Achievable Rates for Multiple Access Channels with State Information Known at One Encoder

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Abstract

We consider a multiple access channel (MAC) with channel state information, or side information, available at one encoder. Focusing on the case of one informed encoder and one uninformed encoder, we develop achievable rate regions in the discrete memoryless and Gaussian memoryless cases. The Gaussian case combines the techniques of partial state cancellation and dirty paper coding by the informed encoder. Both encoders benefit in terms of achievable rates from the informed encoder's use of the channel state. Although this region is larger than that obtained by dirty paper coding alone, the state cannot be perfectly canceled as in the case when both encoders are informed.

1 Introduction

A class of discrete memoryless channels $\{\mathcal{X}, p(y|x,s), \mathcal{Y}, \mathcal{S}\}$, with state information S^n known at the encoder, has been studied by several groups of researchers, including Gel'fand and Pinsker [1], and Heegard and El Gamal [2]. Here, we extend results in [1, 2, 3] to discrete memoryless multiple access channel (MAC) with state information S^n known non-causally at only one encoder.

The communication system under investigation in this paper is shown in Figure 1. A memoryless MAC denoted by $\{\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1,x_2,s), \mathcal{Y}, \mathcal{S}\}$ consists of four sets $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{Y}$, and $\mathcal{S}$ and also a transition probability $p(y|x_1,x_2,s)$. We can interpret this as $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$ being the channel inputs, $S \in \mathcal{S}$ being the state of the channel, and $Y \in \mathcal{Y}$ being the channel output. We consider scenarios in which the MAC is embedded in some environment with state information S^n available non-causally at only encoder 1, as shown in Figure 1. We assume that S_i's are i.i.d. random variables $\sim p(s)$, $i = 1, 2, \ldots, n$. The output Y^n has conditional probability distribution

$$p(y^n|x_1^n, x_2^n, s^n) = \prod_{i=1}^{n} p(y_i|x_{1,i}, x_{2,i}, s_i)$$

The message sources at encoder 1 and encoder 2 produce random integers

$$W_1 \in \{1, 2, \ldots, M_1\} \quad \text{and} \quad W_2 \in \{1, 2, \ldots, M_2\},$$

respectively, at the beginning of each block of n channel uses. We assume that the probability of each pair of messages $(W_1 = w_1, W_2 = w_2)$ is given by $\frac{1}{M_1 M_2}$. Encoder

1 $X_1^n(W_1, S^n)$, encoder 2 $X_2^n(W_2)$, and decoder $\hat{W}(Y^n) = (\hat{W}_1, \hat{W}_2)$, are defining a $((M_1, M_2), n)$ code. The resulting probability of error is $P_e^n = \Pr\{\hat{W}(Y^n) \neq (W_1, W_2)\}$. A rate pair (R_1, R_2), where $R_1 = \frac{1}{n}\log_2 M_1$ and $R_2 = \frac{1}{n}\log_2 M_2$, is achievable if there exists sequence of $((M_1, M_2), n)$ codes with $P_e^n \to 0$ as $n \to \infty$. An achievable rate region of the MAC is the closure of the convex hull of set of all achievable (R_1, R_2) rate pairs.

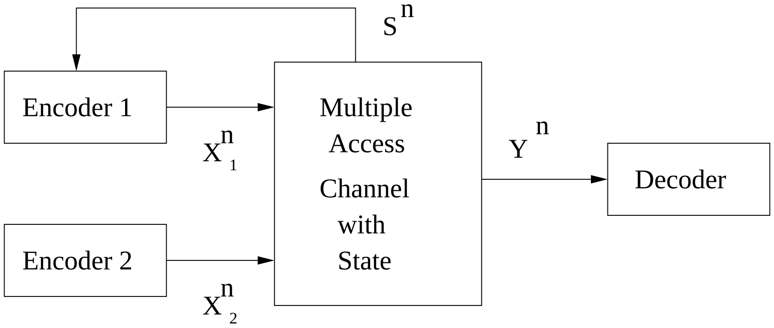

Figure 1: Multiple Access Channel model with state known at Encoder 1

We derive an achievable rate region for the discrete memoryless MAC with state information known at only one encoder in Section 2. In Section 3, we derive an achievable rate region for the Gaussian MAC with state information known at one encoder. In the case of the Gaussian MAC, the encoder that has channel state information allocates some of its available power to partially cancel the state of the channel, and the remainder of its available power to transmit pure information using dirty paper coding [3]. The achievable rate region for the Gaussian MAC obtained in this way is larger than that obtained from dirty paper coding alone. The encoder that has channel state information can help the other encoders by allocating some of its available power to cancel the state of the channel. However, in contrast to the case of side information available at both encoders [4], it appears that the effect of the state can not be eliminated completely from the point of view of both encoders if only one of them obtains side information. The results obtained in this paper can be extended to MAC with more than two encoders and state information available at one of the encoders.

2 Discrete Memoryless Case

We derive an achievable rate region of discrete memoryless MAC denoted by $\{\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1, x_2, s), \mathcal{Y}, \mathcal{S}\}$ consists of four finite sets $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{Y}$, and $\mathcal{S}$ and also a transition probability $p(y|x_1, x_2, s)$.

Theorem 1 *An achievable rate region of discrete memoryless MAC with state information S^n, S_i's are i.i.d. $\sim p(s)$, which is available non-causally at only encoder 1, is the closure of the convex hull of all (R_1, R_2) satisfying*

$$R_1 < I(U_1; Y|X_2) - I(U_1; S), \tag{1}$$

$$R_2 < I(X_2; Y|U_1), \tag{2}$$

$$R_1 + R_2 < I(U_1, X_2; Y) - I(U_1; S) \tag{3}$$

where U_1 is a finite alphabet auxiliary random variable, X_2, Y, and S are finite alphabet random variables. All mutual informations in the above equations are calculated using joint distributions of the form $p(s)p(u_1, x_1|s)p(x_2)p(y|x_1, x_2, s)$.

2.1 Proof of Theorem 1

The proof in this section closely follows the proofs in [1, 5] and uses the concepts of strong typicality [6]. We are given that S^n, S_i's i.i.d. $\sim p(s)$, is available non-causally at encoder 1. Now consider an $((M_1, M_2), n)$ code with a mapping

$$X_1^n : \{1, 2, \ldots, M_1\} \times \mathcal{S}_1^n \to \mathcal{X}_1^n,$$

a mapping

$$X_2^n : \{1, 2, \ldots, M_2\} \to \mathcal{X}_2^n,$$

and a mapping

$$\hat{W} : \mathcal{Y}^n \to \{1, 2, \ldots, M_1\} \times \{1, 2, \ldots, M_2\}.$$

The average probability of error P_e^n is defined as

$$P_e^n = \frac{1}{M_1 M_2} \sum_{i,j,s^n} p(s^n) \Pr[\hat{W}(Y^n) \neq (i,j) | W_1 = i, W_2 = j, S^n = s^n]$$

Encoding: The encoding strategy at the two encoders is as follows. Fix $\epsilon > 0$. Let $M_1 = 2^{n(R_1 - 4\epsilon)}$ and $J = 2^{n(I(U_1;S)+2\epsilon)}$. At Encoder 1, where the channel state is available, generate JM_1 i.i.d.sequences $U_1^n(m_1, j)$, $1 \leq m_1 \leq M_1$, $1 \leq j \leq J$, according to distribution $\prod_{i=1}^n p(u_{1,i})$. Here, m_1 indexes bins and j indexes sequences within a particular bin. For encoding, given state S^n and message $W_1 \in \{1, 2, \ldots, M_1\}$, look in a bin W_1 for sequence $U_1^n(W_1, j), 1 \leq j \leq J$, such that $(U_1^n(W_1, j), S^n)$ is jointly strongly typical. Then, encoder 1 chooses X_1^n for the given jointly typical pair according to the probability density function $p(x_1|u_1, s)$ which satisfies $p(x_1, u_1|s) = p(u_1|s)p(x_1|u_1, s)$. Encoder 2 sends the codeword $X_2^n(W_2)$ corresponding to $W_2 \in \{1, 2, \ldots, M_2\}$, where $M_2 = 2^{n(R_2 - 2\epsilon)}$. Given the inputs and channel state vectors, the decoder receives Y^n according to conditional distribution $\prod_i p(y_i|x_{1,i}, x_{2,i}, s_i)$.

Decoding: The decoder chooses the pair $(U_1^n(m_1, j),\ X_2^n(m_2))$, $1 \leq m_1 \leq M_1$, $1 \leq j \leq J$, and $1 \leq m_2 \leq M_2$ such that $(U_1^n(m_1, j), X_2^n(m_2), Y^n)$ is jointly strongly typical. If such a pair exists and is unique, the decoder declares that $\hat{W}_1$ is the bin index of this sequence $U_1^n(m_1, j)$, and $\hat{W}_2$ is the index of the codeword $X_2^n(m_2)$. Otherwise, the decoder declares an error.

Analysis of Probability of Error: By the symmetry of the random code construction, the conditional probability of error given a pair of message indices and state of the channel does not depend on the pair of indices sent and the channel state. Thus, without loss of generality, we can assume that $(W_1, W_2) = (1, 1)$ was sent for any given $S^n = s^n$, and the probability of error is given by the conditional probability of error given $(W_1, W_2) = (1, 1)$ and $S^n = s^n$. We can also assume that, for a given $S^n = s^n$, a particular sequence $U_1^n(1, 1)$ in bin 1 is jointly strongly typical with s^n. Thus, codewords $X_1^n(1, s^n)$ corresponding to the pair $(U_1^n(1, 1), s^n)$ and $X_2^n(1)$ are sent from encoder 1 and encoder 2, respectively

There are five possible sources of error. Let us denote these sources of error by E_l, $l \in \{1, 2, 3, 4, 5\}$. Let E_1 be the event that there is no jointly strongly typical pair $(U_1^n(W_1, j), S^n)$ for a given $S^n = s^n$ and message $W_1 = 1$ at encoder1. Let E_2 be the event that $(U_1^n(1, 1), X_2^n(1), Y^n)$ is not jointly strongly typical. Let E_3 be the event that $(U_1^n(m_1, j), X_2^n(1), Y^n)$ is jointly strongly typical for $m_1 \neq 1$ and $1 \leq j \leq J$, and E_4 be the event that $(U_1^n(1, 1), X_2^n(l), Y^n)$ is jointly strongly typical for $l \neq 1$. Finally, let E_5 be the event that $(U_1^n(m_1, j), X_2^n(l), Y^n)$ is jointly strongly typical for $m_1 \neq 1$, $1 \leq j \leq J$, and $l \neq 1$. In terms of these events, the probability of error is given by

$$P_e = \Pr[E_1 \cup (E_2 \cap E_1^c) \cup (E_3 \cap E_1^c \cap E_2^c) \cup (E_4 \cap E_1^c \cap E_2^c) \cup (E_5 \cap E_1^c \cap E_2^c)],$$

which can be upper-bounded via the union bound, and the fact that probabilities are less than one, as

$$P_e \leq \Pr[E_1] + \Pr[E_2|E_1^c] + \Pr[E_3|E_1^c, E_2^c] + \Pr[E_4|E_1^c, E_2^c] + \Pr[E_5|E_1^c, E_2^c] \tag{4}$$

Error event E_1 is analyzed first. The probability that any pair (U_1^n, S^n), with U_1^n and S^n generated independently according to $\prod p(u_{1i})$ and $\prod p(s_i)$, respectively, is jointly strongly typical is greater than $(1-\epsilon)2^{-n(I(U_1;S)+\epsilon)}$ for n sufficiently large. The probability of event E_1 is that there is no U_1^n for a given S^n in a particular bin. There are a total of J such U_1^n's in each bin. Thus, the probability of event E_1 is given by

$$\begin{aligned} \Pr[E_1] &\leq [1-(1-\epsilon)2^{-n(I(U_1;S)+\epsilon)}]^J \\ &= [1-(1-\epsilon)2^{-n(I(U_1;S)+\epsilon)}]^{2^{n(I(U_1;S)+2\epsilon)}} \end{aligned} \tag{5}$$

Taking the natural logarithm on both sides of (5), we obtain

$$\begin{aligned} \ln(\Pr[E_1]) &\leq 2^{n(I(U_1;S)+2\epsilon)} \ln[1-(1-\epsilon)2^{-n(I(U_1;S)+\epsilon)}] \\ &\overset{(a)}{\leq} -2^{n(I(U_1;S)+2\epsilon)}(1-\epsilon)2^{-n(I(U_1;S)+\epsilon)} \\ &= -(1-\epsilon)2^{n\epsilon} \end{aligned} \tag{6}$$

where (a) follows from the inequality $\ln(q) \leq (q-1)$. From (6), $\Pr[E_1] \to 0$ as $n \to \infty$. Thus, for n sufficiently large, we can ensure that

$$\Pr[E_1] \leq \epsilon/5 \tag{7}$$

For the probability of event $E_2 \cap E_1^c$, the Markov lemma [6] ensures jointly strong typicality of that $(U_1^n(1,1), X_2^n(1), S^n, Y^n)$ with high probability if $(U_1^n(k,1), S^n)$ is jointly strongly typical and $X_2^n(1)$ is strongly typical. We can conclude that $\Pr[E_2|E_1^c] \to 0$ $n \to \infty$. Thus, for n sufficiently large, we can ensure that

$$\Pr[E_2|E_1^c] \leq \epsilon/5. \tag{8}$$

For the probability of event $E_3 \cap (E_1^c \cap E_2^c)$, the probability that $(U_1^n(m_1, j), X_2^n(1), Y^n)$ for $m_1 \neq 1$, $1 \leq j \leq J$, is jointly strongly typical is less than $2^{-n(I(U_1;Y|X_2)-\epsilon)}$ for sufficiently large n. There are approximately JM_1 (exactly $J(M_1-1)$) such U^n sequences at encoder 1. Thus, the probability of this event is upper bounded by

$$\begin{aligned} \Pr[E_3|E_1^c, E_2^c] &\leq (JM_1)2^{-n(I(U_1;Y|X_2)-\epsilon)} \\ &= 2^{-n((I(U_1;Y|X_2)-I(U_1;S))-R_1)+\epsilon)}, \end{aligned} \tag{9}$$

where (9) follows from $J = 2^{n(I(U_1;S)+2\epsilon)}$ and $M_1 = 2^{n(R_1-4\epsilon)}$. From (9), $\Pr\{E_2|E_1^c, E_2^c\} \to 0$ as $n \to \infty$ since $R_1 < I(U_1;Y|X_2) - I(U_1;S)$ and $\epsilon > 0$. Thus, for n sufficiently large, we can ensure that

$$\Pr[E_3|E_1^c, E_2^c] \leq \epsilon/5 \tag{10}$$

For the probability of event $E_4 \cap (E_1^c \cap E_2^c)$, the probability that $(U_1^n(1,1), X_2^n(l), Y^n)$ is jointly strongly typical for any $l \neq 1$ is less than $2^{-n(I(X_2;Y|U_1)-\epsilon)}$ for sufficiently large n. There are approximately $M_2 = 2^{n(R_2-2\epsilon)}$ such X_2^n sequences at encoder 2. Thus, the probability of this event is given by

$$\begin{aligned}\Pr[E_4|E_1^c, E_2^c] &\leq (M_2)2^{-n(I(X_2;Y|U_1)-\epsilon)} \\ &= 2^{-n(I(X_2;Y|U_1)-R_2+\epsilon)}, \qquad (11)\end{aligned}$$

where (11) follows from $M_2 = 2^{n(R_2-2\epsilon)}$. From (11), the $\Pr[E_4|E_1^c, E_2^c] \to 0$ as $n \to \infty$ since $R_2 < I(X_2;Y|U_1)$. Thus, for n sufficiently large, we can ensure that

$$\Pr[E_4|E_1^c, E_2^c] \leq \epsilon/5 \qquad (12)$$

In the case of error event $E_5 \cap (E_1^c \cap E_2^c)$, the probability that $(U_1^n(m_1, j), X_2^n(l), Y^n)$ is jointly strongly typical for any $m_1 \neq 1$, $1 \leq j \leq J$, and $l \neq 1$ is less than $2^{-n(I(U_1,X_2;Y)-\epsilon)}$, for sufficiently large n. There are approximately JM_1 sequences U^n at encoder 1 and M_2 sequences X_2^n at encoder 2. The probability of this event is given by

$$\begin{aligned}\Pr[E_5|E_1^c, E_2^c] &\leq (JM_1M_2)2^{-n(I(U_1,X_2;Y)-\epsilon)} \\ &= 2^{-n((I(U_1,X_2;Y)-I(U_1;S))-(R_1+R_2)+3\epsilon)} \qquad (13)\end{aligned}$$

where (13) follows from $J = 2^{n(I(U_1;S)+2\epsilon)}$, $M_1 = 2^{n(R_1-4\epsilon)}$, and $M_2 = 2^{n(R_2-2\epsilon)}$. From (13), $\Pr[E_3|E_1^c, E_2^c] \to 0$ as $n \to \infty$ since $R_1 + R_2 < I(U_1, X_2; Y) - I(U_1; S)$. Thus, for n sufficiently large, the probability of this event is given by

$$\Pr[E_5|E_1^c, E_2^c] \leq \epsilon/5 \qquad (14)$$

Substituting (7), (8), (10), (12), and (14) in to (4), we have therefore shown that

$$P_e \leq \epsilon \qquad (15)$$

for n sufficiently large. This completes the proof of the Theorem 1.

3 Gaussian Memoryless Case

This section develops an achievable rate region of additive white Gaussian MAC. The model of Gaussian MAC with state known at encoder 1 is shown in Figure 2. The output of the channel is given by $Y^n = X_1^n + X_2^n + S^n + Z^n$ where, X_1^n and X_2^n are inputs of the channel from encoder 1 and encoder 2, respectively, S^n is the state of the channel, and Z^n is additive noise in the channel. The state S^n and noise Z^n are two independent Gaussian random vectors. The individual elements of S^n and Z^n are independent and identically distributed random variables according to distributions $\mathcal{N}(0, Q)$ and $\mathcal{N}(0, N)$, respectively. The inputs X_1^n and X_2^n must satisfy the power constraints $(1/n)\sum_{i=1}^n X_{1,i}^2 \leq P_1$ and $(1/n)\sum_{i=1}^n X_{2,i}^2 \leq P_2$, respectively, with probability one.

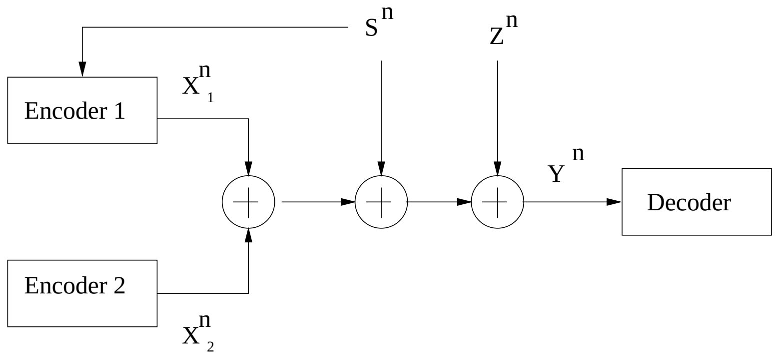

Figure 2: Gaussian multiple access channel model with state known at encoder 1

Theorem 2 *An achievable rate region of the above mentioned Gaussian multiple access channel with state information known at encoder 1 is given by*

$$\mathcal{R} = \bigcup_{\beta} \bigcup_{\alpha \in \mathcal{A}} \{(R_1, R_2) : R_1 < r_1(\beta,\alpha), R_2 < r_2(\beta,\alpha), R_1 + R_2 < r_3(\beta,\alpha)\} \tag{16}$$

where

$$r_1(\beta,\alpha) = \frac{1}{2}\ln\left(\frac{\overline{\beta}P_1[\overline{\beta}P_1 + (\sqrt{Q}-\sqrt{\beta P_1})^2 + N]}{\overline{\beta}P_1(\sqrt{Q}-\sqrt{\beta P_1})^2(1-\alpha)^2 + N(\overline{\beta}P_1 + \alpha^2(\sqrt{Q}-\sqrt{\beta P_1})^2)}\right) \tag{17}$$

$$r_2(\beta,\alpha) = \frac{1}{2}\ln\left(1 + \frac{P_2[\overline{\beta}P_1 + \alpha^2(\sqrt{Q}-\sqrt{\beta P_1})^2]}{\overline{\beta}P_1(\sqrt{Q}-\sqrt{\beta P_1})^2(1-\alpha)^2 + N[\overline{\beta}P_1 + \alpha^2(\sqrt{Q}-\sqrt{\beta P_1})^2]}\right) \tag{18}$$

$$r_3(\beta,\alpha) = \frac{1}{2}\ln\left(\frac{\overline{\beta}P_1[\overline{\beta}P_1 + P_2 + (\sqrt{Q}-\sqrt{\beta P_1})^2 + N]}{\overline{\beta}P_1(\sqrt{Q}-\sqrt{\beta P_1})^2(1-\alpha)^2 + N(\overline{\beta}P_1 + \alpha^2(\sqrt{Q}-\sqrt{\beta P_1})^2)}\right) \tag{19}$$

and $0 \leq \beta < \min(1, \frac{Q}{P_1})$, $\overline{\beta} = (1-\beta)$, $\mathcal{A} = \{x : x \in \mathbb{R}, r_1(\beta,x) \geq 0, r_2(\beta,x) \geq 0, r_3(\beta,x) \geq 0\}$,

3.1 Proof of Theorem 2

According to the above given channel, the output random variable of the channel is given by $Y = X_1 + X_2 + S + Z$, where X_1 and X_2 have power P_1 and P_2, respectively, and S and Z are Gaussian random variables distributed according to $\mathcal{N}(0,Q)$ and $\mathcal{N}(0,N)$, respectively. The results of the discrete memoryless MAC channel can be readily extended to memoryless channels with discrete time and continuous alphabets using techniques in [7].

Encoder 1, which has channel state information, uses power-sharing techniques as in [8] for encoding. In this case, the available transmitter power P_1 is allocated between state cancellation and pure information transmission. Fix a power sharing parameter $0 \leq \beta < \min(1, \frac{Q}{P_1})$ and divide the available power at encoder 1 in to βP_1 and $(1-\beta)P_1$. Then the codeword X_1^n can be represented as sum of two components X_{1s}^n and X_{1w}^n. The component X_{1s}^n is used to partially cancel the channel state so that the other encoder can benefit from this cancellation. Since the channel state S^n is available at encoder 1 non-causally, then $X_{1s}^n = -\sqrt{\frac{\beta P_1}{Q}}S^n$ can be generated with power βP_1. With such partial cancellation, channel becomes $Y^n = X_{1w}^n + X_2^n + (1 - \sqrt{\frac{\beta P_1}{Q}})S^n + Z$. Define $\overline{\beta} = 1-\beta$, $P_1' = \overline{\beta}P_1$ and $Q' = (\sqrt{Q} - \sqrt{\beta P_1})^2$. We can interpret this equivalent channel as a

Gaussian MAC with state $S'^n = (1 - \sqrt{\frac{\beta P_1}{Q}})S^n$, $S'_i \sim \mathcal{N}(0, Q')$, Z^n is Gaussian random vector, and inputs X_{1w}^n and X_2^n satisfying the power constraints $(1/n)\sum_{i=1}^n X_{1w,i}^2 \leq P'_1$ and $(1/n)\sum_{i=1}^n X_{2,i}^2 \leq P_2$, respectively

Now the pure information encoding is similar to encoding of information in [3]. Specifically, we consider the auxiliary random variable $U_1 = X_{1w} + \alpha S'$, where, α is any real number, X_{1w} and S' are independent random variables distributed according to $\mathcal{N}(0, P'_1)$ and $\mathcal{N}(0, Q')$, respectively. The exact range of α will be discussed later. Fix $\epsilon > 0$. First we generate $2^{n(I(U_1;Y)-2\epsilon)}$ sequences U^n with components independently drawn according to $\mathcal{N}(0, P'_1 + \alpha^2 Q')$. Then, we place these sequences randomly in to $2^{n(R_1-4\epsilon)}$ bins in such a way that each bin contains the same number of sequences. This codebook is revealed to both encoders and decoder. Given a state vector $S'^n = s'^n$ and a message $W_1 = m_1$, the encoder looks for a jointly typical pair (U_1^n, s'^n) in bin m_1. Encoder 1 declares an error if no such sequence U_1^n found. Assume that $U_1^n = u_1^n$ is jointly typical with s'^n. Then encoder calculates $x_{1w}^n = u_1^n - \alpha s'^n$. If x_{1w}^n is not typical, then encoder declares an error. In this way, information can be encoded at encoder 1.

At encoder 2, we generate $2^{n(R_2-2\epsilon)}$ sequences X_2^n with components independently drawn according to $\mathcal{N}(0, P_2)$ and index these sequences. This codebook is also revealed to both encoders and the decoder. To send $W_2 = m_2$, encoder 2 sends $x_2^n(m_2)$.

At the decoder, $y^n = x_{1w}^n + x_2^n + s'^n + z^n$ is received. Then the decoder looks for pair (U_1^n, X_2^n) jointly typical with the sequence y^n. If a unique pair exists, then the decoder's estimate $(\hat{W}_1, \hat{W}_2)$ is the bin index of the sequence U_1^n and index of the sequence X_2^n. Otherwise, the decoder declares an error.

We can calculate an achievable region for a given α and β by using the achievable region of discrete memoryless MAC mentioned in (1),(2), and (3) and distributions of random variables S', U_1, X_2, Z, and Y . Now we calculate the right hand side expressions of (1), (2), and (3). First, we calculate the right hand side expression of (1) which is given by

$$r_1(\beta, \alpha) \overset{(\triangle)}{=} I(U_1; Y|X_2) - I(U_1; S'),$$

where

$$\begin{aligned} I(U_1; Y|X_2) &= H(X_{1w} + X_2 + S' + Z|X_2) - H(X_{1w} + X_2 + S_1 + Z|X_1 + \alpha S', X_2), \\ I(U_1 : S') &= H(X_{1w} + S') - H(X_{1w} + S'|S') \end{aligned}$$

Then $r_1(\beta, \alpha)$ can be simplified to

$$r_1(\beta, \alpha) = \frac{1}{2} \ln \left(\frac{P'_1[P'_1 + Q' + N]}{P'_1 Q'(1-\alpha)^2 + N(P'_1 + \alpha^2 Q')} \right) \tag{20}$$

Similarly, right hand expressions in (2) and (3) can be simplified to

$$\begin{aligned} r_2(\beta, \alpha) &\overset{(\triangle)}{=} I(X_2; Y|U_1) \\ &= I(X_2; X_{1w} + X_2 + S' + Z|X_{1w} + \alpha S') \\ &= \frac{1}{2} \ln \left(1 + \frac{P_2(P'_1 + \alpha^2 Q')}{P'_1 Q'(1-\alpha)^2 + N(P'_1 + \alpha^2 Q')} \right) \end{aligned} \tag{21}$$

$$\begin{aligned} r_3(\beta, \alpha) &\overset{(\triangle)}{=} I(U_1, X_2; Y) - I(U_1; S_1) \\ &= \frac{1}{2} \ln \left(\frac{P'_1(P'_1 + P_2 + Q' + N)}{P'_1 Q'(1-\alpha)^2 + N(P'_1 + \alpha^2 Q')} \right) \end{aligned} \tag{22}$$

An achievable rate region for a given β and α is thus given by

$$\mathcal{R}(\beta,\alpha) = \{(R_1,R_2) : R_1 < r_1(\beta,\alpha), R_2 < r_2(\beta,\alpha), R_1 + R_2 < r_3(\beta,\alpha)\} \tag{23}$$

Also note that we restrict α to the $\mathcal{A} = \{x : x \in \mathbb{R}, r_1(\beta,x) \geq 0, r_2(\beta,x) \geq 0, r_3(\beta,x) \geq 0\}$ for the choice of α for a given β. By varying β and α, we can get different achievable rate regions $\mathcal{R}(\beta,\alpha)$. The union of all achievable rate regions $\mathcal{R}(\beta,\alpha)$ obtained by varying β and α gives an achievable rate region of Gaussian MAC. This completes the proof of Theorem 2.

3.2 Discussion

The main advantage of the above mentioned coding scheme is that encoder 1, with channel state information available, can help encoder 2 which does not have channel state information and suffers from the interfering channel state. Even though state information is known at only one encoder, both the encoders benefit from this situation by using both dirty paper coding alone as well as cancellation and dirty paper coding together. The achievable rate region $\mathcal{R}(0,\alpha)$, which is obtained by applying dirty paper coding with α as parameter alone to all the available power at encoder 1, is always contained in the $\mathcal{R}$. This shows that, in contrast to the case of side information available to both encoders [4], dirty paper coding alone may be insufficient. The achievable rate regions obtained by using dirty paper coding alone as well as cancellation and dirty paper coding together are larger than the achievable regions obtained when state information is not available at both encoders.

3.3 Numerical Example

This section illustrates the achievable rate region of Gaussian MAC with the help of an example. We illustrate the effect of applying a cancellation and dirty paper coding together with a concrete example. In this example, we set $P_1 = 20$, $P_2 = 50$, $Q = 20$, and $N = 60$. Figure 3 compares the achievable rate regions obtained when the state is known at one encoder by using dirty paper coding alone as well as by using cancellation and dirty paper coding together, when the state is known at both encoders [4], and when the state is known at neither encoder. Figure 3 shows that this power sharing technique between partial state cancellation and pure information transmission yields a larger achievable rate region than applying only dirty paper coding at encoder 1. This figure shows that the two encoders benefit in terms of the achievable rates even though only one encoder has knowledge of state of the channel. It also suggests that the full rate region achieved when both encoders obtain state information may not be achievable when only one of them is informed of the channel state.

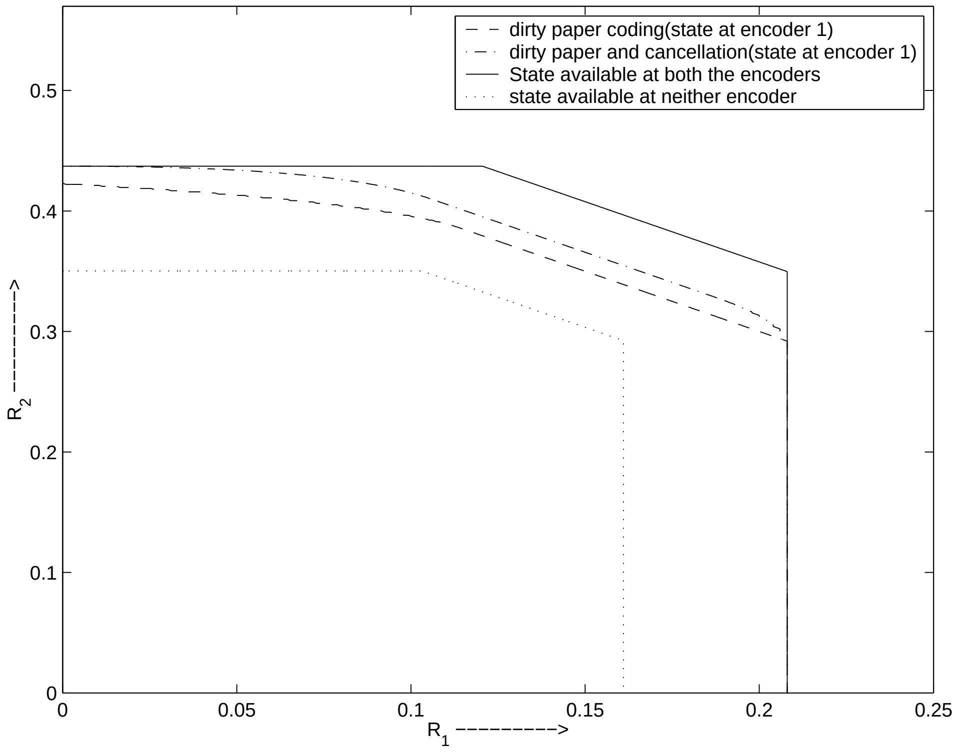

Figure 3: An achievable region of Gaussian MAC with $P_1 = 20$, $P_2 = 50$, $Q = 20$, and $N = 60$

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