		3.16(+2)
18	448.60	4.86(-3)	4.84(+2)	5.76(+2)	7.19(+2)
19	415.30	6.94(-3)	8.71(+2)		1.02(+3)
20	385.10	9.67(-3)	1.52(+3)	1.76(+3)	1.75(+3)
21	357.49	1.32(-2)	2.59(+3)		2.93(+3)
22	332.10	1.76(-2)	4.32(+3)		4.80(+3)
23	308.64	2.30(-2)	7.04(+3)		7.73(+3)
24	286.87	2.96(-2)	1.13(+4)		1.22(+4)
25	266.59	3.75(-2)	1.78(+4)		1.90(+4)
26	247.67	4.66(-2)	2.76(+4)	3.03(+4)	2.91(+4)
Z	$WL^a$	$S^a$	$A^a$	$A^b$	$A^d$
Transition ${}^3P_1^{-1}P_1$					
6	2093.4	5.63(-7)	5.52(-4)	5.68(-3)	1.66(-3)
7	1624.1	2.58(-6)	5.42(-3)	3.72(-2)	1.20(-2)
8	1331.1	8.59(-6)	3.28(-2)	1.63(-1)	5.83(-2)
9	1129.0	2.32(-5)	1.45(-1)	5.49(-1)	2.33(-1)
10	980.40	5.42(-5)	5.17(-1)	1.63(+0)	7.81(-1)
11	866.01	1.13(-4)	1.57(+0)		2.23(+0)
12	774.94	2.18(-4)	4.22(+0)	2.05(+0)	5.71(+0)
13	700.46	3.93(-4)	1.03(+1)		1.34(+1)
14	638.22	6.69(-4)	2.32(+1)	1.36(+1)	2.91(+1)
15	585.23	1.09(-3)	4.88(+1)		5.98(+1)
16	539.41	1.70(-3)	9.74(+1)		1.16(+2)
17	499.22	2.56(-3)	1.85(+2)		2.18(+2)
18	463.54	3.75(-3)	3.39(+2)	2.33(+2)	3.91(+2)
19	431.50	5.34(-3)	5.98(+2)		6.80(+2)
20	402.44	7.42(-3)	1.02(+3)	1.02(+3)	1.03(+3)
21	375.84	1.01(-2)	1.71(+3)		1.89(+3)
22	351.29	1.34(-2)	2.78(+3)		3.04(+3)
23	328.47	1.75(-2)	4.44(+3)		4.81(+3)
24	307.13	2.24(-2)	6.95(+3)		7.46(+3)
25	287.07	2.82(-2)	1.07(+4)		1.14(+4)
26	268.14	3.48(-2)	1.63(+4)	1.23(+4)	1.72(+4)

TABLE V. Wavelengths  $\lambda$  ( $\text{\AA}$ ), line strengths  $S$  (a.u.), and transition probabilities  $A$  ( $\text{s}^{-1}$ ) for  $M1$  transition in  $1s^2 2p^2$  and  $1s^2 2s^2$  configurations as function of  $Z$ . Comparison present calculations (a) with other theoretical results (b)- Glass (1983), (c)-Oboladze *et al* (1980).

$Z$	$\lambda^a$	$S^a$	$A^a$	$A^b$	
Transition $2s^2 \ ^1S_0-2p^2 \ ^3P_1$					
6	730.61	2.83(-8)	6.53(-4)	5.33(-4)	
7	570.81	1.20(-7)	5.82(-3)	4.63(-3)	
8	468.81	3.78(-7)	3.30(-2)	2.69(-2)	
9	397.75	9.79(-7)	1.40(-1)	1.17(-1)	
10	345.24	2.21(-6)	4.83(-1)	4.41(-1)	
11	304.72	4.49(-6)	1.43(+0)		
12	272.44	8.43(-6)	3.75(+0)	5.06(+0)	
13	246.04	1.48(-5)	8.94(+0)		
14	223.99	2.47(-5)	1.97(+1)	2.55(+1)	
15	205.27	3.93(-5)	4.09(+1)		
16	189.12	6.02(-5)	8.00(+1)		
17	175.02	8.91(-5)	1.49(+2)		
18	162.57	1.28(-4)	2.68(+2)	3.36(+2)	
19	151.48	1.79(-4)	4.63(+2)		
20	141.50	2.45(-4)	7.77(+2)	8.91(+2)	
21	132.48	3.27(-4)	1.26(+3)		
22	124.25	4.29(-4)	2.01(+3)		
23	116.71	5.52(-4)	3.12(+3)		
24	109.77	6.98(-4)	4.75(+3)		
25	103.34	8.70(-4)	7.09(+3)		
26	97.37	1.07(-3)	1.04(+4)	1.36(+4)	
$Z$	$WL^a$	$S^a$	$A^a$	$A^b$	$A^c$
Transition $2p^2 \ ^3P_0-2p^2 \ ^3P_1$					
6	3410000	2.00(+0)	4.52(-7)	4.09(-7)	4.4 (-7)
7	1350000	2.00(+0)	7.29(-6)	6.85(-6)	7.0 (-6)
8	637800	2.00(+0)	6.93(-5)	6.61(-5)	6.82(-5)
9	338100	2.00(+0)	4.65(-4)	4.47(-4)	4.56(-4)
10	194700	2.00(+0)	2.44(-3)	2.36(-3)	2.40(-3)
11	119400	2.00(+0)	1.06(-2)		1.05(-2)
12	76750	2.00(+0)	3.97(-2)	4.04(-2)	3.93(-2)
13	51260	2.00(+0)	1.33(-1)		1.32(-1)
14	35290	2.00(+0)	4.08(-1)	4.27(-1)	3.82(-1)
15	24920	1.99(+0)	1.16(+0)		1.14(+0)
16	17960	1.99(+0)	3.09(+0)		3.04(+0)
17	13190	1.99(+0)	7.79(+0)		7.66(+0)
18	9829.4	1.98(+0)	1.88(+1)	2.09(+1)	1.84(+1)
19	7427.9	1.97(+0)	4.33(+1)		4.23(+1)
20	5682.2	1.97(+0)	9.64(+1)	1.10(+2)	9.37(+1)
21	4395.7	1.96(+0)	2.07(+2)		2.00(+2)
22	3435.9	1.94(+0)	4.31(+2)		4.14(+2)
23	2712.0	1.93(+0)	8.69(+2)		8.32(+2)
24	2160.5	1.91(+0)	1.70(+3)		1.62(+3)
25	1736.5	1.89(+0)	3.25(+3)		3.07(+3)
26	1407.7	1.87(+0)	6.03(+3)	7.89(+3)	5.67(+3)
$Z$	$WL^a$	$S^a$	$A^a$	$A^b$	$A^c$
Transition $2p^2 \ ^3P_1-2p^2 \ ^1S_0$					
6	2311.2	2.14(-6)	4.68(-3)	1.92(-2)	2.0 (-2)
7	1718.2	1.04(-5)	5.51(-2)	1.57(-1)	1.5 (-1)
8	1370.9	3.52(-5)	3.69(-1)	8.31(-1)	8.0 (-1)
9	1141.6	9.63(-5)	1.75(+0)	3.36(+0)	3.21(+0)
10	978.25	2.26(-4)	6.52(+0)	1.13(+1)	1.09(+1)
11	855.58	4.75(-4)	2.05(+1)		3.16(+1)
12	759.81	9.15(-4)	5.62(+1)	7.34(+1)	8.18(+1)
13	682.74	1.64(-3)	1.39(+2)		1.93(+2)
14	619.18	2.79(-3)	3.17(+2)	3.97(+2)	4.22(+2)

15	565.66	4.51(-3)	6.72(+2)		8.16(+2)
16	519.80	7.00(-3)	1.34(+3)		1.68(+3)
17	479.87	1.05(-2)	2.55(+3)		3.11(+3)
18	444.62	1.52(-2)	4.65(+3)	5.58(+3)	5.53(+3)
19	413.10	2.13(-2)	8.15(+3)		9.49(+3)
20	384.59	2.92(-2)	1.38(+4)	1.69(+4)	1.58(+4)
21	358.54	3.89(-2)	2.28(+4)		2.55(+4)
22	334.51	5.07(-2)	3.65(+4)		4.02(+4)
23	312.19	6.47(-2)	5.73(+4)		6.20(+4)
24	291.31	8.07(-2)	8.81(+4)		9.31(+4)
25	271.70	9.88(-2)	1.33(+5)		1.39(+5)
26	253.22	1.19(-1)	1.97(+5)	2.31(+5)	2.03(+5)
Z	WL <sup>a</sup>	S <sup>a</sup>	A <sup>a</sup>	A <sup>b</sup>	A <sup>c</sup>
Transition $2p^2\ ^3P_1-2p^2\ ^3P_2$					
6	1980000	2.50(+0)	1.73(-6)	1.32(-6)	1.8 (-6)
7	782500	2.50(+0)	2.81(-5)	2.44(-5)	2.25(-5)
8	367100	2.50(+0)	2.73(-4)	2.46(-4)	2.89(-4)
9	194500	2.50(+0)	1.83(-3)	1.69(-3)	1.93(-3)
10	112750	2.50(+0)	9.40(-3)	8.38(-3)	9.82(-3)
11	69974	2.50(+0)	3.93(-2)		4.05(-2)
12	45859	2.50(+0)	1.40(-1)	1.37(-1)	1.42(-1)
13	31417	2.49(+0)	4.34(-1)		4.39(-1)
14	22339	2.49(+0)	1.20(+0)	1.20(+0)	1.21(+0)
15	16400	2.48(+0)	3.03(+0)		3.03(+0)
16	12382	2.47(+0)	7.02(+0)		6.96(+0)
17	9584.9	2.45(+0)	1.50(+1)		1.48(+1)
18	7590.8	2.43(+0)	3.00(+1)	3.09(+1)	2.93(+1)
19	6139.3	2.40(+0)	5.60(+1)		5.43(+1)
20	5063.8	2.36(+0)	9.81(+1)	1.01(+2)	9.48(+1)
21	4254.3	2.31(+0)	1.62(+2)		1.55(+2)
22	3636.6	2.25(+0)	2.52(+2)		2.41(+2)
23	3159.0	2.17(+0)	3.72(+2)		3.53(+2)
24	2785.2	2.09(+0)	5.21(+2)		4.94(+2)
25	2488.9	2.00(+0)	6.98(+2)		6.61(+2)
26	2251.1	1.90(+0)	8.99(+2)	9.58(+2)	8.51(+2)
Z	WL <sup>a</sup>	S <sup>a</sup>	A <sup>a</sup>	A <sup>b</sup>	A <sup>c</sup>
Transition $2p^2\ ^3P_1-2p^2\ ^1D_2$					
6	15939.	8.74(-6)	1.16(-5)	1.98(-4)	2.7 (-4)
7	8640.4	3.98(-5)	3.33(-4)	2.05(-3)	2.41(-3)
8	5989.1	1.31(-4)	3.30(-3)	1.25(-2)	1.40(-2)
9	4597.2	3.53(-4)	1.96(-2)	5.61(-2)	6.10(-2)
10	3730.5	8.23(-4)	8.55(-2)	2.04(-1)	2.16(-1)
11	3133.9	1.73(-3)	3.03(-1)		6.67(-1)
12	2694.7	3.36(-3)	9.27(-1)	1.72(+0)	1.83(+0)
13	2355.1	6.13(-3)	2.53(+0)		4.60(+0)
14	2082.5	1.06(-2)	6.34(+0)	1.05(+1)	1.08(+1)
15	1856.8	1.76(-2)	1.49(+1)		2.39(+1)
16	1665.2	2.82(-2)	3.29(+1)		5.06(+1)
17	1498.9	4.37(-2)	6.99(+1)		1.03(+2)
18	1351.9	6.56(-2)	1.43(+2)	2.09(+2)	2.03(+2)
19	1220.0	9.57(-2)	2.84(+2)		3.89(+2)
20	1100.3	1.36(-1)	5.50(+2)	7.72(+2)	7.05(+2)
21	990.68	1.87(-1)	1.04(+3)		1.32(+3)
22	890.01	2.50(-1)	1.92(+3)		2.36(+3)
23	797.48	3.25(-1)	3.46(+3)		4.11(+3)
24	712.68	4.09(-1)	6.10(+3)		6.87(+3)
25	635.36	5.00(-1)	1.05(+4)		1.17(+4)
26	565.33	5.93(-1)	1.77(+4)	2.22(+4)	1.92(+4)
Z	WL <sup>a</sup>	S <sup>a</sup>	A <sup>a</sup>	A <sup>b</sup>	A <sup>c</sup>

Transition $2p^2\ ^3P_2-2p^2\ ^1D_2$					
6	16068.	2.80(-5)	3.64(-5)	6.30(-4)	8.1(-4)
7	8736.9	1.25(-4)	1.01(-3)	6.59(-3)	7.0(-3)
8	6088.4	4.09(-4)	9.79(-3)	3.97(-2)	4.02(-2)
9	4708.4	1.09(-3)	5.65(-2)	1.74(-1)	1.71(-1)
10	3858.2	2.54(-3)	2.38(-1)	6.09(-1)	5.89(-1)
11	3280.8	5.31(-3)	8.12(-1)		1.75(+0)
12	2862.9	1.03(-2)	2.36(+0)	3.99(+0)	4.57(+0)
13	2546.0	1.87(-2)	6.11(+0)		1.09(+1)
14	2296.6	3.23(-2)	1.44(+1)	2.19(+1)	2.12(+1)
15	2093.9	5.33(-2)	3.13(+1)		4.97(+1)
16	1923.9	8.48(-2)	6.42(+1)		9.74(+1)
17	1776.7	1.30(-1)	1.25(+2)		1.82(+2)
18	1644.8	1.94(-1)	2.35(+2)	3.19(+2)	3.29(+2)
19	1522.6	2.79(-1)	4.27(+2)		5.73(+2)
20	1405.7	3.89(-1)	7.55(+2)	1.00(+3)	9.50(+2)
21	1291.4	5.24(-1)	1.31(+3)		1.63(+3)
22	1178.4	6.81(-1)	2.25(+3)		2.69(+3)
23	1066.8	8.54(-1)	3.79(+3)		4.37(+3)
24	957.74	1.03(+0)	6.34(+3)		6.84(+3)
25	853.16	1.20(+0)	1.05(+4)		1.12(+4)
26	754.92	1.36(+0)	1.71(+4)	1.95(+4)	1.78(+4)