Basic Limits on Protocol Information in Slotted Communication Networks

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***Abstract*—We investigate the amount of protocol information required for a communication network to meet an average delay constraint for the delivery of messages that arrive according to a Bernoulli random process. We obtain a lower bound on this overhead as a function of the arrival rate and average delay. Our model is a discrete-time analog of the Poisson arrival process considered by Gallager, and we show that in the limit as slot duration goes to zero, Gallager's bound is recovered.**

I. INTRODUCTION

Communication systems typically exhibit some form of underlying cost to achieving small delays or latency. Given a particular protocol we can sometimes bound the relationship between delay and other system parameters; however, we have largely fallen short of understanding the fundamental limits for all protocols, and are uncertain if and when they can be achieved.

In [1], Gallager studies a model for the amount of protocol information that is necessary in a communication network to meet an average delay constraint for messages with continuous-time arrivals. In practice, communication links are often slotted in nature, and messages only arrive at discrete time instances. It can be the case that the destination recovers timing information in a lossless fashion. This would be impossible for a continuous arrival process, because it would require an infinite amount of protocol information to be sent over the network. We consider the protocol overhead required for a slotted system to meet an average delay constraint. We assume that messages arrive according to a Bernoulli random process with parameter p. We derive the first-order rate-distortion function for this source, and give a closed-form expression for sufficiently small delays. Finally, we verify that, in the limit as slot duration approaches zero, our bound converges to Gallager's, demonstrating that in this regime the models are consistent.

Gallager introduced the idea of using a rate-distortion formulation for protocol overhead in [1]. Gallager's work on this subject was discussed more recently in [2], where a thorough review of networking overhead is given. Although we focus on the overhead associated with meeting a delay constraint, quite recently this framework has been used to study other network costs, such as routing overhead [3].

Our work is presented as follows: In Section II we give the problem formulation, followed by the derivation of the first-order rate-distortion function—a lower bound on protocol overhead—in Section III. In Section IV we plot the overhead for slotted arrivals for various slot durations and illustrate that our bound converges to the one for continuous arrivals given in [1]. Finally, in Section V we give some concluding remarks. Before proceeding, we introduce our notation. Random variables are denoted using a special font, e.g., X, with corresponding sample values denoted x. The (ensemble average) mutual information and entropy are defined in the standard way and are denoted as $\mathbb{I}(\cdot;\cdot)$ and $\mathbb{H}(\cdot)$, respectively.

II. PROBLEM FORMULATION

Our main result in Section III is a lower bound on protocol overhead due to the transfer of timing information about message arrivals. Specifically, we consider a discrete time communication system in which messages are independently generated in each slot with probably p, and messages must be sent to the destination with average delay (taken over all messages) less than or equal to d. Let the discrete-time stochastic process $\mathsf{X} = \mathsf{X}_0, \mathsf{X}_1, \ldots$ be a sequence of i.i.d. Bernoulli random variables with parameter $(0 < p \leq 1)$. Assume that for

$k = 0, 1, \ldots$, if $\mathsf{X}_k = 1$, then a message M is generated during the kth slot according to the discrete distribution $P_{\mathsf{M}}[\mathsf{m}]$. For the nth such $\mathsf{X}_k = 1$ $(1 \leq n \leq N)$ denote the corresponding arrival time as $\mathsf{K}_n = k$, and the message delivery time as $\hat{\mathsf{K}}_n$ $(1 \leq n \leq N)$. Recall that, because message arrival times form a Bernoulli process, the message interarrival times $(\mathsf{K}_{n+1} - \mathsf{K}_n)$ are i.i.d. geometric random variables with parameter $(0 < p \leq 1)$, i.e. for $n = 1, \ldots, N-1$, $(\mathsf{K}_{n+1} - \mathsf{K}_n) \sim (1-p)^{(\mathsf{k}-1)} p$, $\mathsf{k} = 1, 2, \ldots$. For notational convenience in Theorem 1, we have defined slot indexing starting from zero, so the arrival time for the first message follows a 'shifted' geometric distribution of $P_{\mathsf{K}_1}[\mathsf{k}] = p(1-p)^{\mathsf{k}}$.

For groups of N consecutive messages, denote the message arrival times as $\mathsf{K}^N = (\mathsf{K}_1, \mathsf{K}_2, \ldots, \mathsf{K}_N)$, and the corresponding delivery times as $\hat{\mathsf{K}}^N = (\hat{\mathsf{K}}_1, \hat{\mathsf{K}}_2, \ldots, \hat{\mathsf{K}}_N)$. Define the delay for the nth message as $\mathsf{D}_n := \hat{\mathsf{K}}_n - \mathsf{K}_n$, with corresponding expectation $d_n := \mathbb{E}[\mathsf{D}_n]$, and the average delay for a group of N messages as $1/N \sum_{n=1}^{N} \mathbb{E}[\mathsf{D}_n]$. Let $\mathcal{P}_N(d)$ denote the set of joint probability measures on K^N and $\hat{\mathsf{K}}^N$ that:

- have marginal distribution for K^N satisfying the Bernoulli arrival process model,
- result in $\mathsf{D}_n \geq 0$ $\forall n$ with probability 1, and,
- satisfy the average delay constraint $1/N \sum_{n=1}^{N} \mathbb{E}[\mathsf{D}_n] \leq d$.

For any joint distribution on a pair of random vectors, the mutual information between these two random vectors is a well-defined mathematical quantity. As such, for any network or protocol, we can compute the mutual information $\mathbb{I}(\mathsf{K}^N; \hat{\mathsf{K}}^N)$ induced by the joint distribution on message arrival and delivery times. We now give more operation significance to this mutual information for our problem.

To motivate the idea that $\mathbb{I}(\mathsf{K}^N; \hat{\mathsf{K}}^N)$ represents the amount of information being sent to the destination about message arrival times, we now summarize the perspective introduced in [1]. A key observation is that if messages are delivered within some average delay d, the destination is able to form a (perhaps noisy) estimate of the sequence of message arrival times. By virtue of this fact alone, whatever form of data the destination receives, it must contain the information necessary to form an estimate of message arrival times. If the destination's estimate is better than it would be from guessing randomly accordingly to the marginal distribution on arrival times, the protocol overhead is non-zero. This statement can be made more precise by introducing a rate-distortion formulation, which yields a mathematical characterization of the minimum amount of timing information being sent to the destination. To this end, we define the N-th order rate-distortion function as

$$R_N(d) := \inf_{\mathcal{P}_N(d)} \frac{1}{N} \mathbb{I}(\mathsf{K}^N; \hat{\mathsf{K}}^N), \tag{1}$$

and the corresponding rate-distortion function as

$$R(d) := \liminf_{N \to \infty} R_N(d). \tag{2}$$

Note that it may very well be that, even in theory, no physical system or communication protocol can result in a joint distribution required to meet $R(d)$. Nonetheless, practical systems could only do worse, so $R(d)$ is a lower bound on protocol overhead.

III. Protocol Overhead for Slotted Arrivals

In order to prove our main result, we will require the following lemma.

Lemma 1: For messages that arrive according to the Bernoulli process described in Section II and the N-th order rate-distortion functions defined by (1), it holds that

$$R_1(d) \leq R_N(d), \quad \forall N. \tag{3}$$

For completeness, a proof of Lemma 1 is given in the Appendix; however, because the interarrival times meet the conditions necessary for the first part of the proof of Theorem 3 in [1], the proof given here is conceptually analogous. Armed with Lemma 1 and the definition for $R(d)$ in (2), we see that $R_1(d)$ is a lower bound on $R(d)$, and therefore a lower bound on the protocol overhead that is *somehow* communicated to the destination. We now compute $R_1(d)$.

Theorem 1: For Bernoulli arrivals with parameter p, the first-order rate-distortion function $R_1(d)$ for timing information about message arrivals is given by

$$\begin{aligned} R_1(d) = \sup_{\nu \geq 0} \Bigg\{ & -\nu d + \nu k_0 - (k_0+1)\log(1-p) \\ & - \left[(1-p)^{(k_0+1)} - 1\right] \log\left[(1-p)^{-(k_0+1)} - 1\right] \\ & + (1-p)^{(k_0+1)} \log\left[\frac{(e^{\nu}-1)(1-p)}{p}\right] \\ & - \frac{\nu + (1-p)^{k_0}\left[\nu(p-1) + \log(1-p)\right]}{e^{\nu}-1} \Bigg\}, \end{aligned} \tag{4}$$

where

$$k_0(p, \nu) := \left\lceil \frac{\log\left(\frac{e^{\nu}-1}{e^{\nu}+p-1}\right)}{\log(1-p)} - 1 \right\rceil^{+}. \tag{5}$$

Proof: (Theorem 1) The computation of $R_1(d)$ is similar to the continuous case [1]; however there are differences due to the discrete nature of the problem that are sufficient to preclude a closed form analytic expression. Instead, we arrive at an analytic expression as the supremum over a Lagrange multiplier for the delay constraint, ν, and then solve for it numerically.

Following optimization techniques from [4], we find a sequence of Lagrange multipliers ψ_{k} satisfying

$$\sum_{\mathsf{k}} \psi_{\mathsf{k}} e^{-\nu d(\mathsf{k};\hat{\mathsf{k}})} \le 1, \quad \forall \hat{\mathsf{k}} \tag{6}$$

and a probability distribution $P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}]$ satisfying

$$P_{\mathsf{K}}[\mathsf{k}] = \psi_{\mathsf{k}} \sum_{\hat{\mathsf{k}}} P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}] e^{-\nu d(\mathsf{k};\hat{\mathsf{k}})}, \quad \forall \mathsf{k}. \tag{7}$$

As a Corollary to [4, Theorem 9.4.1], if, for a given sequence ψ_{k} satisfying (6), there exists a valid probability distribution $P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}]$ satisfying (7) for that ψ_{k}, and (6) is met with equality for all $\hat{\mathsf{k}}$ with $P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}] > 0$, then $R_1(d)$ is given by

$$R_1(d) = \sup_{\nu \ge 0} \left[\sum_{\mathsf{k}} P_{\mathsf{K}}[\mathsf{k}] \log \frac{\psi_{\mathsf{k}}}{P_{\mathsf{K}}[\mathsf{k}]} - \nu d \right], \tag{8}$$

where ν is the Lagrange multiplier for the delay constraint.

Using the distortion measure

$$d(\mathsf{k};\hat{\mathsf{k}}) := \begin{cases} \hat{\mathsf{k}} - \mathsf{k} & \text{for } \hat{\mathsf{k}} \ge \mathsf{k}, \\ \infty & \text{else,} \end{cases} \tag{9}$$

(6) and (7) become

$$e^{-\nu \hat{\mathsf{k}}} \sum_{\mathsf{k}=0}^{\hat{\mathsf{k}}} \psi_{\mathsf{k}} e^{\nu \mathsf{k}} \le 1, \quad \forall \hat{\mathsf{k}} \ge 0 \tag{10}$$

and

$$\frac{e^{-\nu \mathsf{k}} P_{\mathsf{K}}[\mathsf{k}]}{\psi_{\mathsf{k}}} = \sum_{\hat{\mathsf{k}}=\mathsf{k}}^{\infty} P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}] e^{-\nu \hat{\mathsf{k}}}, \quad \forall \mathsf{k} \ge 0, \tag{11}$$

respectively.

For reasons similar to those in [1], we conjecture that there exists an integer k_0 such that $P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}] = 0$ for $\hat{\mathsf{k}} < k_0$ and $P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}] > 0$ for $\hat{\mathsf{k}} \ge k_0$. Since for all $\hat{\mathsf{k}}$ with $P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}] > 0$ (10) must be met with equality, the difference between the left hand side of (10) for any two values of $\hat{\mathsf{k}} \ge k_0$ must be 0. Choosing two consecutive values of $\hat{\mathsf{k}} \ge k_0$ gives

$$\psi_{\mathsf{k}} = 1 - e^{-\nu}, \quad \text{for } \mathsf{k} \ge k_0 + 1. \tag{12}$$

In a similar manner, using these values of ψ_{k} in (11), along with $P_{\mathsf{K}}[\mathsf{k}] = p(1-p)^{\mathsf{k}}$, we find

$$P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}] = \frac{p(1-p)^{\hat{\mathsf{k}}}(e^{\nu} + p - 1)}{e^{\nu} - 1}, \quad \hat{\mathsf{k}} \ge k_0 + 1. \tag{13}$$

Using the fact that $P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}]$ is a probability distribution and therefore $\sum_{\hat{\mathsf{k}}} P_{\hat{\mathsf{K}}}[\hat{\mathsf{k}}] = 1$ we have

$$k_0(p,\nu) = \left\lceil \frac{\log\left(\frac{e^{\nu}-1}{e^{\nu}+p-1}\right)}{\log(1-p)} - 1 \right\rceil^{+}, \tag{14}$$

from which it follows that

$$P_{\hat{\mathsf{K}}}[k_0] = 1 - \frac{e^{\nu} + p - 1}{e^{\nu} - 1}(1-p)^{(k_0+1)}. \tag{15}$$

Using (13) and (15) in (11) we obtain

$$\psi_{\mathsf{k}} = \frac{p(1-p)^{\mathsf{k}} e^{\nu(k_0 - \mathsf{k})}}{1 - (1-p)^{(k_0+1)}}, \quad \text{for } \mathsf{k} \le k_0. \tag{16}$$

Next, we show that this ψ_{k} satisfies (10) for all $\hat{\mathsf{k}}$, and that (10) is met with equality for $\hat{\mathsf{k}} \ge k_0$. By construction, (10) is met with equality for $\hat{\mathsf{k}} \ge k_0 + 1$. For $\hat{\mathsf{k}} = k_0$, equality is verified by using (16) in (10). A sufficient condition for (10) to be satisfied for $\hat{\mathsf{k}} < k_0$ is that the left hand side of (10) is non-decreasing for all $\hat{\mathsf{k}} < k_0$. Taking the difference for consecutive values of $\hat{\mathsf{k}}$ leads to the sufficient condition

$$\nu \le \log\left[\frac{1 - (1-p)^{(\hat{\mathsf{k}}+2)}}{1 - (1-p)^{(\hat{\mathsf{k}}+1)}}\right], \quad \forall \hat{\mathsf{k}} < k_0. \tag{17}$$

Note that (17) is met with equality for $\hat{\mathsf{k}}$ equal to the argument of the ceiling function in $k_0(p,\nu)$, i.e.,

$$\hat{\mathsf{k}} = \frac{\log\left(\frac{e^{\nu}-1}{e^{\nu}+p-1}\right)}{\log(1-p)} - 1. \tag{18}$$

Since the right hand side of (17) is monotone decreasing in $\hat{\mathsf{k}}$ over $\hat{\mathsf{k}} \ge 0$, and $k_0 - 1$ is strictly less than (18), the inequality is satisfied for all $\hat{\mathsf{k}} < k_0$, as required. Therefore, the unique[1] optimal sequence ψ_{k} is the one given by (12) and (16). Computing $\sum_{\mathsf{k}} P_{\mathsf{K}}[\mathsf{k}] \log \psi_{\mathsf{k}}/P_{\mathsf{K}}[\mathsf{k}]$ in (8) with this sequence of ψ_{k} leads to (4), thereby completing the proof. ■

For sufficiently small d, $k_0 = 0$ and (4) becomes

$$\begin{aligned} R_1(d<1) = \sup_{\nu \ge 0} \Big[&(1-p)\log(1-e^{-\nu}) - \log p \\ &- \frac{1-p}{p}\log(1-p) - \nu d \Big]. \end{aligned} \tag{19}$$

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[1] Uniqueness follows from the strict convexity of $\sum_{\mathsf{k}} P_{\mathsf{K}}[\mathsf{k}] \log \frac{\psi_{\mathsf{k}}}{P_{\mathsf{K}}[\mathsf{k}]}$. See [4, p. 460] for details.

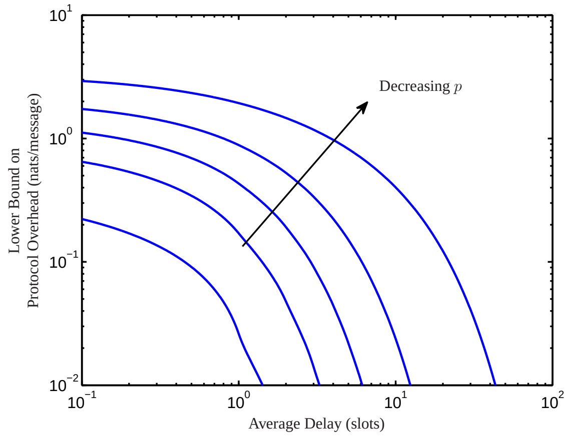

Fig. 1. Lower bound on protocol overhead as a function of delay for a Bernoulli arrival process with parameter $p \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$.

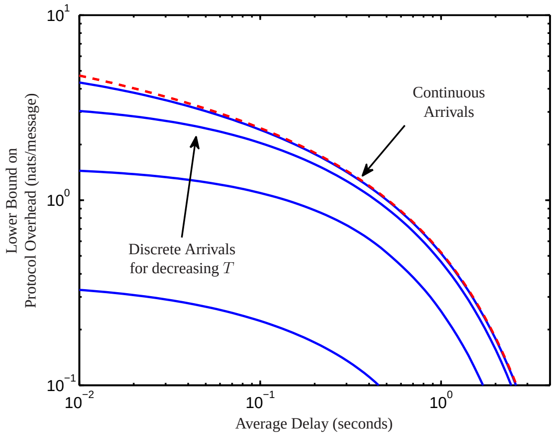

Fig. 2. Lower bound on protocol overhead for continuous (– –) and discrete (—) message arrivals. The average message arrival rate is $\lambda = 0.9$ messages/second, and for discrete arrivals the slot time is $0.01 \leq T \leq 1$ seconds.

In this regime the optimal ν is given by $\nu = \log[(d+1-p)/d]$, which leads to

$$R_1(d<1) = (1-p)\log\left(\frac{1-p}{d+1-p}\right) - \frac{1-p}{p}\log(1-p) - \log p. \tag{20}$$

Figure 1 shows (4) evaluated numerically for a range of p. Because the arrival process is disrete, as $d \rightarrow 0$, $R_1(d)$ approaches the entropy of the source. Note that this contrasts the continuous case in which the protocol information grows without bound as $d \rightarrow 0$.

IV. RELATIONSHIPS BETWEEN CONTINUOUS AND DISCRETE-TIME PROTOCOL OVERHEAD

Although the discussion has so far been for a discrete arrival process with delay measured in slots, for sufficiently small time intervals, a Bernoulli process can serve as a good approximation to a continous-time Poisson process. Accordingly, one would expect that as the slot duration becomes small, the two rate-distortion functions would converge. In [1], Gallager considered a model similar to ours, but for messages that arrive according to a Poisson process of rate λ. Because of the Poisson process assumption, the message interarrival times and the arrival time for the first message are i.i.d. exponential random variables with parameter λ. Gallager derived the first-order rate-distortion function given by

$$R_1(d) = -\log(1-e^{-\lambda d}), \tag{21}$$

and showed it is a lower bound on protocol overhead. Using the normalization $p = \lambda T$ for various slot times, Figure 2 illustrates that our bound on protocol overhead for discrete arrivals converges to the lower bound on protocol overhead for the continuous case.

V. CONCLUSIONS

In this paper we formulated a rate-distortion problem quantifying a fundamental lower limit on the amount of protocol information necessary to meet an average delay constraint. A key difference between our work and previous work on the subject is that we assumed messages can only arrive at the source and be delivered to the destination at discrete time instances corresponding to slots in a communication system. This situation was motivated by the fact that most practical systems operate with some level of time discretization, and many communication links are designed to be effectively slotted. In the limit as slot time goes to zero, we showed that our lower bound on protocol overhead coincides with Gallager's lower bound.

APPENDIX

Proof: (Lemma 1) From the definition of $\mathcal{P}_N(d)$, any $P_{\mathsf{K}^N,\hat{\mathsf{K}}^N}(\mathsf{k}^N, \hat{\mathsf{k}}^N) \in \mathcal{P}_N(d)$ satisfies the following properties:

- the corresponding marginal distribution for K^N satisfies the Bernoulli arrival process assumption,
- $\mathsf{D}_n := \hat{\mathsf{K}}_n - \mathsf{K}_n \geq 0 \ \forall n$ with probability 1, and,
- $1/N \sum_{n=1}^{N} d_n \leq d$, where $d_n := \mathbb{E}\left[\mathsf{D}_n\right]$.

Define U_n and V_n as

$$\mathsf{U}_n := \mathsf{K}_n - \mathsf{K}_{n-1}, \tag{22}$$

$$\mathsf{V}_n := \hat{\mathsf{K}}_n - \mathsf{K}_{n-1}, \tag{23}$$

for $n = 2, \dots, N$. Note that from the definition of d_n, we have $d_n = \mathbb{E}\left[\mathsf{V}_n - \mathsf{U}_n\right]$. Therefore, for any joint distribution $P_{\mathsf{K}^N, \hat{\mathsf{K}}^N}(\mathsf{k}^N, \hat{\mathsf{k}}^N) \in \mathcal{P}_N(d)$, the corresponding distribution on U_n and V_n satisfies $P_{\mathsf{U}_n, \mathsf{V}_n}(\mathsf{u}_n, \mathsf{v}_n) \in \mathcal{P}_1(d)$ for $n = 2, \dots, N$. Now consider any joint distribution $P_{\mathsf{K}^N, \hat{\mathsf{K}}^N}(\mathsf{k}^N, \hat{\mathsf{k}}^N) \in \mathcal{P}_N(d)$, and the following chain of (in)equalities:

$$\begin{aligned}
&\mathbb{I}(\mathsf{K}^N; \hat{\mathsf{K}}^N) && (24)\\
&\overset{(a)}{=} \mathbb{H}\left(\mathsf{K}^N\right) - \mathbb{H}\left(\mathsf{K}^N | \hat{\mathsf{K}}^N\right), && (25)\\
&\overset{(b)}{=} \mathbb{H}\left(\mathsf{K}_1\right) + \sum_{n=2}^{N} \mathbb{H}\left(\mathsf{K}_n | \mathsf{K}_1^{n-1}\right) \\
&\qquad - \left[\mathbb{H}\left(\mathsf{K}_1 | \hat{\mathsf{K}}^N\right) + \sum_{n=2}^{N} \mathbb{H}\left(\mathsf{K}_n | \hat{\mathsf{K}}^N \mathsf{K}_1^{n-1}\right)\right], && (26)\\
&\overset{(c)}{\geq} \mathbb{H}\left(\mathsf{K}_1\right) - \mathbb{H}\left(\mathsf{K}_1 | \hat{\mathsf{K}}_1\right) \\
&\qquad + \sum_{n=2}^{N} \left[\mathbb{H}\left(\mathsf{K}_n | \mathsf{K}_{n-1}\right) - \mathbb{H}\left(\mathsf{K}_n | \mathsf{K}_{n-1} \hat{\mathsf{K}}_n\right)\right], && (27)\\
&\overset{(d)}{=} \mathbb{I}(\mathsf{K}_1; \hat{\mathsf{K}}_1) \\
&\qquad + \sum_{n=2}^{N} \left[\mathbb{H}\left(\mathsf{K}_n | \mathsf{K}_{n-1}\right) - \mathbb{H}\left(\mathsf{U}_n | \mathsf{K}_{n-1} \hat{\mathsf{K}}_n \mathsf{V}_n\right)\right], && (28)\\
&\overset{(e)}{\geq} \mathbb{I}(\mathsf{K}_1; \hat{\mathsf{K}}_1) + \sum_{n=2}^{N} \left[\mathbb{H}\left(\mathsf{U}_n\right) - \mathbb{H}\left(\mathsf{U}_n | \mathsf{V}_n\right)\right], && (29)\\
&\overset{(f)}{=} \mathbb{I}(\mathsf{K}_1; \hat{\mathsf{K}}_1) + \sum_{n=2}^{N} \mathbb{I}(\mathsf{U}_n; \mathsf{V}_n), && (30)\\
&\overset{(g)}{\geq} \sum_{n=1}^{N} R_1(d_n), && (31)\\
&\overset{(h)}{\geq} N R_1\left(\frac{1}{N} \sum_{n=1}^{N} d_n\right), && (32)\\
&\overset{(i)}{\geq} N R_1(d), && (33)
\end{aligned}$$

where each are justified as follows:

(a) definition of mutual information,
(b) chain rule,
(c) conditioning cannot increase entropy; by the Bernoulli arrival process assumption, when conditioned on K_{n-1}, K_n is independent of K_1^{n-2},
(d) definition of mutual information; U_n is a translation of K_n when conditioned on K_{n-1},
(e) the conditional entropy of K_n given K_{n-1} is the same as the entropy of U_n; conditioning cannot increase entropy,
(f) definition of mutual information,
(g) using definition of $R_1(d)$ and $P_{\mathsf{U}_n, \mathsf{V}_n}(\mathsf{u}_n, \mathsf{v}_n) \in \mathcal{P}_1(d)\ \forall n$,
(h) convexity of $R_1(d)$,
(i) using $1/N \sum_{n=1}^{N} d_n \leq d$ and the fact that $R_1(d)$ is non-increasing in d.

Because the choice of $P_{\mathsf{K}^N, \hat{\mathsf{K}}^N}(\mathsf{k}^N, \hat{\mathsf{k}}^N) \in \mathcal{P}_N(d)$ was arbitrary, we have

$$\inf_{\mathcal{P}_N(d)} \frac{1}{N} \mathbb{I}(\mathsf{K}^N; \hat{\mathsf{K}}^N) \geq R_1(d), \tag{34}$$

and therefore $R_N(d) \geq R_1(d)$, concluding the proof. ■

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