User Cooperation Diversity

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Abstract

This report summarize some results from [1, 2, 3] in multiple-access channel (MAC) with user cooperative diversity. Also, a brief discussion over the relationship between this model and other channels, (e.g. MAC with feedback, relay channel), is presented and expected to inspire further discussion on the class as well as help to gain some insight over this particular model.

Keywords: MAC, User Cooperation, Diversity

1 Illustrative Idea

The basic idea behind MAC with user cooperative diversity is to introduce spatial diversity without extra cost for multi-antennas. Spatial diversity relies on the principle that signals transmitted from geographically separated transmitters, and/or to geographically separated receivers, experience fading that is independent. Therefore, if a single user decide to exploit this diversity to increase the data rates and guarantee the QoS(Quality of Service), it is desirable to have multi-antennas in separated spot. In the uplink of a cellular system, this is impractical due to the size and cost constraint of the mobile unit. To overcome this limitation, Sendonaris, etc. [1, 2], proposed a new form of spatial diversity where diversity gains are achieved via the coopearation of in-cell users. That is, each user would have some partners. As a group, the partners are responsible for transmitting both their own information and the information of their partners which they receive and detect. For one particular user, it can view

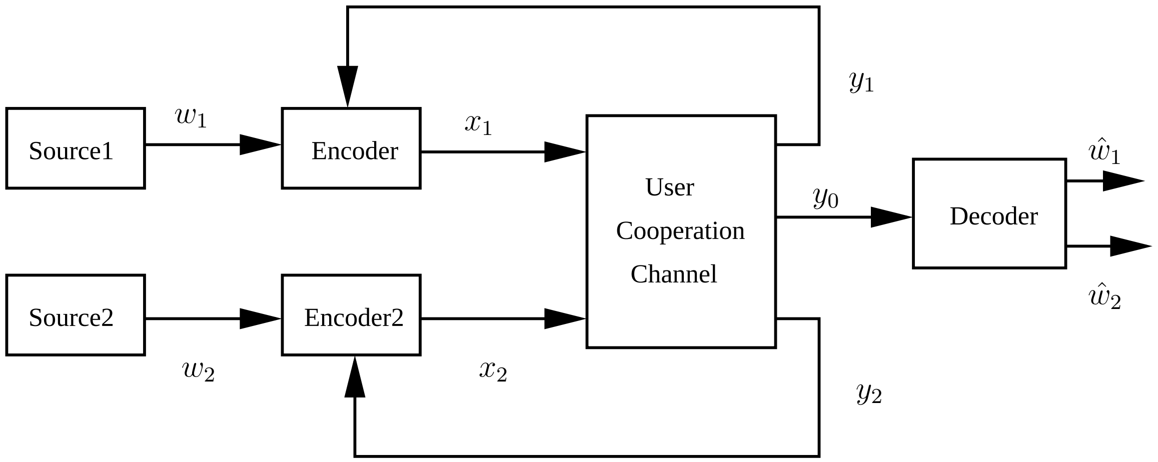

Figure 1: Multiple Access Channel with User Cooperation.

other partners as its virtual antennas and thus can expect the receiver can decode its information better with the help from information given by other partners.

Several issues contribute to the complexity of this problem. An immediate while hard-to-answer issue is how the users form a group. In the following discussion, we would assume that the users already decide to cooperate together and forget how they may make such decision. Eventually, the discussion would reveal some help hints for group policy. The problem is also complicated by the fact that the inter-user channel is noisy and partners need to send its own information too (they are not so selfless like relay channel).

One of the particular attractive application for this cooperative diversity would be in wireless ad-hoc network, where users usually have to cooperate to avoid collision anyway.

2 Model

A general model for MAC with two-user cooperation is illustrated in Figure-1. Two sources generate independent messages w_1 and w_2. These messages are encoded into codewords x_1 and x_2. One of the important characterics of the codewords is that it can also depend causally on the channel outputs y_1 and y_2, respectively. The decoder observes the channel output y_0 and estimates the source messages as $\hat{w}_1$ and $\hat{w}_2$, respectively.

A discrete-time,Guassian fading, baseband equivalent channel model can be descibed as, for each sample time k,

$$\begin{bmatrix} y_0[k] \\ y_1[k] \\ y_2[k] \end{bmatrix} = \begin{bmatrix} \alpha_{0,1}[k] & \alpha_{0,2}[k] \\ \alpha_{1,1}[k] & \alpha_{1,2}[k] \\ \alpha_{2,1}[k] & \alpha_{2,2}[k] \end{bmatrix} \times \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} z_0[k] \\ z_1[k] \\ z_2[k] \end{bmatrix} \tag{1}$$

where $\alpha_{i,j}[k]$ captures the effects of attenuation and multipath fading between output i and input j, and $z_j[k]$ represents the additive noise. In the complex Gaussian case, it is modeled as mutually independent, zero-mean, circularly -symmetric complex Gaussian white noise processes, each with variance N_j. The fading coefficients, $\alpha_{i,j}$, are assumed to be known at both encoders and decoder. In [1], it is modeled as stationary and ergodic random process. However, the way to deal with the randomness of the coefficients providing by [1] is still to get the corresponding bound under the condition of fixed coefficients and then take expectation. Another useful parameter is the signal-to-noise ratio (SNRs) between encoder i and channel output j

$$s_{j,i} = |\alpha_{j,i}|^2 \frac{P_i}{N_j}, \tag{2}$$

where P_i represents power constraint at transmitter i.

An implicit and inherent assumption in this model is that the encoders and the decoders are perfectly synchronized. That would be possible in view of the wide bandwidth supplied by today's CDMA technology. Even with a low bandwidth, this is possible when the encoder and decoder have knowledge of the phase of the fading parameter between them and correponding transmitters. Under this assumption, we can simply change our model from complex domain to real domain with corresponding distribution changed and all the argument still apply. In [2], it is shown that cooperation is still beneficiary even this assumption is violated.

Also, encoders are allowed to transmit and receive simultaneously which explains that why $x_1[k]$ affects $y_1[k]$. This is acceptable if certain medium-access-control(MAC, too) is implemented and divide the channels into orthogonal subchannels. The same reasoning applies to the decoder. In [1], it is assumed that each encoder knows its channel input so that it can cancel the effects of the input from the corresponding channel out and results in,

$$\begin{aligned} y_1[k] &= \alpha_{1,2}[k]x_2[k] + z_1[k] \\ y_2[k] &= \alpha_{2,1}[k]x_1[k] + z_2[k] \end{aligned} \tag{3}$$

As we will seen, this would results in the same outer and inner bound.

Though this model can be extended to more than two users, the discussion is limited to the two-user case for simplicity of exposition. It is not clear that the results obtained for two-user case can be naturally extened to fit more-than-two-user cases unless otherwise stated.

3 Related Channel Models

One of the interesting thing here is the similarity between this channel and other well known channel models. Various special cases can be indentified from this model. We would borrow some coding strategies from some work on those particular cases. Actually a simple way to understand the capacity region for cooperative channel is to make use of MAC channel theorem [1].

The first look over (1) shows the extreme similarity with MIMO channel. The key issue here is the dependence of $x_1[k], x_2[k]$ over the channel output y_1, y_2. In this sense, it can be viewed as MIMO channel with feedback.

If we simply remove two feedback, i.e., let $y_1 = y_2 = 0$, this is the well-known Gaussian multiple-access channel.

Suppose $y_1 = y_2 = y_0$, by which the information received by decoder is all revealed to encoders, this is a two-user MAC channel with feeback. As pointed out in the previous class, capacity regions have been established for Guassian MAC and some other specific case, but not for general MAC. The same thing happens here. In this sense, the MAC with user cooperative diversity is sometimes refered to as MAC with "Generalized" feedback, where feedback is not necessarily the exact information received by the decoder.

If let only one encoder transmits information and not receive any feedback, the other encoder does not transmit any information and receive the same channel output, i.e. $y_1 = 0, x_2 = 0$, this can be viewed as a broadcast channel.

If one encoder, e.g. Encoder 2, is transmitting no messages of its own and the other, i.e., Encoder 1, does not receive feedback,i.e., $w_2 = 0, y_1 = 0$, this gives rise to relay channel. In this case, Encoder 2 is refered to as "Relay". A strategy called "cooperation" for this relay channel is proposed by Cover and El Gamal [4]. The relay fully decodes the source message w_1 from its channel output y_2. With the message available to both the encoder and relay, the two cooperatively transmit correlated signals that coherently combine at the decoder. This method is appealing when $s_{0,2}$ is not too small and $s_{2,1}$ is large, which means that the channel condition from Relay(Encoder 2) to Decoder is not so bad to provide extra useful information and the channel condition from Encoder 1 to Relay(Encoder 2) is good enough to ensure the successful decoding at Relay. It is adopted and provide a "decode -and-forward" strategy for the two-user MAC with cooperative diversity as illustrated in the proof of achievability [3].

Furthermore, an intuitive expectation is that if the encoders know both the amplitude and phase of the fading, it is possible that we can employ some type of waterfilling to allocate power depending on the different fading states while maintaining their averate power constraint.

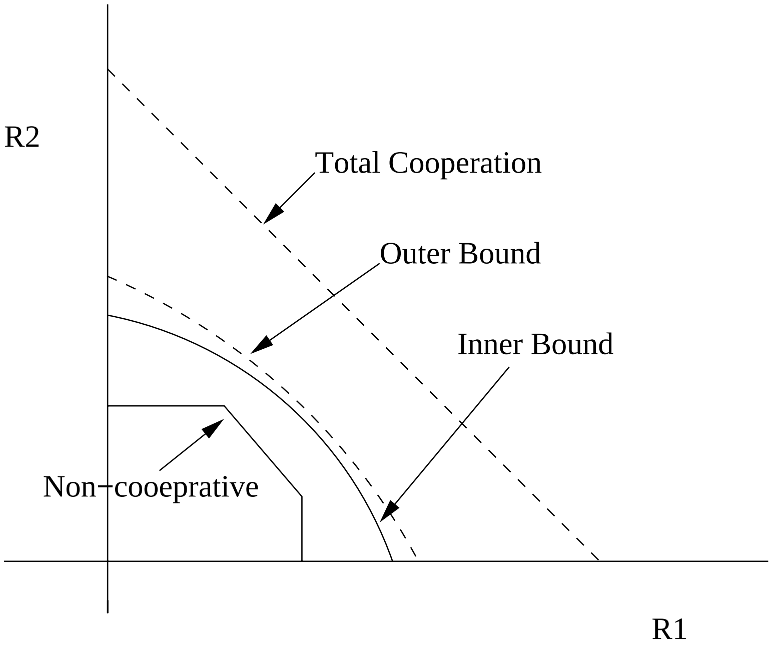

Figure 2: Outer/Inner Bound of MAC with user cooperative diversity.

4 Outer Bound

To see whether the cooperative diversity is beneficial at all, we examine the outer bound [3] on the capacity region.

Theorem 4.1 *(Outer bound on the capacity region) For the Gaussian memoryless multiple -access channel with cooperative diversity, if the rate pair (R_1,R_2) is achievable, then there exists a* $0 \leq |\rho| \leq 1$ *such that* [1]

$$R_1 \leq \log(1 + [1 - |\rho|^2][s_{0,1} + s_{2,1}]) \tag{4}$$

$$R_2 \leq \log(1 + [1 - |\rho|^2][s_{0,2} + s_{1,2}]) \tag{5}$$

$$R_1 + R_2 \leq \log(1 + s_{0,1} + s_{0,2} + 2|\rho|\sqrt{s_{0,1}s_{0,2}}) \tag{6}$$

To prove this theorem, we directly make use of Theorem 14.10.1 in [5], which is stated here for convenience, readers might refer to [5] for details.

Theorem 4.2 *(Outer Bound for General MultiTerminal Networks) If the information rates* $\{R^{ij}\}$ *are achievable, then there exists some joint probability distribution*

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[1] All logrithms are to base-2 unless stated otherwise

$p(x^{(1)}, x^{(2)}, \ldots, x^{(m)})$, such that

$$\sum_{i\in S, j\in S^c} R^{(ij)} \leq I(X^{(S)}; Y^{(S^c)}|X^{(S^c)}) \tag{7}$$

for all $S \subset \{1, 2, \ldots, m\}$. Thus the total rate of flow of information across cut-sets is bounded by the conditional mutual information.

Starting from this theorem, we immediately have

$$R_1 \leq I(\mathsf{x}_1; \mathsf{y}_0, \mathsf{y}_2|\mathsf{x}_2) \tag{8}$$

$$R_2 \leq I(\mathsf{x}_2; \mathsf{y}_0, \mathsf{y}_1|\mathsf{x}_1) \tag{9}$$

$$R_1 + R_2 \leq I(\mathsf{x}_1, \mathsf{x}_2; \mathsf{y}0) \tag{10}$$

Through some mathematical manipulation, noting the fact that the Gaussian distribution maximizes entropy subject to a covariance constraint and utilizing the power constraint, we can prove Theorem 4.1., It is noted that it follows quite similar steps as [6] except that explicit expression of matrix is pluged in. The similarity is not surprising due to the similarity between this cooperative Channel model and MIMO channel.

If the encoders do not utilize their observations y_1 and y_2, the model reduces to the classical multiple-access channel and is refered to as non-cooperative transmission [3]. The well-known capacity region for Guassian MAC is as following,

Theorem 4.3 *The set of achievable rates for non-cooperative transmission over a memoryless Gaussian multiple-access channel with cooperative diversity is given by the set of all (R_1,R_2) satisfying*

$$R_1 \leq \log(1 + s_{0,1}) \tag{11}$$

$$R_2 \leq \log(1 + s_{0,2}) \tag{12}$$

$$R_1 + R_2 \leq \log(1 + s_{0,1} + s_{0,2}) \tag{13}$$

Fig.2 illustrate bounds given by Theorem 4.1 and Theorem 4.3 as well as other region stated later. To compare the achievable sum rate (6) with the outer bound (13) [3], let $\overline{s} = (s_{0,1} + s_{0,2})/2$ be the arithmetic mean of the SNRs from the encoders to the decoder. Then the non-cooperative transmission achieves sum-rate $\log(1+2\overline{s})$. The outer bound on the sum rate (6) satisfies

$$\log(1 + 2\overline{s} + 2|\rho|\sqrt{s_{0,1}s_{0,2}}) \leq \log(1 + 4\overline{s}), \tag{14}$$

Now as $\overline{s} \to 0$, which means that the direct transmission between encoders and decoder are extremely bad, the ratio

$$\log(1 + 4\overline{s})/\log(1 + 2\overline{s}) \to 2, \tag{15}$$

so that the cooperative diversity increases the sum rate by at most a factor of two for low average SNR. On the other hand, as $\overline{s} \to \infty$, the difference

$$\log(1+4\overline{s}) - \log(1+2\overline{s}) \to 1, \tag{16}$$

which shows that the cooperative diversity increases the sum rate by at most 1b/s/Hz for high average SNR. It can be shown that for M encoders, that cooperative diversity provides sum rate no more than $\log(1+M^2\overline{s})$, while non-cooperative transmission provides sum rate at most $\log(1+M\overline{s})$, leading to gains in sum rate of at most a factor of M for low SNR and an additional $\log(M)$ b/s/Hz for high SNR [3]. This result can give some hints to grouping policy, i.e., user cooperation is most useful when the channels between two users is comparatively better with the channels between users and the receivers.

5 Achievability

Theorem 5.1 *The set of achievable rates for decode-and-forward transmission over a discrete-memoryless multiple-access channel with cooperative diversity is given by the closure of the convex hull of all (R_1,R_2) satisfying*

$$R_1 \leq I(\mathsf{x}_1;\mathsf{y}_2|\mathsf{x}_2,\mathsf{u}), \tag{17}$$

$$R_2 \leq I(\mathsf{x}_2;\mathsf{y}_1|\mathsf{x}_1,\mathsf{u}), \tag{18}$$

$$R_1 + R_2 \leq I(\mathsf{x}_1,\mathsf{x}_2;\mathsf{y}), \tag{19}$$

for some distribution $p_{\mathsf{u}}(u)p_{\mathsf{x}_1|\mathsf{u}}(x_1|u)p_{\mathsf{x}_2|\mathsf{u}}(x_2|u)$ *on* $\mathcal{U} \times \mathcal{X}_1 \times \mathcal{X}_2$

One Simple way to understand this theorem is that (18,19) are going to ensure successful decoding at the encoder. Given that the encoder decode the other's cooperative information without any error, the model can be viewed as a point-to-point MIMO channel and (19) follows directly from [6]. The outline of the achievability proof is presented as the following. Details are in [3]. The main feature is to utilize superposition block-Markov coding and backward decoding.

Sketch of Proof:

1. **Codebook Generation** :

   Fix blocklength n, Let messages $\mathsf{w}_i \in \mathcal{M}_i = 1,2,\ldots,M_i$, with $M_i = 2^{\lceil nR_i \rceil}, i = 1,2$. Suppose $w_i \in \mathcal{M}_i, i = 1,2$, and $w_0 \in \mathcal{M}_1 \times \mathcal{M}_2$. Fix distribution $p_{\mathsf{u}}(u)p_{\mathsf{x}_1|u}(x_1|u)p_{\mathsf{x}_2|u}(x_2|u)$. Generate $M_1 \cdot M_2$ codewords $\underline{\mathbf{u}}(w_0)$ i.i.d according to $p_{\underline{\mathbf{u}}}(\underline{\mathbf{u}})$. For each $\underline{\mathbf{u}}(w_0)$, generate M_1 codewords $\underline{\mathbf{x}}_1(w_1,w_0)$ i.i.d according to $p_{\underline{\mathbf{x_1|u}}}(\underline{\mathbf{x_1|u}}(w_0))$.

Block 1	Block2	...	Block B-1	Block B
$\underline{\mathbf{x_1}}(\mathsf{w}_{1,1},(1,1))$ $\underline{\mathbf{x_2}}(\mathsf{w}_{2,1},(1,1))$	$\underline{\mathbf{x_1}}(\mathsf{w}_{1,2},(\mathsf{w}_{1,1},\hat{\hat{\mathsf{w}}}_{2,1}))$ $\underline{\mathbf{x_2}}(\mathsf{w}_{2,2},(\mathsf{w}_{1,1},\hat{\hat{\mathsf{w}}}_{2,1}))$	... ...	$\underline{\mathbf{x_1}}(\mathsf{w}_{1,B-1},(\mathsf{w}_{1,B-2},\hat{\hat{\mathsf{w}}}_{2,B-2}))$ $\underline{\mathbf{x_2}}(\mathsf{w}_{2,B-1},(\mathsf{w}_{1,B-2},\hat{\hat{\mathsf{w}}}_{2,B-2}))$	$\underline{\mathbf{x_1}}(1,(\mathsf{w}_{1,B-2},\hat{\hat{\mathsf{w}}}_{2,B-2}))$ $\underline{\mathbf{x_2}}(1,(\mathsf{w}_{2,B-2},\hat{\hat{\mathsf{w}}}_{2,B-2}))$

Table 1: Block-Markove encoding structure for decode-and-forward transmission.

2. **Encoding & Decoding At Encoders:**

   We encode information into B blocks each of length n channel uses. Table. 2 shows what the encoders send in the respective blocks. Each encoder estimate the other's fresh information ,denoted as $\underline{\hat{\hat{\mathbf{w}}}}_{2,b}$ and $\underline{\hat{\hat{\mathbf{w}}}}_{1,b}$ at encoders 1 and 2, from block $b-1$ to determine the refinement information for block b.

3. **Backwards Decoding :**

   The decoder waits for all B blocks to be received , and then begins decoding with block B with the usual typical sequence idea. In general, the decoder looks for a unique $w_0 \in \mathcal{M}_1 \times \mathcal{M}_2$ such that the event $(\underline{\mathbf{u}}(w_0), \underline{\mathbf{x}}_1(\hat{\mathsf{w}}_{1,b}, w_0), \underline{\mathbf{x}}_2(\hat{\mathsf{w}}_{2,b}, w_0), \underline{\mathbf{y}}_0) \in A_\varepsilon^{(n)}(\mathsf{u}, \mathsf{x}_1, \mathsf{x}_2, \mathsf{y}_0)$ occurs.

4. **Probability of Error and Achievable Rates :**

   To bound the overall probability of error, we actually separate the event that the encoders make errors from the event that the decoder makes error given perfect encoders. Using basic properties of sets and probabilities, we can indivially prove that each of them goes to infinity as $n \to 0$ under the rates list in Theorem 5.1.

For the Gaussian case, we can have

Theorem 5.2 *The set of achievable rates for decode-and-forward transmission over a discrete-memoryless Gaussian multiple-access channel with cooperative diversity is given by the set of all (R_1,R_2) satisfying*

$$R_1 \leq \log(1 + \alpha_1 s_{2,1}), \tag{20}$$

$$R_2 \leq \log(1 + \alpha_2 s_{1,2}), \tag{21}$$

$$R_1 + R_2 \leq \log(1 + s_{0,1} + s_{0,2} + 2\sqrt{(1-\alpha_1)(1-\alpha_1)s_{0,1}s_{0,2}}), \tag{22}$$

for some $0 \leq \alpha_i \leq 1$,$i = 1, 2$.

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