Cooperative Diversity for Wireless Fading Channels without Channel State Information

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***Abstract*— Relaying and cooperative diversity allow multiple wireless radios to effectively share their antennas and create a virtual antenna array, thereby leveraging the spatial diversity benefits of multiple-input, multiple-output (MIMO) antenna systems. This paper examines the benefits of cooperative diversity for scenarios in which the receivers cannot exploit accurate channel state information (CSI). In particular, noncoherent demodulation is explored for two classes of relay processing, namely, detect-and-forward and amplify-and-forward. A complete maximum likelihood (ML) framework for noncoherent demodulation is developed for detect-and-forward, and is shown to naturally extend the corresponding framework for coherent demodulation. By contrast, the intractability of ML demodulation for noncoherent amplify-and-forward is demonstrated, suggesting a disconnect from the well-developed framework for coherent demodulation. Simulation results exhibit the diversity benefits of the detect-and-forward algorithms.**

I. Introduction

Cooperative diversity allows a collection of radios to relay signals for each other and effectively create a virtual antenna array for combating multipath fading in wireless channels. Fig. 1 depicts a simple example of such a communication scenario. Many studies of cooperative diversity focus on an information-theoretic perspective, employing either Shannon capacity (see [1], [2] and references therein) or outage capacity (see [3]–[5] and references therein) as performance measures for ergodic or non-ergodic channel environments, respectively. In real networks, especially those such as sensor networks with delay-constrained applications and complexity-constrained radios, practical codes with finite blocklength must be considered.

To obtain the best performance from practical decoding algorithms such as maximum-likelihood decoding or iterative decoding, it becomes necessary to take into account the effects of relay processing when implementing the destination decoding algorithm. In the extreme case, one can consider uncoded transmissions and study modulation and demodulation for various kinds of relay processing. The processing elements and performance analysis obtained for uncoded symbols can then be extended to the analysis of coded systems as is done for classical channel models.

If the relays perform some sort of detection and/or decoding, the constraint of limited blocklength requires the cooperative protocol to cope with relay decision errors. One way of dealing with the issue is to employ an outer cyclic-redundancy check (CRC) code to screen for block decision errors at the relays [6]. Another possibility is to explicitly take the effects of relay decision errors into account in the destination decoding algorithm [7], [8]. Our expectation is that some combination of both approaches is required in practical systems.

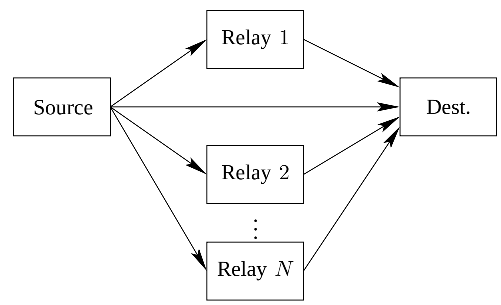

Fig. 1. Communication between a source and destination with multiple relays.

With this motivation, we turn to the study of modulation and demodulation for cooperative wireless systems. Some algorithms for coherent demodulation are introduced and partially analyzed in [7]. In particular, relay processing takes two simple forms: *detect-and-forward*, in which the relay demodulates and remodulates the signal transmitted by the source, and *amplify-and-forward*, in which the relay simply amplifies its received signal. Similar schemes are also considered in [9]. Further analysis of coherent amplify-and-forward is developed [10]–[12].

A majority of the work on cooperative diversity has focused on scenarios in which the receivers, and perhaps the transmitters, obtain channel state information (CSI) in the form of accurate estimates of the fading coefficients. The receivers can utilize available CSI for coherent reception, and the transmitters can utilize available CSI for power control and coherent beamforming. This paper summarizes some results for coherent demodulation with CSI at the receivers only [7], and extends the framework to noncoherent demodulation without CSI at the transmitters or receivers. The case of binary transmission is analyzed in more depth in [8]. We note that, even when CSI is available at the receivers, noncoherent modulation and demodulation may be required for low-complexity, low-power hardware implementations.

II. System Model

For simplicity of exposition, we focus on the communication model depicted by Fig. 1, with one source denoted s, one destination denoted d, and one or more relays denoted $r = 1, 2, \ldots, N$. The relays must satisfy a half-duplex constraint, *i.e.*, they cannot transmit and receive simultaneously in any given frequency band. To study the demodulation issues in their simplest setting, we further constrain the radios to relay orthogonally, *e.g.*, in time or in frequency. Optimization within this model, in the form of power and bandwidth allocation, as well as extensions to more general models, are certainly important but beyond the scope of this paper. As we will see, even these simple models generate quite challenging detection problems.

A. Channel Model

In the scenario described above, a baseband-equivalent, discrete-time channel model is as follows.[1] We consider an input "symbol" (modulation symbol, channel codeword, and so forth) as a block of K complex channel uses, and collect time varying signals into vectors, so that, for example, $\mathsf{x} = [\mathsf{x}[1]\ \mathsf{x}[2]\ \ldots\ \mathsf{x}[K]]^T$. To isolate the benefits of spatial diversity, our model captures the effects of block fading, either in time or in frequency. For a given symbol, the source transmission x_s is received by the destination as

$$\mathsf{y}_{d,s} = \mathsf{a}_{d,s}\mathsf{x}_s + \mathsf{z}_{d,s} \tag{1}$$

and by the relays as

$$\mathsf{y}_{r,s} = \mathsf{a}_{r,s}\mathsf{x}_s + \mathsf{z}_{r,s}\ , \quad r = 1, 2, \ldots, N\ . \tag{2}$$

After processing their received signals, the relays transmit signals x_r, $r = 1, 2, \ldots, N$, to the destination over the channels

$$\mathsf{y}_{d,r} = \mathsf{a}_{d,r}\mathsf{x}_r + \mathsf{z}_{d,r}\ , \quad r = 1, 2, \ldots, N\ . \tag{3}$$

We denote the average energy per symbol for the source and relays as $\mathcal{E}_s := \mathrm{E}[\mathsf{x}_s^\dagger \mathsf{x}_s]$ and $\mathcal{E}_r := \mathrm{E}[\mathsf{x}_r^\dagger \mathsf{x}_r]$, for $r = 1, 2, \ldots, N$, respectively.

In (1)–(3), $\mathsf{a}_{i,j}$ captures the effects of narrowband fading, and $\mathsf{z}_{i,j}$ captures the effects of additive noise and other interference in the system. We model $\mathsf{a}_{i,j}$ as being mutually independent zero-mean, complex Gaussian random variables with variances $\sigma^2_{\mathsf{a}_{i,j}}$, and $\mathsf{z}_{i,j}$ as mutually independent, zero-mean, complex Gaussian random vectors with covariance matrices $\mathcal{N}_i \mathbf{I}_K$, where $\mathbf{I}_K$ is the $K \times K$ identify matrix. Furthermore, the fading coefficients $\mathsf{a}_{i,j}$, noise vectors $\mathsf{z}_{i,j}$, and transmitted signals x_s and x_r are all modeled as mutually independent. Among other possible parameterizations, we define the average received signal-to-noise ratio (SNR) between transmitter j and receiver i as

$$\mathrm{SNR}_{i,j} = \frac{\sigma^2_{\mathsf{a}_{i,j}}\mathcal{E}_j}{\mathcal{N}_i}\ . \tag{4}$$

[1] San serif fonts denote random variables, *e.g.*, x, and serif fonts denote deterministic quantities, *e.g.*, x. Vectors are denoted in the appropriate bold face, so that x denotes a random vector, and $\mathbf{x}$ denotes a deterministic vector.

B. Maximum-Likelihood (ML) Demodulation

In the sequel, we consider modulation and demodulation, with an emphasis wherever possible on maximum-likelihood (ML) demodulation. We develop our results for general M-ary signaling, *i.e.*, transmit signal take values $\mathbf{x}_m$, for $m = 1, 2, \ldots, M$ with equal probability. To provide specific simulation results, we specialize the results to the case of M-ary orthogonal signaling, so that the symbol duration $K = M$, and the signal vectors take values

$$\mathbf{x}_m = \sqrt{\mathcal{E}}\mathbf{i}_m\ , \quad m = 1, 2, \ldots, M\ , \tag{5}$$

where $\mathbf{i}_m$ is a unit vector with 1 as its m-th element and 0 as all its other elements. If the fading captured by our model is flat across frequency, then the orthogonal signal set corresponds to frequency-shift keying (FSK). If the fading captured by our model flat is across time, then the orthogonal signal set corresponds to pulse position modulation (PPM).

Because the fading coefficients are modeled as being mutually independent, and the relays do not interact, the destination received signals $\mathsf{y}_{d,s}$ and $\mathsf{y}_{d,r}$, $r = 1, 2, \ldots, N$, are conditionally independent given x_s. It is therefore natural, when considering ML detection at the destination, to define log-likelihood ratios in order to simplify analysis of the problem. Specifically, let

$$\ell^s_m(\mathbf{y}) := \ln \frac{p_{\mathsf{y}_{d,s}|\mathsf{x}_s}(\mathbf{y}|\mathbf{x}_m)}{p_{\mathsf{y}_{d,s}|\mathsf{x}_s}(\mathbf{y}|\mathbf{x}_1)} \tag{6}$$

be the log likelihood ratio (LLR) for $\mathsf{y}_{d,s}$, the vector received at the destination from the source, given the source transmits symbol $\mathbf{x}_m$. Note that, although normalization by the likelihood given the first symbol $\mathbf{x}_1$ is not necessary, it is convenient for obtaining simplified LLRs. For example, we have $\ell^s_1(\mathbf{y}) = 0$. Similarly, let

$$\ell^r_m(\mathbf{y}) := \ln \frac{p_{\mathsf{y}_{d,r}|\mathsf{x}_s}(\mathbf{y}|\mathbf{x}_m)}{p_{\mathsf{y}_{d,r}|\mathsf{x}_s}(\mathbf{y}|\mathbf{x}_1)} \tag{7}$$

be the LLR for $\mathsf{y}_{d,r}$, the vector received at the destination from the relay, given the *source* transmits symbol $\mathbf{x}_m$. Again, we have $\ell^r_1(\mathbf{y}) = 0$, for $r = 1, 2, \ldots, N$. With these definitions, the destination ML decision rule can be compactly written as

$$\hat{\mathsf{m}} = \arg \max_{m=1,2,\ldots,M} \ell^s_m(\mathsf{y}_{d,s}) + \sum_{r=1}^{N} \ell^r_m(\mathsf{y}_{d,r})\ . \tag{8}$$

With this high-level structure in place for the destination receiver, we now specialize (6) and (7) to various types of relay processing, including detect-and-forward and amplify-and-forward, as well as various types of demodulation, including coherent and noncoherent.

III. Detect-and-Forward

We begin our discussion with detect-and-forward processing at the relays. First, we draw some conclusions about the general structure of the ML receiver for detect-and-forward. We point out how the results specialize to the more well-known cases of noncoherent spatial diversity with receive antenna arrays. We then specialize the results to the coherent and noncoherent cases, respectively.

A. General Formulation and Observations

Detect-and-forward proceeds as follows. The source transmits signal $\mathbf{x}_s$. The relays make decision errors, so that $\mathbf{x}_r/\sqrt{\mathcal{E}_r} \neq \mathbf{x}_s/\sqrt{\mathcal{E}_s}$ with nonzero probability. Although it is possible to find the ML detector, its nonlinear form makes detailed analysis of the probability of error quite difficult.

For a given channel model, (6) is relatively straightforward to compute. What remains is to determine (7). To this end, let

$$\hat{\ell}^r_{m'}(\mathbf{y}) := \ln \frac{p_{\mathbf{y}_{d,r}|\mathbf{x}_r}(\mathbf{y}|\mathbf{x}_{m'})}{p_{\mathbf{y}_{d,r}|\mathbf{x}_r}(\mathbf{y}|\mathbf{x}_1)} \tag{9}$$

be the LLR for $\mathbf{y}_{d,r}$, the vector received at the destination from the relay, given the *relay* transmits symbol $\mathbf{x}_{m'}$. As before, $\hat{\ell}^r_1(\mathbf{y}) = 0$, $r = 1, 2, \ldots, N$. To write an expression for (7) in terms of (9), we need to know the conditional probability law for each relay transmit symbol given the source transmit symbol. In particular, let

$$P^r_{m',m} := \Pr[\mathbf{x}_r = \mathbf{x}_{m'}|\mathbf{x}_s = \mathbf{x}_m] \tag{10}$$

be the transition probabilities that capture the effects of relay decisions. Then we can readily show that, for each $m = 1, 2, \ldots, M$,

$$\ell^r_m(\mathbf{y}) = \ln \left[\frac{\sum_{m'=1}^M P^r_{m',m} \exp(\hat{\ell}^r_{m'}(\mathbf{y}))}{\sum_{m'=1}^M P^r_{m',1} \exp(\hat{\ell}^r_{m'}(\mathbf{y}))} \right] \ . \tag{11}$$

Suppose it were possible for the relays to avoid making decision errors, *i.e.*, $\mathbf{x}_r/\mathcal{E}_r = \mathbf{x}_s/\mathcal{E}_s$ with probability one for $r = 1, 2, \ldots, N$. Then $P_{m',m} = \delta_{m',m}$ so that (11) reduces to $\ell^r_m(\mathbf{y}) = \hat{\ell}^r_m(\mathbf{y})$. This fictional scenario is equivalent to removing the noisy, faded paths between the source and relays in Fig. 1 and replacing them with noise-free paths. If we maintain orthogonal relaying, the detection problem becomes equivalent to one for receive antenna diversity, possibly with different branch SNRs [13]. In fact, the corresponding receive antenna diversity problem provides a convenient way of developing a lower bound on performance. However, as we might expect, this lower bound is only tight for scenarios in which the paths between the source and relays are very strong.

For more general channel conditions, (8), in combination with (9)–(11), tell us that it is possible for the destination to explicitly take into account the effects of relay decision errors. However, a caveat is that computational complexity may limit the use of this approach to signal sets of moderate size. To see this, we observe from (8) and (11) that we require $O(M^2N)$ multiplications and additions, $O(M^2N)$ exponentiations, and $O(MN)$ logarithms to find the ML decision at the destination. Although the number of relays N may be reasonably small, M can become quite large for coded modulations. For example, a binary linear block code of rate R and blocklength K has $M = 2^{RK}$ codewords. One alternative for reducing the computational burden, but presumably at the expense of degraded performance, would be to break a codeword of length $K = K'L$ into L symbols of length K', and apply the detect-and-forward strategy to these shorter symbols. The exact tradeoffs involved, as well as code designs for detect-and-forward relaying, represent an interesting area for further research.

Another caveat is that, for M-ary signal sets, the relay transition probabilities $P^r_{m',m}$ can be difficult to obtain without resorting to bounds or Monte Carlo integration. This issue can be partially alleviated when the signal sets exhibit some kind of symmetry such are geometrically uniformity [14]. Phase-shift keying (PSK) and orthogonal signal sets such as FSK and PPM can have this property, depending upon the channel model; general quadrature amplitude modulation (QAM) constellations often are not geometrically uniform.

A final caveat is that, because of the non-linearities in (11), analyzing the performance of the detector (8), for example, in terms of symbol-error rate, is quite challenging. Some progress can be made in the case of symmetric binary transmissions, by observing that (11) reduces in this case to $\ell^r_1(\mathbf{y}) = 0$ and

$$\ell^r_2(\mathbf{y}) = \ln \left[\frac{\epsilon_r + (1-\epsilon_r)\exp(\hat{\ell}^r_2(\mathbf{y}))}{(1-\epsilon_r) + \epsilon_r \exp(\hat{\ell}^r_2(\mathbf{y}))} \right] \ , \tag{12}$$

where $\epsilon_r = P^2_{1,2} = P^r_{2,1}$ is the relay decision error probability. That is, $\ell^r_2(\mathbf{y}) = f_r(\hat{\ell}^r_2(\mathbf{y}))$, where

$$f_r(t) = \ln \left[\frac{\epsilon_r + (1-\epsilon_r)e^t}{(1-\epsilon_r) + \epsilon_r e^t} \right] \approx \begin{cases} T_r \ , & t > T_r \\ t \ , & -T_r \leq t \leq T_r \\ -T_r \ , & t < -T_r \end{cases} \ , \tag{13}$$

and $T_r = \ln[(1-\epsilon_r)/\epsilon_r]$. The sigmoidal behavior of $f_r(t)$ suggests the piecewise linear approximation in (13). This is appealing because it eliminates the logarithm and exponentiations in (12), is amenable to analysis in some cases [7], [8], and provides tight approximations to performance.

B. Coherent Demodulation

For coherent demodulation, we assume the destination receiver can obtain accurate estimates of the realizations of the fading coefficients $a_{d,s}$ and $a_{d,r}$, for $r = 1, 2, \ldots, N$. Given $a_{d,s}$ and $\mathbf{x}_s$, $\mathbf{y}_{d,s}$ is conditionally complex Gaussian with density

$$\mathbb{CN}(\mathbf{y}; a_{d,s}\mathbf{x}_s, \mathcal{N}_d \mathbf{I}_K) \ , \tag{14}$$

where $\mathbb{CN}(\mathbf{y}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes a K-dimensional complex Gaussian probability density function with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Thus, the LLR (6) simplifies to[2]

$$\ell^s_m(\mathbf{y}) = \frac{2\mathrm{Re}\{a^*_{d,s}(\mathbf{x}_m - \mathbf{x}_1)^\dagger \mathbf{y}\} + |a_{d,s}|^2(\mathbf{x}_1^\dagger \mathbf{x}_1 - \mathbf{x}_m^\dagger \mathbf{x}_m)}{\mathcal{N}_d} \ . \tag{15}$$

We recognize the first term of (15) as the appropriate matched-filter operation for our problem. Note that the relays employ

[2]There is a slight abuse of notation here. The LLR in (6) involves a ratio of likelihoods without conditioning on receiver CSI, but (15) assumes conditioning on receiver CSI. The appropriate interpretation should be clear from the context.

similar demodulators to form their decisions, with substitution of the appropriate fading realizations and noise variances.

Similarly, given $a_{d,r}$ and $\mathbf{x}_r$, $\mathbf{y}_{d,r}$ is conditionally complex Gaussian with density

$$\mathbb{CN}\left(\mathbf{y}; a_{d,r}\mathbf{x}_r, \mathcal{N}_d \mathbf{I}_K\right) \ , \tag{16}$$

Thus the LLR (9) simplifies to

$$\hat{\ell}^r_{m'}(\mathbf{y}) = \frac{2\mathrm{Re}\{a^*_{d,r}(\mathbf{x}_{m'} - \mathbf{x}_1)^\dagger \mathbf{y}\} + |a_{d,r}|^2(\mathbf{x}_1^\dagger \mathbf{x}_1 - \mathbf{x}_{m'}^\dagger \mathbf{x}_{m'})}{\mathcal{N}_d} \ . \tag{17}$$

Although (17) is a linear operation, substitution into (11) to form $\ell^r_m(\mathbf{y})$ in general results in a nonlinear operation on the received signal.

C. Noncoherent Demodulation

For noncoherent demodulation, the destination receiver cannot obtain accurate estimates of the realizations of the fading coefficients $a_{d,s}$ and $a_{d,r}$, for $r = 1, 2, \ldots, N$. Given $\mathbf{x}_s$, the signal $\mathbf{y}_{d,s}$ received by the destination directly from the source is conditionally complex Gaussian with density

$$\mathbb{CN}\left(\mathbf{y}; \mathbf{0}, \sigma^2_{a_{d,s}} \mathbf{x}_s \mathbf{x}_s^\dagger + \mathcal{N}_d \mathbf{I}_K\right) \ . \tag{18}$$

Thus, the LLR (6) simplifies to

$$\begin{aligned} \ell^s_m(\mathbf{y}) &= \frac{\sigma^2_{a_{d,s}}}{\mathcal{N}_d^2} \mathbf{y}^\dagger \left[\frac{\mathbf{x}_m \mathbf{x}_m^\dagger}{1 + \frac{\sigma^2_{a_{d,s}}}{\mathcal{N}_d} \mathbf{x}_m^\dagger \mathbf{x}_m} - \frac{\mathbf{x}_1 \mathbf{x}_1^\dagger}{1 + \frac{\sigma^2_{a_{d,s}}}{\mathcal{N}_d} \mathbf{x}_1^\dagger \mathbf{x}_1} \right] \mathbf{y} \\ &\quad + \ln \left[\frac{\sigma^2_{a_{d,s}} \mathbf{x}_1^\dagger \mathbf{x}_1 + \mathcal{N}_d}{\sigma^2_{a_{d,s}} \mathbf{x}_m^\dagger \mathbf{x}_m + \mathcal{N}_d} \right] \end{aligned} \tag{19}$$

We recognize the first term of (19) as a generalized energy detector. Again, note that the relays utilize similar LLRs in making their decisions, with substitution of the appropriate fading and noise variances.

Similarly, given $\mathbf{x}_r$, $\mathbf{y}_{d,r}$ is conditionally complex Gaussian with density

$$\mathbb{CN}\left(\mathbf{y}; \mathbf{0}, \sigma^2_{a_{d,r}} \mathbf{x}_r \mathbf{x}_r^\dagger + \mathcal{N}_d \mathbf{I}_K\right) \ . \tag{20}$$

Thus, the LLR (9) simplifies to

$$\begin{aligned} \hat{\ell}^r_{m'}(\mathbf{y}) &= \frac{\sigma^2_{a_{d,r}}}{\mathcal{N}_d^2} \mathbf{y}^\dagger \left[\frac{\mathbf{x}_{m'} \mathbf{x}_{m'}^\dagger}{1 + \frac{\sigma^2_{a_{d,r}}}{\mathcal{N}_d} \mathbf{x}_{m'}^\dagger \mathbf{x}_{m'}} - \frac{\mathbf{x}_1 \mathbf{x}_1^\dagger}{1 + \frac{\sigma^2_{a_{d,r}}}{\mathcal{N}_d} \mathbf{x}_1^\dagger \mathbf{x}_1} \right] \mathbf{y} \\ &\quad + \ln \left[\frac{\sigma^2_{a_{d,r}} \mathbf{x}_1^\dagger \mathbf{x}_1 + \mathcal{N}_d}{\sigma^2_{a_{d,r}} \mathbf{x}_{m'}^\dagger \mathbf{x}_{m'} + \mathcal{N}_d} \right] \end{aligned} \tag{21}$$

Although (21) is a quadratic form in $\mathbf{y}$, substitution into (11) to form $\ell^r_m(\mathbf{y})$ in general results in a non-quadratic form in $\mathbf{y}$.

IV. Amplify-and-Forward

For completeness, we include some discussion of demodulation for amplify-and-forward relay processing. Under coherent reception, amplify-and-forward has received considerable attention. By contrast, we have found no results for noncoherent amplify-and-forward. As we will see in this section, there is likely a good reason for this: it is impossible to obtain the ML detector without resorting to numeric integration. Moreover, simple linear combiners inspired by optimal combining and equal-gain combining (EGC) for noncoherent receive arrays [13] appear to perform worse than direct transmission.

Under amplify-and-forward, each relay scales its received signal, *i.e.*,

$$\mathbf{x}_r = \beta_r \mathbf{y}_{r,s} \ ,$$

where β_r is the scaling factor at relay r. To satisfy an average output energy constraint per symbol, several constraints can be imposed at the relay, *e.g.*,

$$\beta_r^2 \le \frac{\mathcal{E}_r}{||\mathbf{y}_{r,s}||^2} \tag{22}$$

$$\beta_r^2 \le \frac{\mathcal{E}_r}{|a_{s,r}|^2 \mathcal{E}_s + K \cdot N_r} \tag{23}$$

$$\beta_r^2 \le \frac{\mathcal{E}_r}{\sigma^2_{a_{s,r}} \mathcal{E}_s + K \cdot N_r} \ . \tag{24}$$

The first constraint, (22), ensures that the relay output energy per symbol is no larger than $\mathcal{E}_r$ with probability one. This power constraint is suitable for both coherent and noncoherent scenarios, but yields models that are very difficult to analyze. The second constraint, (23), uses receive CSI from the source-relay link to ensure that an average output energy per symbol is maintained for each realization of $a_{r,s}$. This power constraint is suitable for full or partially coherent scenarios in which each relay obtains accurate receiver CSI for at least its source-relay fading magnitude. The third and final constraint, (24), only ensures that an average output energy per symbol is maintained, but allows for the instantaneous output power to be much larger than the average. This power constraint is also suitable for both coherent and noncoherent scenarios. It is particularly convenient for purposes of analysis because the relay scaling factor β_r is a constant instead of a random variable, as in (22) and (23).

In the following sections, we summarize the existing results for coherent amplify-and-forward and discuss how challenging it is to extend these ideas to noncoherent amplify-and-forward. The key differences lie in the density of the signal received at the destination through a relay path, *i.e.*,

$$\mathbf{y}_{d,r} = a_{d,r}\beta_r(a_{r,s}\mathbf{x}_s + \mathbf{z}_{r,s}) + \mathbf{z}_{d,r} \ . \tag{25}$$

A. Coherent Demodulation

For coherent demodulation under (23), we assume the destination receiver can obtain accurate estimates of the realizations of the fading coefficients $a_{d,s}$, $a_{r,s}$, $a_{d,r}$, and the relay scaling factors β_r, for $r = 1, 2, \ldots, N$. The signal $\mathbf{y}_{d,s}$ is identical to the case of coherent detect-and-forward, so that its density is given by (14) and the LLR (6) again reduces to (15).

Given all the CSI along with $\mathbf{x}_s$, and under the power constraints (23) or (24), the received signals $\mathbf{y}_{d,r}$, $r = 1, 2, \ldots, N$, are conditionally independent and complex Gaussian with densities

$$\mathcal{CN}\left(\mathbf{y}; a_{d,r}\beta_r a_{r,s}\mathbf{x}_s, (|a_{d,r}|^2\beta_r^2\mathcal{N}_r + \mathcal{N}_d)\mathbf{I}_K\right) \ , \qquad (26)$$

respectively. Thus, the LLR (7) simplifies to

$$\ell_m^s(\mathbf{y}) = \frac{2\mathrm{Re}\{g^*(\mathbf{x}_m - \mathbf{x}_1)^\dagger\mathbf{y}\} + |g|^2(\mathbf{x}_1^\dagger\mathbf{x}_1 - \mathbf{x}_m^\dagger\mathbf{x}_m)}{|a_{d,r}|^2\beta_r\mathcal{N}_r + \mathcal{N}_d} \ , \qquad (27)$$

where $g = a_{d,r}\beta_r a_{r,s}$. Unlike coherent detect-and-forward, the ML demodulator for coherent amplify-and-forward is equivalent to a linear operation on the received signals. For the Gaussian noise channel model, performance conditioned on the receive CSI is relatively straightforward to obtain; however, because (27) involves nonlinear operations on the fading coefficients, the main challenge for coherent amplify-and-forward is in averaging performance over the densities for the fading coefficients. Some progress in this direction has been obtained in [10]–[12].

B. Noncoherent Demodulation

Under noncoherent demodulation, neither the amplitude nor the phase of the effective fading coefficients are known to the appropriate relay or destination receivers. Thus, only the power constraints (22) and (24) are allowed. We focus on (24) throughout this section to simplify the discussion.

The signal $\mathbf{y}_{d,s}$ is identical to the case of noncoherent detect-and-forward, so that its density is given by (18) and the LLR (6) reduces to (19).

None of signals $\mathbf{y}_{d,r}$ received through the relays (*cf.* (25)) are conditionally Gaussian given only the transmitted signal. To see this, we can condition on the transmitted signal $\mathbf{x}_s$ and the fading coefficient $a_{d,r}$ to obtain a conditionally complex Gaussian random vector with zero-mean and covariance matrix

$$\begin{aligned} &\mathrm{E}[\mathbf{y}_{d,r}\mathbf{y}_{d,r}^\dagger|\mathbf{x}_s, a_{d,r}] \\ &\quad = \ |a_{d,r}|^2\beta_r^2(\sigma_{a_{r,s}}^2\mathbf{x}_s\mathbf{x}_s^\dagger + \mathcal{N}_r\mathbf{I}_K) + \mathcal{N}_d\mathbf{I}_K \end{aligned} \qquad (28)$$

For Rayleigh fading, $|a_{d,r}|^2$ is exponentially distributed with parameter $\lambda = \sigma_{a_{d,r}}^{-2}$. Thus, the conditional density of $\mathbf{y}_{d,r}$ given only $\mathbf{x}_s$ is the average of a complex Gaussian vector with zero-mean and covariance matrix (28) over an exponential density,

$$\begin{aligned} &p_{\mathbf{y}_{d,r}|\mathbf{x}_s}(\mathbf{y}|\mathbf{x}) \\ &\quad = \ \int_0^\infty \mathcal{CN}(\mathbf{y}; \mathbf{0}, \mathrm{E}[\mathbf{y}_{d,r}\mathbf{y}_{d,r}^\dagger|\mathbf{x}, a])\lambda e^{-\lambda a} da \ . \end{aligned} \qquad (29)$$

As far as we know, there is no closed form solution to the integral (29), even for the case of a single dimension, *i.e.*, $K = 1$ so that $\mathbf{y}_{d,r}$ is a scalar, or for orthogonal signal sets (5). In principle, the LLR (7) can be implemented as the logarithm of the ratio of two numerically computed integrals of the form (29). However, this essentially makes the ML detector unsuitable for practical implementation.

Since ML detection is too complex for analysis and implementation, we could consider suboptimal diversity combiners inspired by (21). Specifically, one can consider

$$\begin{aligned} \ell_m^r(\mathbf{y}) \ = \ & \frac{\sigma_{a_\mathrm{eff}}^2}{\mathcal{N}_\mathrm{eff}^2}\mathbf{y}^\dagger\left[\frac{\mathbf{x}_m\mathbf{x}_m^\dagger}{1+\frac{\sigma_{a_\mathrm{eff}}^2}{\mathcal{N}_\mathrm{eff}}\mathbf{x}_m^\dagger\mathbf{x}_m} - \frac{\mathbf{x}_1\mathbf{x}_1^\dagger}{1+\frac{\sigma_{a_\mathrm{eff}}^2}{\mathcal{N}_\mathrm{eff}}\mathbf{x}_1^\dagger\mathbf{x}_1}\right]\mathbf{y} \\ & + \ln\left[\frac{\sigma_{a_\mathrm{eff}}^2\mathbf{x}_1^\dagger\mathbf{x}_1 + \mathcal{N}_\mathrm{eff}}{\sigma_{a_\mathrm{eff}}^2\mathbf{x}_m^\dagger\mathbf{x}_m + \mathcal{N}_\mathrm{eff}}\right] \ , \end{aligned} \qquad (30)$$

where the effective fading and noise variances are

$$\sigma_{a_\mathrm{eff}}^2 := \sigma_{a_{d,r}}^2\beta_r^2\sigma_{a_{r,s}}^2 \qquad (31)$$

$$\mathcal{N}_\mathrm{eff} := \sigma_{a_{d,r}}^2\beta_r^2\mathcal{N}_r + \mathcal{N}_d \ . \qquad (32)$$

Of course, employing (30) in (8) sacrifices optimality of the demodulator. More concerning, however, is the fact that numerical investigation suggests the demodulator based upon (30) appears to perform worse than the case of no relays, *i.e.*, direct transmission from the source to the destination. Similar empirical conclusions have been obtained for demodulators based upon EGC.

V. SIMULATIONS

This section provides empirical simulation results for noncoherent, detect-and-forward cooperative diversity with up to three relays. We specialize the signal schemes and demodulation algorithms to the case of M-ary orthogonal signaling (*cf.* (5)) to illustrate the results. In particular, this choice is convenient because the signals are geometrically uniform [14], and because a closed form expression exists for the symbol error rate for these signals transmitted over noncoherent fading channels [13]. Thus, (10) is readily computable without having to resort to extensive simulations.

The simulation conditions follow the same lines as in [7], [8]. Specifically, the coordinates of the whole communication network are normalized by the distance $l_{d,s}$ between the source and destination transceivers. Without loss of generality, the source is assumed to be located at $(0, 0)$, and the destination located at $(1, 0)$. For simplicity of exposition, the relays are assumed to be located at $(l, 0)$, $0 < l < 1$. The fading variances $\sigma_{a_{i,j}}^2$ are assigned using a path-loss model of the form $\sigma_{a_{i,j}}^2 \propto l_{i,j}^{-\alpha}$, where $l_{i,j}$ is the distance from node i to node j, and α is the path-loss exponent, chosen as $\alpha = 4$ for our results. The total network energy per transmitted symbol is also normalized to unity. Specifically, we set $\mathcal{E}_s = 1$ for direct transmission; for cooperative diversity transmission, we assign equal energy among the source and relays, so that $\mathcal{E}_s = \mathcal{E}_r = 1/(N + 1)$, $r = 1, 2, \ldots, N$. We stress that this power allocation need not be optimal in general.

Fig. 2 displays simulation results for the symbol error rate (SER) of M-ary orthogonal signaling, with $M = 4$, and $N = 0, 1, 2, 3$ relays. Diversity benefits of cooperative transmission appear as faster decay of the SER with SNR. Similar observations about diversity gains can also be made for

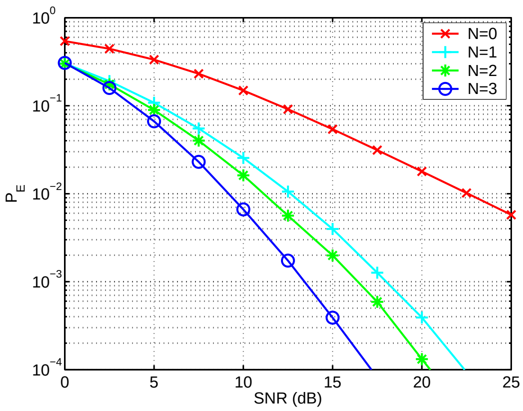

Fig. 2. 4-FSK with noncoherent detect-and-forward and $N = 0, 1, 2, 3$ relays. Relays located in the middle.

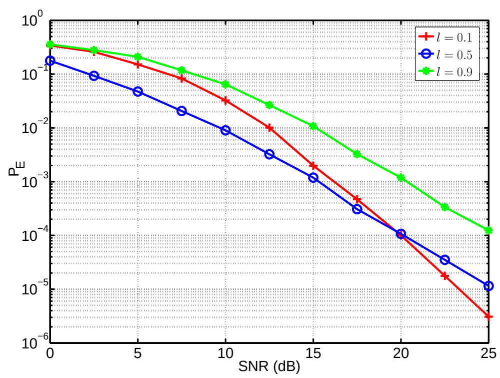

Fig. 3. Binary FSK with noncoherent detect-and-forward and $N = 2$ relays.

general M. Since the analysis for the general M-ary signaling is much more involved than binary FSK, the observations here suggests that the insight about diversity order provided in the context of BFSK [8] may apply to general MFSK.

Fig. 3 shows SER for noncoherent BFSK and $N = 2$ relays with different relay locations, *i.e.*, close to the source, in the middle between the source and destination, and close to the destination. It can be observed from Fig. 3 that, at high SNR, the cooperative transmission scheme with relays located close to the source outperforms the one with relays located in the middle. Therefore, the asymptotically optimum location for relays with noncoherent detect-and-forward is not necessarily in the middle between the source and destination. This observation differs from that for coherent amplify-and-forward, in which the optimum relay locations are in the middle between the source and destination [11]. However, for moderate values of SNR, Fig. 3 suggests that the mid-point between the source and destination is a reasonable choice for the relays due to the coding gains provided.

VI. Conclusions

This paper explores relay processing and destination demodulation for cooperative diversity in wireless networks, focusing on the case of noncoherent demodulation without receiver CSI. For amplify-and-forward relay processing, little insights have been obtained in the noncoherent case. In the coherent case, amplify-and-forward is convenient because the receiver processing is linear, and there is only extra additive noise in the transmitted signal. On the other hand, diversity benefits of coherent amplify-and-forward are only available with orthogonal relaying for the single source-destination model that we consider. Detect-and-forward relay processing is convenient because it integrates better with existing network protocol stacks and allows for more bandwidth efficient operation via non-orthogonal relaying. On the other hand, taking into account the effects of relay demodulation errors becomes intractable for moderate to large blocklengths. Both approaches produce thorny detection problems: either the ML detection rule is impossible to find or intractable to implement, and analysis of bit-error probability is quite involved. Much more insight is needed for a more complete understanding of these demodulation problems.

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