Statistics Part I – Introduction

Joe Nahas

University of Notre Dame

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A Very Simple Example: A Pair of Die

- A pair of six sided die
  - Values for each die: 1, 2, 3, 4, 5, 6.
  - Values for the pair: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
  - When tossed, each side has a probability of 1/6.
- What is the Mean value of a throw of the dice?
- What is the Variance/Standard Deviation?

Experiment with Die

Collect data from fifteen throws of a pair of die.

Dice Experiment – 15 throws

Trial	a	b	x=a+b	x-μ	(x-μ)^2	
1	6	5	11	3.133	9.818	
2	6	6	12	4.133	17.084	
3	6	2	8	0.133	0.018	
4	2	6	8	0.133	0.018	
5	3	4	7	-0.867	0.751	
6	2	6	8	0.133	0.018	
7	4	2	6	-1.867	3.484	
8	2	1	3	-4.867	23.684	
9	3	1	4	-3.867	14.951	
10	6	3	9	1.133	1.284	
11	4	5	9	1.133	1.284	
12	5	2	7	-0.867	0.751	
13	6	6	12	4.133	17.084	
14	6	1	7	-0.867	0.751	
15	2	5	7	-0.867	0.751	
						"Standard
	Count		"Mean"		"Variance"	Deviation"
	15		7.867		6.552	2.560

Dice Experiment – 100 throws

Trial	a	b	x=a+b	x-μ	(x-μ)^2	
1	2	6	8	1.050	1.103	
2	3	5	8	1.050	1.103	
3	1	1	2	-4.950	24.503	
4	4	3	7	0.050	0.002	
5	6	1	7	0.050	0.002	
95	1	6	7	0.050	0.002	
96	2	1	3	-3.950	15.603	
97	6	4	10	3.050	9.303	
98	5	6	11	4.050	16.403	
99	6	4	10	3.050	9.303	
100	6	4	10	3.050	9.303	
						"Standard
	Count		"Mean"		"Variance"	Deviation"
	100		6.950		5.806	2.409

[Link: A Excel Spreadsheet Experiment]

A Very Simple Example: A Pair of Die

- What is the Mean value of a throw of the dice?
  - Theoretically?
- What is the Variance and Standard Deviation of the the values from a pair of thrown die?
  - Theoretically?

Dice Theoretical Analysis

Value	Probability	Mean μ			Variance σ^2	Standard Deviation
x	P	x*P	x -	$(x - \mu)^2$	$P*(x - \mu)^2$	σ
2	1/36	2/36	-5	25	1*25/36	
3	2/36	6/36	-4	16	2*16/36	
4	3/36	12/36	-3	9	3*9/36	
5	4/36	20/36	-2	4	4*4/36	
6	5/36	30/36	-1	1	5*1/36	
7	6/36	42/36	0	0	6*0/36	
8	5/36	40/36	1	1	5*1/36	
9	4/36	36/36	2	4	4*4/36	
10	3/36	30/36	3	9	3*9/36	
11	2/36	22/36	4	16	2*16/36	
12	1/36	12/36	5	25	1*25/36	
Sum	36/36=1	μ =252/36=			210/36=5.83	2.42

A Very Simple Example: A Pair of Die

- What is the Mean value of a throw of the dice?
  - Experimentally?
  - Theoretically?
  - What is the difference?
- What is the Variance and Standard Deviation of the the values from a pair of thrown die?
  - Experimentally?
  - Theoretically?
- What is the difference between the theoretical result and the experimental result?

Ideal vs Experiment

- Measures from theory and experiment have different names.
- Measures of Location:
  - μ - Mean – Measure of the center of the distribution based on the full population of the random variable.
  - $\bar{x}$ - Estimate of the Mean (Average) - Measure of the center of the distribution based on a sample of the random variable.
  - Very seldom is the Mean actually calculated because the full population is usually unknown; it is the Estimate of the Mean or Average that is calculated.
- Measures of Spread:
  - σ^2 – Variance – Measure of the spread of the distribution based on the full population.
  - σ – Standard Deviation
  - s^2 – Variance Estimate – Based on a sample of the population.
  - s – the Standard Deviation Estimate

Notation

Measure	Population		Sample	
Location	Mean	μ	Estimate of the Mean, Average	$\bar{x}$
Spread	Variance	σ^2	Sample Variance	s^2
	Standard Deviation	σ	Sample Standard Deviation	s
Correlation	Correlation Coefficient	ρ	Sample Correlation Coefficient	r

A Look Ahead:
What can we learn about the mean from the average?

- **Confidence Limits**
  - Sometimes called Error Bars

$$\mu \subset \bar{x} \pm t_{1-\alpha/2,N-1}\frac{s}{\sqrt{N}}$$

  - CL ∝ s
    - i.e. proportional to the estimate of the standard deviation.
  - CL ∝ $1/N^{1/2}$
    - proportional to the inverse square root of the number of samples.
    - If you want better estimates, take more samples.
  - CL ∝ $t_{1-\alpha/2,\ N-1}$
    - where $t_{1-\alpha/2,\ N-1}$ is Student's t distribution. (Student, Biometrika 1908)
    - where α is the desired significance level, e.g. 95%
    - where N is the number of samples and N-1 is the degrees of freedom for the distribution.
- **Question: What does Guinness Beer have to do with Confidence Limits?**

15 Samples

Trial	a	b	x=a+b	x-μ	(x-μ)^2	
1	5	6	11	3.200	10.240	
2	5	2	7	-0.800	0.640	
3	4	1	5	-2.800	7.840	
4	6	6	12	4.200	17.640	
5	5	5	10	2.200	4.840	
6	6	2	8	0.200	0.040	
7	6	6	12	4.200	17.640	
8	4	5	9	1.200	1.440	
9	3	1	4	-3.800	14.440	
10	3	1	4	-3.800	14.440	
11	2	1	3	-4.800	23.040	
12	4	4	8	0.200	0.040	
13	5	1	6	-1.800	3.240	
14	5	4	9	1.200	1.440	
15	5	4	9	1.200	1.440	
						"Standard
	Count		"Mean"		"Variance"	Deviation"
	15		7.800		8.457	2.908
Confidence		95%	Mean	7.800	+/-1.610	

100 Samples

Trial	a	b	x=a+b	x-μ	(x-μ)^2	
1	6	5	11	4.120	16.974	
2	1	5	6	-0.880	0.774	
3	5	2	7	0.120	0.014	
4	4	3	7	0.120	0.014	
5	1	6	7	0.120	0.014	
95	4	6	10	3.120	9.734	
96	3	3	6	-0.880	0.774	
97	2	3	5	-1.880	3.534	
98	3	5	8	1.120	1.254	
99	2	5	7	0.120	0.014	
100	6	2	8	1.120	1.254	
						"Standard
	Count		"Mean"		"Variance"	Deviation"
	100		6.880		5.339	2.311
Confidence		95%	Mean	6.880	+/-0.458	

Note smaller confidence range

[Link: A Excel Spreadsheet Experiment]

What Have We Learned

- We most often do not know the distribution of the underlying population of the random variable.
- We can estimate parameters such as mean and standard deviation from a sample of data but we cannot know their actual values.
- We can estimate a range of possibilities for the parameters with a certain degree of confidence.

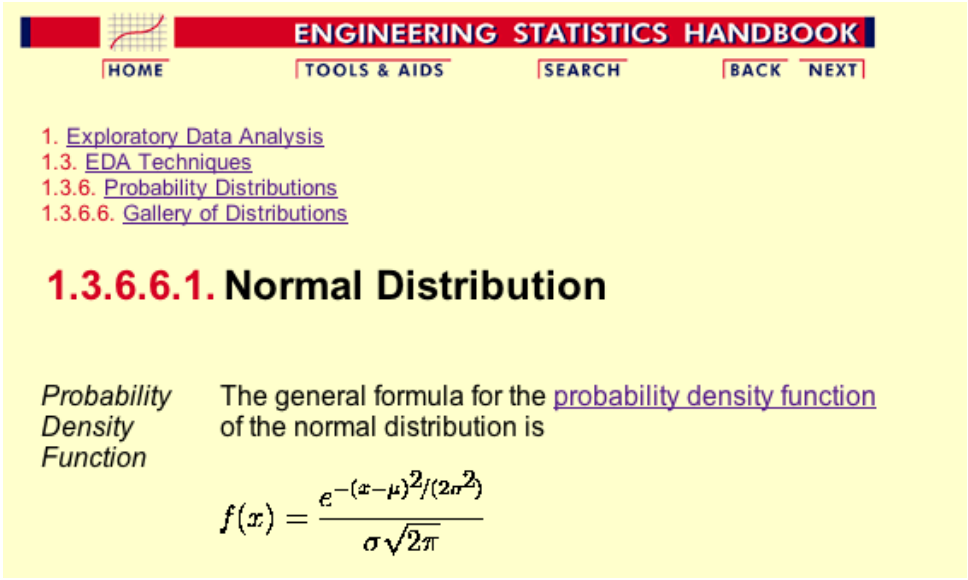
Why Statistics?

Why do we need to study statistics to do good research?

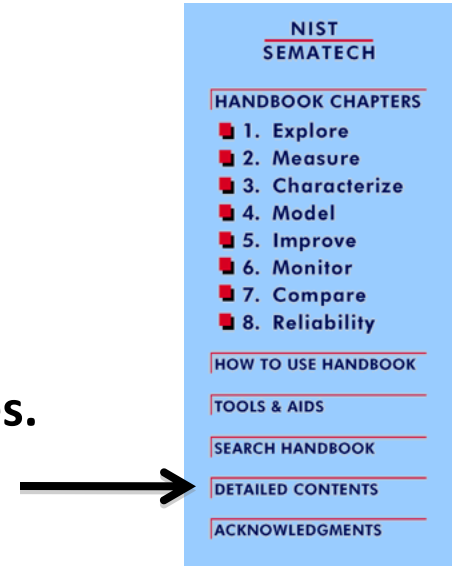
- **Variability of process parameters is prevalent.**
- **Ultimate determination of variability is through experimental measurements.**
- **Patterns of variability are complex.**
  - **i.e. contribution of various sources to the total variation.**
- **Separate signal from noise.**
  - **e.g. the contribution of the measurement system.**
- **To be able to make informed decisions in the presence of uncertainty.**
- **To properly interpret, model, and use the data, we need an understanding of formal statistical techniques.**

Online References

- **NIST/SEMATECH Engineering Statistics Handbook (NIST ESH)**
  - **http://www.itl.nist.gov/div898/handbook/index.htm**
  - **Slides will contain section references in the handbook**
    - **E. g. :For the slides on the Normal Distribution: NIST ESH 1.3.6.6.2**



  - **Click on Detailed Contents on Home page to find the pages.**



References on Slides

References and links to the NIST Engineering Statistics Handbook will be placed here.

Sources

- **Lectures contain material from:**
  - **Prof. Michael Orshansky, ECE, UT Austin**
  - **Profs. Kameshwar Poolla, and Costas J. Spanos, EECS, UC Berkeley**
  - **Patricia A. Nahas**

Statistic Outline

1. Background:
   - A. Why Study Statistics and Statistical Experimental Design?
   - B. References
2. Basic Statistical Theory
   - A. Basic Statistical Definitions
     - i. Distributions
     - ii. Statistical Measures
     - iii. Independence/Dependence
       - a. Correlation Coefficient
       - b. Correlation Coefficient and Variance
       - c. Correlation Example
   - B. Basic Distributions
     - i. Discrete vs. Continuous Distributions
     - ii. Binomial Distribution
     - iii. Normal Distribution
     - iv. The Central Limit Theorem
       - a. Definition
       - b. Dice as an example

Statistic Outline (cont.)

3. Graphical Display of Data
   - A. Histogram
   - B. Box Plot
   - C. Normal Probability Plot
   - D. Scatter Plot
   - E. MatLab Plotting
4. Confidence Limits and Hypothesis Testing
   - A. Student's t Distribution
     - i. Who is "Student"
     - ii. Definitions
   - B. Confidence Limits for the Mean
   - C. Equivalence of two Means