Window Decoding for the Multiaccess Channel with Generalized Feedback

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Abstract — **Low-delay decoding schemes based on window decoding are developed for the multi-access channel with generalized feedback (MAC-GF). It is shown that window decoding sometimes incurs a rate loss as compared to backward decoding. Wireless cases are found for which the achievable rates with backward or window decoding give the capacity region.**

I. Introduction

The MAC-GF [1, 2] has three terminals, two inputs x_1, x_2, three outputs y_0, y_1, y_2, and a conditional probability distribution $p(y_0, y_1, y_2|x_1, x_2)$. The input x_i is a function of a message w_i and the past outputs y_i seen by encoder $i = 1, 2$. This channel models scenarios in which two (or more) encoders causally obtain separate feedback signals. The channel includes several others as special cases, including the MAC, the MAC with noiseless feedback [3], and the relay channel [4, 5].

Similar strategies based upon regular block-Markov encoding for the MAC-GF are developed in [1, 2]. For decoding, we consider the sliding window technique described in [1, 6]. Specifically, data is communicated in B blocks, indexed by $1 \leq b \leq B$, with N channel uses each. The random coding scheme fixes a joint distribution on $(u, v_1, v_2, x_1, x_2, y_0, y_1, y_2)$ of the form

$$p(u)p(v_1|u)p(v_2|u)p(x_1|v_1,u)p(x_2|v_2,u)p(y_0,y_1,y_2|x_1,x_2)\,, \quad (1)$$

where u, v_1, and v_2 are auxiliary random variables. In block b, codebooks for v_1 and v_2 convey information to encoders 2 and 1 at rates $R_{1,2}$ and $R_{2,1}$, respectively; in block $b+1$, the encoders re-encode this cooperative information at rate $R_{1,2} + R_{2,1}$ using the joint codebook for u. The codebooks for x_1 and x_2 convey information directly to the decoder at rates $R_{1,1}$ and $R_{2,2}$, respectively. The decoder estimates the messages of u (or v_1 and v_2), x_1, and x_2 using a sliding window of the past two received blocks.

II. Window Decoding

Backward decoding [2] waits for all B blocks to be received and therefore induces substantial decoding delay. By contrast, window decoding [1, 6] can begin after a delay of only one block, and can be more amenable to practical implementation. These benefits come at a cost, however: window decoding does not achieve the same rates as backward decoding except in certain special cases.

We derive two achievable rate regions for decoders based upon window decoding. The first method, called *window decoding with stripping*, decodes the cooperative information and then treats it as known interference to decode the direct information. The second method, called *window decoding without stripping*, decodes the cooperative and direct information jointly.

(A) Window Decoding with Stripping	(B) Window Decoding without Stripping
$R_{1,2} \leq I(v_1; y_2 \| x_2, u)$ $R_{2,1} \leq I(v_2; y_1 \| x_1, u)$ $R_{1,1} \leq I(x_1; y_0 \| x_2, v_1, v_2, u)$ $R_{2,2} \leq I(x_2; y_0 \| x_1, v_1, v_2, u)$ $R_{1,1} + R_{2,2} \leq I(x_1, x_2; y_0 \| v_1, v_2, u)$ $R_{1,2} + R_{2,1} + R_{1,1} + R_{2,2} \leq I(x_1, x_2; y_0)$	$R_{1,2} \leq I(v_1; y_2 \| x_2, u)$ $R_{2,1} \leq I(v_2; y_1 \| x_1, u)$ $R_{1,1} \leq I(x_1; y_0 \| x_2, v_1, v_2, u)$ $R_{2,2} \leq I(x_2; y_0 \| x_1, v_1, v_2, u)$ $R_{1,1} + R_{2,2} \leq I(x_1, x_2; y_0 \| v_1, v_2, u)$ $R_{1,2} + R_{2,1} + R_{1,1} + R_{2,2} \leq I(x_1, x_2; y_0)$
$R_{1,2} \leq I(u; y_0) + I(v_1; y_0 \| v_2, u)$ $R_{2,1} \leq I(u; y_0) + I(v_2; y_0 \| v_1, u)$ $R_{1,2} + R_{2,1} \leq I(u, v_1, v_2; y_0)$	$R_{1,2} + R_{1,1} \leq I(u; y_0) + I(x_1; y_0 \| x_2, v_2, u)$ $R_{2,1} + R_{2,2} \leq I(u; y_0) + I(x_2; y_0 \| x_1, v_1, u)$ $R_{1,2} + R_{1,1} + R_{2,2} \leq I(u; y_0) + I(x_1, x_2; y_0 \| v_2, u)$ $R_{2,1} + R_{1,1} + R_{2,2} \leq I(u; y_0) + I(x_1, x_2; y_0 \| v_1, u)$

Table 1: Rate constraints for window decoding.

Theorem 1 *For the MAC-GF, window decoding with and without stripping achieve the convex hulls of the sets of rates* $(R_{1,2}, R_{2,1}, R_{1,1}, R_{2,2})$ *satisfying the constraints in Table 1(A) and Table 1(B), respectively, for distributions of the form* (1).

Because the last three constraints in Table 1(A), and the last four constraints in Table 1(B), are in addition to those required by backward decoding [2], window decoding is generally inferior to backward decoding in terms of achievable rates. There exist special cases for which the three regions are identical, including:

- The MAC with noiseless feedback [3], with $y_1 = y_2 = y_0$, $x_1 = v_1$, and $x_2 = v_2$.
- The relay channel [5], with $y_1 = 0$, $v_1 = q$, $v_2 = 0$, and $x_2 = u$.

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