Joint Power and Bandwidth Allocation in Multihop Wireless Networks

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***Abstract*—This paper considers power and bandwidth allocation to maximize the end-to-end rate in a multihop wireless network. Assuming an orthogonal frequency division multiplexing (OFDM) system, we formulate an optimization problem for joint power and subcarrier allocation in a network with one destination and multiple sources and relays. We then focus on low-complexity algorithms for the special case of a multihop network with only one source. In particular, we develop an algorithm for a two-hop network based upon the observation that the optimal frequency allocation in a two-hop network has a two-band structure under certain conditions. For a network with more than two hops, we propose a greedy approach to subcarrier allocation. Simulation results suggest that the performance of the proposed algorithms closely follow the optimum performance. Moreover, our results suggest that more hops do not always improve the end-to-end spectral efficiency for frequency-selective fading channels.**

I. Introduction

Multihop transmission is being considered to improve coverage and increase throughput for the next generation of wireless networks, *e.g.*, 802.16j mobile multihop relay networks. The addition of relay stations introduces new interactions among layers, *e.g.*, the physical layer, the medium access control layer, and the networking layer. This paper focuses on allocating system resources, *e.g.*, power and bandwidth, for the uplink in a wireless network based upon orthogonal frequency division multiplexing (OFDM).

A. Related Work and Motivation

In [1], an algorithm for joint multiuser OFDM power, subcarrier and rate allocation is developed to minimize the total power consumption. In [2], the problem of assigning discrete frequency bins in a Gaussian multiple-access channel with intersymbol interference (ISI) to maximize a weighed sum rate is studied, and a practical low-complexity algorithm is proposed. Relative to [1] and [2], this paper considers a multihop network, which adds flow conservation constraints to the problem formulation. As we will see, the addition of flow conservation constraints makes the optimization problem much more involved.

In [3]–[5], the trade-off between power and bandwidth is studied for a multihop wireless network assuming a narrowband system. Even though [6] considers a wideband multihop wireless network, a frequency subband is *shared* in a time division fashion by different hops along a path. By contrast, this paper considers allocating frequency bands *exclusively* to different hops along the path, *i.e.*, frequency bands assigned to different hops do not overlap. Furthermore, this paper also considers power allocation jointly with subcarrier allocation.

The remainder of the paper is organized as follows. Section II describes the network and channel models. Section III formulates the joint power and subcarrier allocation problem for wireless networks with one destination and multiple sources and relays. Not surprisingly, it turns out that this problem is a combinatorial optimization problem. To gain some insight about the optimum solutions, we relax the problem to a convex optimization problem by adjusting the requirement on the subcarrier allocation. The relaxed problem provides an upper bound to the solution of the original problem. Moreover, Section IV discusses the Karush-Kuhn-Tucker (KKT) conditions for optimal solutions to the relaxed problem and provides important insights that motivate low-complexity algorithms for solving the original problem in Section V. Specifically, Section V-A proposes a high-SNR approximation algorithm based on the observation that the optimal frequency allocation in a two-hop network has a two-band structure under certain conditions. Section V-B proposes a low-complexity greedy algorithm for a network with more than two hops. Section VI provides numerical simulation results, and Section VII concludes the paper.

II. System Model

In this paper, we formulate a power and subcarrier allocation problem for a wireless network with one base station (BS) and multiple sources and relays. A special case of this type of network is the multihop network with one source, one destination, and one or multiple relays. We assume relay stations do not have their own information to send and the BS is the only information sink for all sources. We represent the network as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the set of nodes $\mathcal{V}$ represents stations in the network and the set of links $\mathcal{E}$ represents potential communication between two stations in the network. For each link $e \in \mathcal{E}$, we use $t(e)$ to represent the transmit end of the link and $r(e)$ to represent the receive end. We denote by $\mathcal{S}$ the set of nodes with information sources and by $\mathcal{R}$ the set of relay stations. We assume that sources do

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not relay information, *i.e.*, $\mathcal{S}\bigcap\mathcal{R}=\emptyset$. The sink is denoted d. Hence, $t(e)\neq d, \forall e\in\mathcal{E}$ and $d\notin\mathcal{R}\bigcup\mathcal{S}$.

We model a frequency-selective fading channel by an M-tap filter model [7]. We denote the filter coefficient for m^{th} tap on link l as $\hat{h}_{l,m}$ and assume $\hat{h}_{l,m}$ to be independently Rayleigh distributed. We consider an OFDM-type system in which the whole frequency band is split into multiple subcarriers to share among different transmissions. We neglect the impact of cyclic prefix of OFDM on transmission rates. We denote by $\mathcal{K}$ the set of available frequency subcarriers. The bandwidth of a subcarrier $k\in\mathcal{K}$ is W_k Hz. Note that in practice, the bandwidth of all subcarriers is often identical. We denote by $h_{l,k}, l\in\mathcal{E}, k\in\mathcal{K}$, the fading coefficient of the kth subcarrier for link l and

$$h_{l,k}=\sum_{m=0}^{M-1}\hat{h}_{l,m}e^{-2\pi mk/K}, \tag{1}$$

where $K=|\mathcal{K}|$ is the total number of subcarriers. We assume fading coefficients of all links are available to a central scheduler to allow for subcarrier allocation. We also assume fading coefficients are available to transmitting nodes for power allocation.

Following [8], we assume that the nodes cannot transmit and receive at the same time at the same frequency. Furthermore, we consider no frequency reuse inside the cell, *i.e.*, in the network we study, any part of the bandwidth cannot be assigned to more than one link.

Each transmit node $i\in\mathcal{V}$ has a total power constraint of P_i, *i.e.*,

$$\sum_{\{l:t(l)=i\}}\sum_{k\in\mathcal{K}}P_{l,k}\leq P_i, \tag{2}$$

where $P_{l,k}$ denotes the allocated power on subcarrier $k\in\mathcal{K}$ of link $l\in\mathcal{E}$.

For simplicity of presentation, we assume the one-sided noise power spectral density N_0 is equal to unity for all subcarriers and all nodes.

III. Uplink Problem Formulation

The goal of the optimization is to maximize the total information received by the sink from the sources subject to transmit power and subcarrier constraints. Furthermore, we impose a flow conservation constraint to reflect the fact that the rate of a multihop route is limited by the rate of the bottleneck link, *i.e.*, the link that has the minimal rate among all intermediate hops [3], [4]. As we will see in the sequel, the problem of allocating power and subcarrier optimally becomes much more involved due to this constraint.

Specifically, we formulate the following optimization problem:

$$C^* \quad = \quad \max\sum_{k\in\mathcal{K}}\sum_{\{l:t(l)\in\mathcal{S}\}}C_{l,k}, \tag{3}$$

subject to the following constraints:

$$C_{l,k}\leq W_{l,k}\log(1+|h_{l,k}|^2P_{l,k}/W_{l,k}), \forall l\in\mathcal{E}, \forall k\in\mathcal{K}, \tag{4a}$$

$$0\leq C_{l,k}, \forall l\in\mathcal{E}, \forall k\in\mathcal{K}, \tag{4b}$$

$$0\leq W_{l,k}\leq W_k, \forall k\in\mathcal{K}, \tag{4c}$$

$$W_{l,k}W_{j,k}=0, l\neq j, \forall k\in\mathcal{K}, \tag{4d}$$

$$\sum_{\{l:t(l)=i\}}\sum_{k\in\mathcal{K}}P_{l,k}\leq P_i, \forall i\in\mathcal{V}, \tag{4e}$$

$$0\leq P_{l,k}, \forall l\in\mathcal{E}, \forall k\in\mathcal{K}, \tag{4f}$$

$$\sum_{k\in\mathcal{K}}\sum_{\{l:t(l)=i\}}C_{l,k}\geq\sum_{k\in\mathcal{K}}\sum_{\{l:r(l)=i\}}C_{l,k}, \forall i\in\mathcal{R}, \tag{4g}$$

where: $C_{l,k}$ is the information rate transmitted on link l at subcarrier k; $W_{l,k}$ denotes whether subcarrier k is used on link l; and $P_{l,k}$ denotes the transmit power of link l at subcarrier k. Constraint (4g) captures the flow conservation constraint at the relay stations, *i.e.*, the rate of flow arriving at the node must be no larger than the rate of flow departing the node. Under this constraint, the rate of a multihop route is limited by the minimum of the rates of intermediate hops, consistent with the information theoretic studies in [3], [4]. Constraint (4e) represents the transmit power constraint at a node, and constraint (4c) reflects the bandwidth constraint of subcarrier k. Finally, constraint (4d) reflects the practical constraint that each subcarrier is allocated to only one link. Constraints (4c) and (4d) suggest that $W_{l,k}$ is either 0 or W_k. We note that a solution to this problem also provides optimal routes if the network graph is fully connected.

Unfortunately, the combination of (4c) and (4d) makes the problem intractable for a large number of subcarriers and links due to its combinatorial nature. In order to utilize convex optimization techniques to gain some insight on the problem, we relax the frequency constraint and allow a subcarrier to be shared by multiple links, *i.e.*, $W_{l,k}$ can take continuous value between 0 and W_k. This relaxation allows further division of each subcarrier's bandwidth. The relaxed problem is

$$C^* \quad = \quad \max\sum_{k\in\mathcal{K}}\sum_{\{l:t(l)\in\mathcal{S}\}}C_{l,k}, \tag{5}$$

subjected to the following constraints:

$$C_{l,k}\leq W_{l,k}\log(1+|h_{l,k}|^2P_{l,k}/W_{l,k}), \forall l\in\mathcal{E}, \forall k\in\mathcal{K}, \tag{6a}$$

$$0\leq C_{l,k}, \forall l\in\mathcal{E}, \forall k\in\mathcal{K}, \tag{6b}$$

$$0\leq W_{l,k}\leq W_k, \forall k\in\mathcal{K}, \tag{6c}$$

$$\sum_{l\in\mathcal{E}}W_{l,k}\leq W_k, \forall k\in\mathcal{K}, \tag{6d}$$

$$\sum_{\{l:t(l)=i\}}\sum_{k\in\mathcal{K}}P_{l,k}\leq P_i, \forall i\in\mathcal{V}, \tag{6e}$$

$$0\leq P_{l,k}, \forall l\in\mathcal{E}, \forall k\in\mathcal{K}, \tag{6f}$$

$$\sum_{k\in\mathcal{K}}\sum_{\{l:t(l)=i\}}C_{l,k}\geq\sum_{k\in\mathcal{K}}\sum_{\{l:r(l)=i\}}C_{l,k}, \forall i\in\mathcal{R}. \tag{6g}$$

We note that (6d) ensures that frequency is not reused, *i.e.*, there is no interference. Thus, (6a) holds.

It can be shown that the new problem is a convex optimization problem. Hence, standard optimization tools such as the KKT conditions can be employed, leading to insights on optimal solutions that can help design suboptimum, yet efficient, numerical algorithms. We note that the constraint set of the new problem, *e.g.*, frequency constraints, is no smaller than that of the original problem; thus, a solution to the new problem provides an upper bound on a solution of the original problem (3).

IV. KKT Conditions After the Relaxation

The Lagrangian for the relaxed optimization problem (5) is

$$\begin{aligned} L &= -\sum_{k\in\mathcal{K}}\sum_{\{l:t(l)\in\mathcal{S}\}} C_{l,k} \\ &+ \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{E}} \lambda_{l,k}[C_{l,k} - W_{l,k}\log(1+|h_{l,k}|^2 P_{l,k}/W_{l,k})] \\ &+ \sum_{k\in\mathcal{K}} \nu_k[-W_k + \sum_{l\in\mathcal{E}} W_{l,k}] \\ &+ \sum_{i\in\mathcal{V}} \tau_i[-P_i + \sum_{k\in\mathcal{K}}\sum_{\{l:t(l)=i\}} P_{l,k}] \\ &+ \sum_{i\in\mathcal{R}} \beta_i[-\sum_{k\in\mathcal{K}}\sum_{\{l:t(l)=i\}} C_{l,k} + \sum_{k\in\mathcal{K}}\sum_{\{l:r(l)=i\}} C_{l,k}]. \end{aligned}$$

where $\lambda_{l,k}$, ν_k, τ_i and β_i are non-negative Lagrange multipliers.

The partial derivatives of the Lagrangian with respect to the optimization parameters, *i.e.*, rate, power and bandwidth, are as follows:

$$\frac{\partial L}{\partial C_{l,k}} = \begin{cases} -\beta_{t(l)} + \beta_{r(l)} + \lambda_{l,k}, & t(l)\notin\mathcal{S}, r(l)\neq d, \\ -\beta_{t(l)} + \lambda_{l,k} & t(l)\notin\mathcal{S}, r(l)=d, \\ -1 + \beta_{r(l)} + \lambda_{l,k} & t(l)\in\mathcal{S}, r(l)\neq d, \\ -1 + \lambda_{l,k} & t(l)\in\mathcal{S}, r(l)=d, \end{cases} \quad (7)$$

$$\frac{\partial L}{\partial P_{l,k}} = -\log e \frac{\lambda_{l,k}|h_{l,k}|^2 W_{l,k}}{W_{l,k} + P_{l,k}|h_{l,k}|^2} + \tau_{t(l)}, \quad (8)$$

$$\begin{aligned} \frac{\partial L}{\partial W_{l,k}} &= -\lambda_{l,k}\log(1+\frac{P_{l,k}|h_{l,k}|^2}{W_{l,k}}) \\ &\quad + \log e \frac{\lambda_{l,k}|h_{l,k}|^2 P_{l,k}}{W_{l,k} + |h_{l,k}|^2 P_{l,k}} + \nu_k, \end{aligned} \quad (9)$$

Therefore, necessary conditions for an optimal solution $(C^*_{l,k}, P^*_{l,k}, W^*_{l,k})$ are that:

$$\frac{\partial L}{\partial C_{l,k}}|_{(C^*_{l,k},P^*_{l,k},W^*_{l,k})} \quad \begin{cases} = 0 & \text{for } C^*_{l,k} > 0, \\ \geq 0 & \text{for } C^*_{l,k} = 0 \end{cases} \quad (10)$$

$$\frac{\partial L}{\partial P_{l,k}}|_{(C^*_{l,k},P^*_{l,k},W^*_{l,k})} \quad \begin{cases} = 0 & \text{for } P^*_{l,k} > 0, \\ \geq 0 & \text{for } P^*_{l,k} = 0, \end{cases} \quad (11)$$

$$\frac{\partial L}{\partial W_{l,k}}|_{(C^*_{l,k},P^*_{l,k},W^*_{l,k})} \quad \begin{cases} = 0 & \text{for } W^*_{l,k} > 0, \\ \geq 0 & \text{for } W^*_{l,k} = 0, \\ \leq 0 & \text{for } W^*_{l,k} = W_k, \end{cases} \quad (12)$$

and

$$\begin{aligned} \lambda_{l,k} &\geq 0, \\ \beta_i &\geq 0, \\ \nu_i &\geq 0, \\ \tau_i &\geq 0. \end{aligned}$$

From (12), if a subcarrier k is assigned exclusively to a link l, *i.e.*, $W^*_{l,k} = W_k$, the necessary condition is

$$\begin{aligned} \nu_k &\leq \lambda_{l,k}\log(1+\frac{P^*_{l,k}|h_{l,k}|^2}{W^*_{l,k}}) \\ &\quad - \log e \frac{\lambda_{l,k}|h_{l,k}|^2 P^*_{l,k}}{W^*_{l,k} + |h_{l,k}|^2 P^*_{l,k}}. \end{aligned} \quad (13)$$

An important observation from (13) is that its left hand side only depends on the subcarrier index k; hence, only the link with the largest right hand side can use the subcarrier exclusively. Thus, the subcarrier assignment problem can be viewed as a bidding process. Consider the right hand side of (13) as the bidding price offered by link l towards subcarrier k for utilizing its bandwidth; the highest bidder wins the exclusive right to use the bandwidth in subcarrier k. When there are multiple highest bidders, the bandwidth of subcarrier k is shared.

Following (11), the optimal power allocation for a node given a subcarrier assignment and Lagrange multipliers is,

$$P^*_{l,k} = W^*_{l,k}\left[\log e \frac{\lambda_{l,k}}{\tau_{t(l)}} - \frac{1}{|h_{l,k}|^2}\right]^+. \quad (14)$$

The power allocation (14) is similar to the standard water-filling solution [9].

From (12) and (7), the following necessary conditions for $C^*_{l,k} \neq 0$ can be inferred,

$$\begin{aligned} \lambda_{l,k} &= \beta_{t(l)} - \beta_{r(l)}, & t(l)\notin\mathcal{S}, r(l)\neq d, \quad (15) \\ \lambda_{l,k} &= \beta_{t(l)}, & t(l)\notin\mathcal{S}, r(l)=d, \quad (16) \\ \lambda_{l,k} &= 1, & t(l)\in\mathcal{S}, r(l)=d, \quad (17) \\ \lambda_{l,k} &= 1-\beta_{r(l)}, & t(l)\in\mathcal{S}, r(l)\neq d. \quad (18) \end{aligned}$$

We can further unifiy (15) – (18) into a single expression,

$$\lambda_{l,k} = \beta_{t(l)} - \beta_{r(l)}, \quad (19)$$

by the following extension of β_i, *i.e.*,

$$\beta_d = 0 \quad , \quad \beta_i = 1, \ \forall i\in\mathcal{S}. \quad (20)$$

Similarly, the necessary condition for $C^*_{l,k} = 0$ is

$$\lambda_{l,k} \geq \beta_{t(l)} - \beta_{r(l)}. \quad (21)$$

An analogy between information flow in the wireless network and water flow through a network of pipes can be drawn by considering β_i as the pressure at node $i\in\mathcal{V}$ and $\lambda_{l,k}$ as the resistance in subcarrier k of link l. According to (21), information will flow through link l via subcarrier k only if the difference in pressure between the transmit node and the received node, *i.e.*, $\beta_{t(l)} - \beta_{r(l)}$ is no smaller than the resistance $\lambda_{l,k}$. Expression (19) further suggests that the pressure inside the network reaches an equilibrium, *i.e.*, the difference in pressure is equal to the resistance, at the optimum solution. Again, it is interesting to observe that the pressure difference between two nodes does not depend on the subcarrier index.

Even though the relaxed problem (5) is convex and can be solved efficiently [10], the computational complexity can still be significant for a large number of subcarriers. Furthermore, solutions to the relaxed problem (5) do not always provide a feasible solution to the original problem (3). To further simply the problem, in the following section, we limit our discussion to a multihop network, *i.e.*, one source, one destination and multiple relays with links between adjacent nodes only. By exploiting the insights from the above discussion, we develop practical, low-complexity algorithms for power and subcarrier allocation for a multihop network. In practice, a multihop network or route can be chosen by routing methods such as in [11], [12].

V. Low-Complexity Algorithms

This section focuses on developing practical, low-complexity algorithms for multihop networks with only one source. In Section V-A, we develop a high-SNR approximation algorithm for a two-hop network motivated by the observation that a two-band partition of bandwidth is an optimum solution to the relaxed problem under certain assumptions. Section V-B proposes a greedy algorithm for networks with more than two hops.

A. High-SNR Approximation Algorithm for Two-Hops

For a two-hop network, we denote the source as node 1 and the relay as node 2, the source-relay link as link l_1 and the relay-destination link as link l_2. We also define the per-subcarrier effective SNR as

$$\hat{\rho}_{l_i,k} \quad := \quad \log e \frac{\lambda_{l_i,k}|h_{l_i,k}|^2}{\tau_i} - 1, \quad i = 1, 2. \qquad (22)$$

It can be verified from (22) and (14) that

$$\hat{\rho}_{l_i,k} = \frac{P_{l_i,k}}{W_{l_i,k}} |h_{l_i,k}|^2 \;\text{ for }\; P_{l_i,k} > 0. \qquad (23)$$

The following Proposition indicates the optimum frequency assignment for the relaxed problem is a two-band partition of the total available bandwidth given a set of Lagrange multipliers, hence suggesting a way to significantly reduce the computation complexity.

Propostion 1: Assuming

1) Lagrange multipliers $\lambda_{l_1,k}, \lambda_{l_2,k}, \forall k$ are known;
2) $\hat{\rho}_{l_i,k} \gg 1$, $\forall k$, $i = 1, 2$;
3) $|h_{l_1,k}|^{2\lambda_{l_1,k}} / |h_{l_2,k}|^{2\lambda_{l_2,k}}$ decreases as k increases;

the optimum subcarrier allocation to (5) is a two-band partition of bandwidth, *i.e.*, there exits $1 \leq L_1 \leq |\mathcal{K}|$, such that

$$\begin{aligned} W_{l_1,k} = W_k, \quad W_{l_2,k} = 0 \quad &\text{for } k < L_1, \\ W_{l_1,k} = 0, \quad W_{l_2,k} = W_k \quad &\text{for } L_1 < k \leq L_2. \end{aligned}$$

The proof of Proposition 1 follows similar lines as Theorem 2 of [2]. We note that Assumption 3 can be easily met by reordering the subcarrier indices. From (23), Assumption 2 can be met if SNR is high or channel gain is large. Also, by the KKT conditions, it can be shown that $\lambda_{l_1,k} = \lambda_1 \geq 0, \lambda_{l_2,k} = \lambda_2 \geq 0, \forall k$, and $\lambda_1 + \lambda_2 = 1$ for the two-hop network.

Combining the above observations with Proposition 1, we propose Algorithm 1 that returns the optimum rate given λ_1. A line search method [13] can then be employed to find λ_1 that maximizes the rate, *i.e.*,

$$\max_{\lambda_1 \in (0,1)} g_\lambda(\lambda_1), \qquad (24)$$

Algorithm 1 Function $g_\lambda(\lambda_1)$ Returns the Optimum Rate of Two-Hop Network Given λ_1

```
initialize an array c[1, ..., K − 1];
relabel subcarrier indices such that |h_{l1,k}|^{2λ1} / |h_{l2,k}|^{2(1−λ1)} decreases as k increases;
for k = 1 to K − 1 do
    Perform water-filling for node 1 using subcarrier 1 to k;
    Perform water-filling for node 2 using subcarrier k + 1 to K;
    Calculate the end-to-end rate based on the above subcarrier assignment and power allocation; store the rate into c[k];
end for
return the maximum element inside the array c.
```

where $g_\lambda(\lambda_1)$ is the function that returns the optimum rate of the two-hop network given λ_1, as defined in Algorithm 1. We refer to this method as the high-SNR approximation algorithm due to Assumption 2. We note that the high-SNR approximation algorithm not only returns a feasible power and subcarrier allocation, but also provides a set of Lagrange multipliers, allowing us to check whether the solution is in fact optimal.

Although the high-SNR approximation algorithm is motivated assuming high per-subcarrier effective SNR, Section VI shows by simulation that it works reasonably well throughout the SNR regime that we simulated. Unfortunately, the algorithm only applies to two-hop networks. For the more general multihop network, we suggest the following greedy algorithm as an alternative suboptimum solution.

B. Greedy Algorithm

The greedy algorithm proposed in this section is motivated by the KKT condition (13), which suggests that nodes bid for exclusive rights to subcarriers. Obviously, the price each node would offer to a subcarrier depends on how much reward this subcarrier brings to the system. The better the channel quality of a subcarrier, the more reward it brings. This idea has been exploited by opportunistic communication in a multi-access channel (MAC) [14], [15]. Opportunistic communication [14], [15] in MAC achieves a higher throughput by assigning resources to the user with the best channel gain. However, straightforward extensions of opportunistic communications to a multihop network, *e.g.*, assigning a subcarrier to the link with the best channel, might result in a waste of bandwidths since the end-to-end rate is limited by the minimum rate of all hops. Instead, we propose a greedy algorithm to assign subcarriers to hops with the best reward in increasing the end-to-end rate for the whole network. Consider that, initially, we have a set of available subcarriers that are not assigned to any links. To increase the end-to-end rate of a route, we choose one available subcarrier that maximizes the rate of the bottleneck link and assign it to the bottleneck link. After this assignment, the bottleneck link that limits the route performance may change and we repeat the above process until all subcarriers are assigned. Note that at each step, the end-to-end rate is nondecreasing. A more precise description of the greedy algorithm is given as Algorithm 2.

Algorithm 2 Greedy Algorithm

Initialize the first link as the bottleneck link l_b;
Initialize the set of unassigned subcarrier $\mathcal{K}'$ as $\mathcal{K}$;
for $k = 1$ to K **do**
  Assign from $\mathcal{K}'$ the subcarrier j with the maximum $|h_{l_b,j}|^2$ to the bottleneck link l_b; remove subcarrier j from $\mathcal{K}'$.
  Each transmit node performs water-filling based on the current subcarrier assignment.
  Calculate each link's rate; designate the link with the smallest rate as the new bottleneck link l_b.
end for
Each transmit node performs water-filling based on the current subcarrier assignment; calculate each link's rate and return the minimum rate.

VI. Simulation Results

This section provides numerical simulation results for a linear uniform multihop network with independent multipath fading among links. Specifically, all nodes are located on a straight line, and the distance between the source and destination is normalized to be 1. Without loss of generality, the source is assumed to be located at $(0, 0)$, the i^{th} relay station is located at $(i/N, 0)$ for $i = 1, ..., N-1$, and the destination is located at $(1, 0)$, where N is the number of hops. To estimate the performance of different power and subcarrier allocation algorithms, we average rates over different fading realizations. We adopt a 4-path Rayleigh fading model with unit power-decay factor and exponential power delay profile for all links [16]. The path-loss factor is assumed to be proportional to $d_{i,j}^{-v}$, where $d_{i,j}$ is the distance between node i and node j, and the path-loss exponent v is a constant, chosen to be 4 in our setup. Each node is assumed to have a transmit power constraint ρ. All subcarriers have the same bandwidth of $1/K$ such that the total system bandwidth is normalized to 1, *i.e.*, $\sum_{k\in\mathcal{K}} W_k = 1$.

We consider a fixed allocation of subcarriers as a baseline reference. In the fixed scheduling algorithm, each subcarrier is assigned to a hop independently of the channel conditions. Specifically, assuming $\mathcal{K} = \{1, ..., K\}$, for n^{th} hop, if $n + (\lfloor \frac{K}{N} \rfloor)N > K$ [1], we assign subcarriers $[n, n+N, ..., n+(\lfloor \frac{K}{N} \rfloor - 1)N]$; and if $n + (\lfloor \frac{K}{N} \rfloor)N \leq K$, we assign subcarriers $[n, n+N, ..., n+(\lfloor \frac{K}{N} \rfloor - 1)N, n+(\lfloor \frac{K}{N} \rfloor)N]$. After the subcarriers are assigned, we perform a water-filling power allocation for each hop, compute the rate for each hop, and find the minimum of the rates of all hops.

Figs. 1 and 2 compare the performance of different algorithms for two-hop and three-hop wireless networks, respectively. The optimum performances in Fig. 1 and Fig. 2 are obtained through exhaustive search among all possible subcarrier allocations. We note that the complexity of exhaustive search grows exponentially as the number of subcarriers increases. These simulations suggest that, compared with the high-SNR approximation algorithm, exhaustive search is about 10 times slower for a two-hop network with 8 subcarriers; but is about 10^3 slower for a two-hop network with 16 subcarriers.

[1] $\lfloor x \rfloor$ rounds x to the nearest integer towards minus infinity.

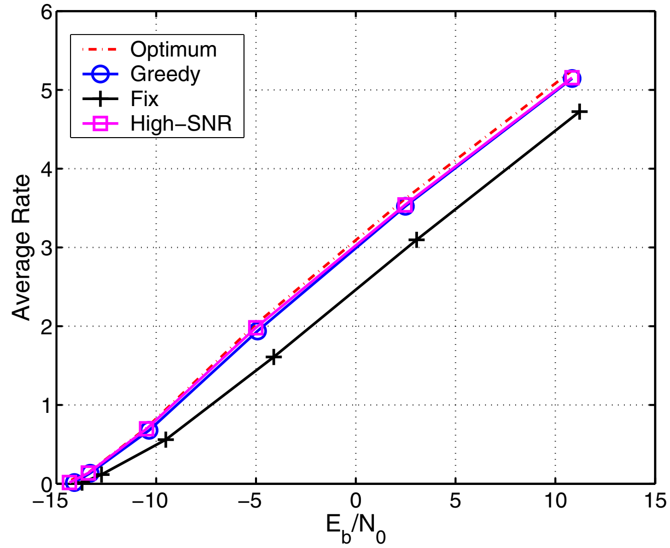

Fig. 1. Average rate versus SNR for a two-hop wireless network with 8 subcarriers.

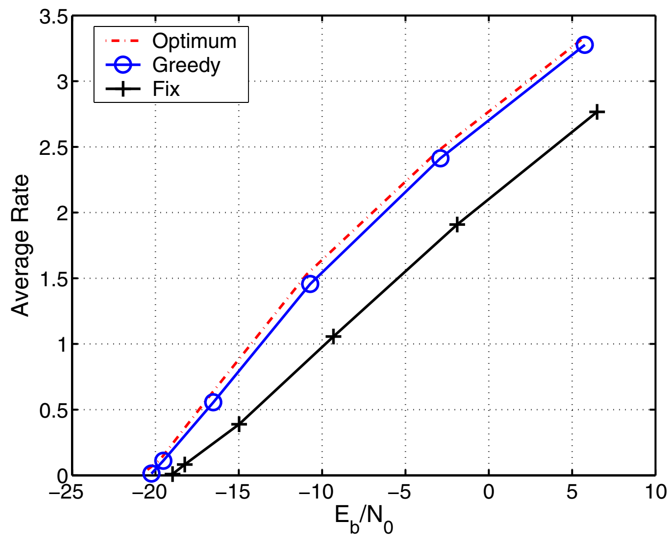

Fig. 2. Average rate versus SNR for a three-hop wireless network with 8 subcarriers.

Figs. 1 and 2 demonstrate that the greedy algorithm offers significant gains, up to 5 dB, compared with fix scheduling in the high SNR regime. This is because a better allocation of subcarriers results in more significant improvement of rates in the bandwidth-limited regime. The average performance of the high-SNR approximation algorithm is slightly better than that of the greedy algorithm. Although the performance of both of these algorithms are close to the optimal performance, the high-SNR approximation algorithm has the advantage of returning the corresponding Lagrange multipliers. We also note that, based upon more extensive simulations not shown

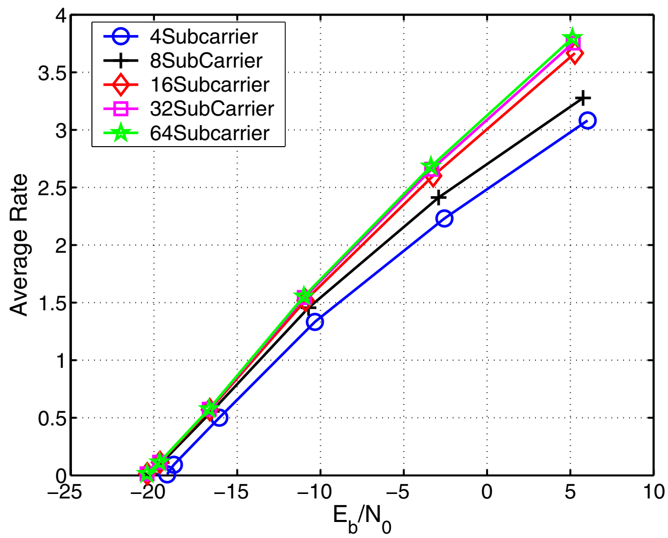

Fig. 3. Average rate versus SNR for a three-hop wireless network with different number of subcarriers. The greedy algorithm is used.

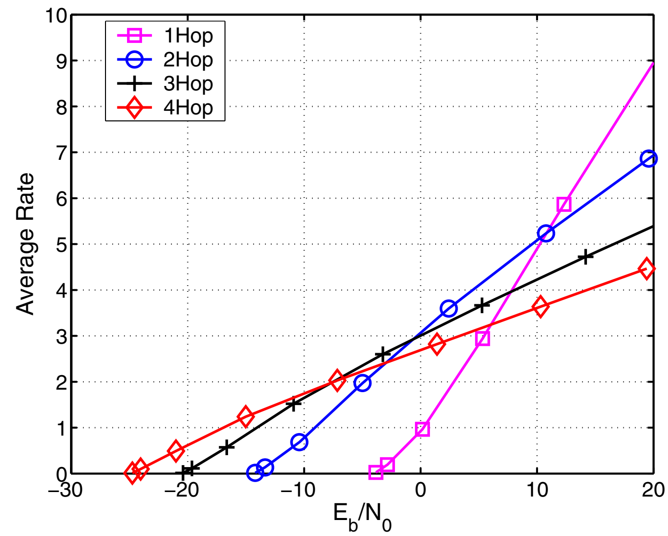

Fig. 4. Average rate versus SNR of networks with different number of hops, 16 subcarriers. The greedy algorithm is used.

here due to space considerations, given the same number of subcarriers, the performance gap between the greedy algorithm and exhaustive search increases as the number of hops increases.

Fig. 3 shows that as the number of subcarriers increases, the performance of the greedy algorithm generally improves due to a finer partition of bandwidth. However, the gain diminishes if the number of subcarriers is large because of increasing correlation among channel states on adjacent subcarriers.

Fig. 4 compares the performance of networks with different number of hops given the same number of subcarriers. It is clear from Fig. 4 that as SNR increases, the optimum number of hops in terms of maximizing the spectral efficiency decreases. We emphasize that even though this observation is made for a broadband OFDM system assuming frequency selective fading channels and channel state information available to transmitters for power allocation, it is consistent in a general sense with similar observations for a narrowband AWGN system [3], [4]. As the channel model varies, only the transition points, in terms of the particular SNR and rate values lying between different values for the optimal number of hops, seem to change.

VII. Conclusion

This paper formulates an optimization problem for the joint power allocation and subcarrier assignment for a multihop wireless network with OFDM. More importantly, the paper proposes two low-complexity algorithms, namely, the high-SNR approximation algorithm and the greedy algorithm, to solve the optimization problem. Simulation results suggest that both algorithms closely track the optimal performance. Furthermore, our simulations results indicate that in a broadband OFDM system, more hops help in the power limited regime, but do not help in the bandwidth limited resume, consistent with previous observations for narrowband systems. Future work includes extending the greedy algorithm to the downlink of a multihop network and the general multiple-source network, and a potential distributed version of the proposed algorithms.

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