Benefits of Correlated MIMO Schemes for Wideband Communication

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I. Introduction

We analyze a wideband non-coherent multi-antenna fading channel model in which the fading process exhibits both spatial and temporal correlation. By studying $\dot{\mathcal{I}}(0)$, the derivative of mutual information at zero signal-to-noise ratio (SNR), we find that significant gain in $\dot{\mathcal{I}}(0)$ is achieved by increasing the number of transmit and receive antennas; moreover, the relative loss in $\dot{\mathcal{I}}(0)$ due to using peak-limited signaling is also considerably reduced as channel correlation increases.

II. Related Work

Our work is motivated by [1], which reveals the fundamental dilemma for average power limited non-coherent channels: unbounded peak power and infinite bandwidth are necessary to achieve first order optimality. In this work, we sacrifice the goal of achieving capacity, and instead investigate the sub-capacity channel behavior.

In a recent work [2], the authors find the capacity per unit energy of scalar (single-antenna) fading channels under a peak power constraint. By extending the proof techniques in [2], we establish the optimality of $\dot{\mathcal{I}}(0)$ for certain special multi-input-multi-output (MIMO) cases, whereas the most general case still remains unsolved.

Among other approaches to exploiting channel correlation, [3] uses flash training to facilitate channel estimation and coherent signaling, and establishes necessary conditions on how channel coherence needs to scale with SNR. By contrast, our scheme uses non-coherent on-off signaling, without explicit training.

III. Channel Model

We consider a MIMO channel with m transmit and n receive antennas. The fading matrix at time index k is modeled as $\mathbf{H}(k) = \Sigma_T^{1/2}\mathbf{H}^0(k)\Sigma_R^{1/2}$, where covariance matrices Σ_T and Σ_R characterize the spatial correlation at the transmit and receive sides, respectively. $\mathbf{H}^0(k)$ has i.i.d. $\mathcal{CN}(0,1)$ entries. Furthermore, covariance matrix Σ_t^K characterizes the temporal correlation among K successive time indexes. The channel over a length-K block can thus be written in vector form

$$\langle \mathbf{x} \rangle = \langle \mathbf{S} \rangle \langle \mathbf{h} \rangle + \langle \mathbf{z} \rangle \ , \tag{1}$$

with fading vector $\langle \mathbf{h} \rangle \sim \mathcal{CN}(0, \Sigma_t^K \otimes \Sigma_R \otimes \Sigma_T)$ and additive noise vector $\langle \mathbf{z} \rangle \sim \mathcal{CN}(0, I_{nK\times nK})$. The channel input $\langle \mathbf{S} \rangle$ is a block diagonal matrix with kth diagonal block $I_{n\times n} \otimes \mathbf{s}^{\mathrm{T}}(k)$, $k = 1, \ldots, K$, where $\mathbf{s}(k)$ is the vector input at time k.

IV. Main Results

1) Without peak power constraints on input signaling, the maximum achievable $\dot{\mathcal{I}}(0)$ is

$$\dot{\mathcal{I}}_{\max}(0) = n\lambda_{\max}(\Sigma_T) \ , \tag{2}$$

achieved when $\{\mathbf{s}(k)\}_{k=1}^K$ are all along the eigen-direction of $\lambda_{\max}(\Sigma_T)$, the maximum eigenvalue of Σ_T; if $\{\mathbf{s}(k)\}_{k=1}^K$ are all restricted to be along $[1, 1, \ldots, 1]_{1\times m}^{\mathrm{T}}$ and to have equal power, *i.e.*, spatially/temporally flat, then

$$\dot{\mathcal{I}}_{\text{flat}}(0) = n\frac{\|\Sigma_T\|_{L_1}}{m} \ , \tag{3}$$

where $\|\Sigma_T\|_{L_1}$ denotes the L_1-norm of Σ_T. Analytical and numerical results illustrate that the gap between (2) and (3) is small, if not vanishing, for many important practical situations. For example, (2) and (3) coincide if $(\Sigma_T)_{ii} = 1$, $(\Sigma_T)_{ij} = \rho \in [0, 1]$, $i \neq j$. In any case, both (2) and (3) indicate that there is a linear gain n, irrespective of Σ_R. Furthermore, as m increases or Σ_T exhibits more correlation (quantified by $\lambda_{\max}(\Sigma_T)$ and $\|\Sigma_T\|_{L_1}/m$), $\dot{\mathcal{I}}_{\max}(0)$ and $\dot{\mathcal{I}}_{\text{flat}}(0)$ also increase.

2) Under finite peak power P per time index and spatially/temporally flat signaling,

$$\dot{\mathcal{I}}(0) = \dot{\mathcal{I}}_{\text{flat}}(0)\,(1 - \mathcal{L}) \ , \tag{4}$$

where

$$\mathcal{L} = \frac{\sum_{i=1}^n \sum_{k=1}^K \log\left(1 + P\frac{\|\Sigma_T\|_{L_1}}{m}\lambda_i(\Sigma_R)\lambda_k(\Sigma_t^K)\right)}{P\frac{\|\Sigma_T\|_{L_1}}{m}nK}$$

quantifies the relative loss due to finite peak power P, and vanishes as $P \to \infty$. For fixed P, $\mathcal{L}$ decreases if Σ_R or Σ_t^K exhibits "stronger" correlation, as quantified by the expression of $\mathcal{L}$. As an extreme example, for block fading model with block length K, $\mathcal{L} \sim \mathcal{O}(\frac{\log K}{K})$, *i.e.*, the effective peak power observed by the receiver is virtually amplified by a factor of K by exploiting the block structure. We also (analytically and numerically) evaluate $\mathcal{L}$ for general Toeplitz correlation structures. Finally, we observe an additional virtual peak amplification factor of $\|\Sigma_T\|_{L_1}/m$ from transmitter correlation.

3) We generalize the result in [2] by showing that, if $\lambda_{\max}(\Sigma_T) = \|\Sigma_T\|_{L_1}/m$, *i.e.*, (2) and (3) coincide, and all $\{\mathbf{s}(k)\}_{k=1}^K$ are restricted to be proportional, then (4) gives the maximum achievable $\dot{\mathcal{I}}(0)$ under peak power P. This holds in the SIMO case. We conjecture that the condition of proportionality among $\{\mathbf{s}(k)\}_{k=1}^K$ can actually be removed.

References

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