# Statistics Part IV
# Confidence Limits and Hypothesis Testing

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# Statistic Outline (cont.)

3. Graphical Display of Data
    A. Histogram
    B. Box Plot
    C. Normal Probability Plot
    D. Scatter Plot
    E. MatLab Plotting
4. Confidence Limits and Hypothesis Testing
    A. Student's t Distribution
        i. Who is "Student"
        ii. Definitions
    B. Confidence Limits for the Mean
    C. Equivalence of two Means
    D. Equivalence of two Variances

# Student’s t Distribution

- Suppose a random sample of size n is drawn from a normal N( $\mu$ , $\sigma$ ) population.
- If $\bar{x}$ is the estimate of the mean from the sample, and s is the sample standard deviation, then

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

  has the t distribution with n-1 degrees of freedom.
- There is a different t distribution for each sample size n as specified in the degrees of freedom.
- WE ARE NOT GOING TO PROVE THIS!

# Who was Student?

- The t-distribution was discovered by William S. Gossett, a statistician employed by the Guinness Brewing Company.
  - He was trying to determine how accurate the data from his small samples were.
- Guinness previously had problems with proprietary information being published so it required Gossett not to publish his discoveries under his own name.
  - Guinness did not want its competitors to know that it was using statistics to improve its beer.
- He published the t-distribution under the pen name "Student" in 1908.
- The distribution is usually referred to as

Student's t Distribution

# Student's t distribution

- The probability density function for the t distribution is:

$$f(x) = \frac{\left(1+\frac{x^2}{\nu}\right)^{\frac{-(\nu+1)}{2}}}{\mathrm{B}(0.5, 0.5\nu)\sqrt{\nu}}$$

where B is the Beta function and $\nu$ is a positive integer shape parameter.

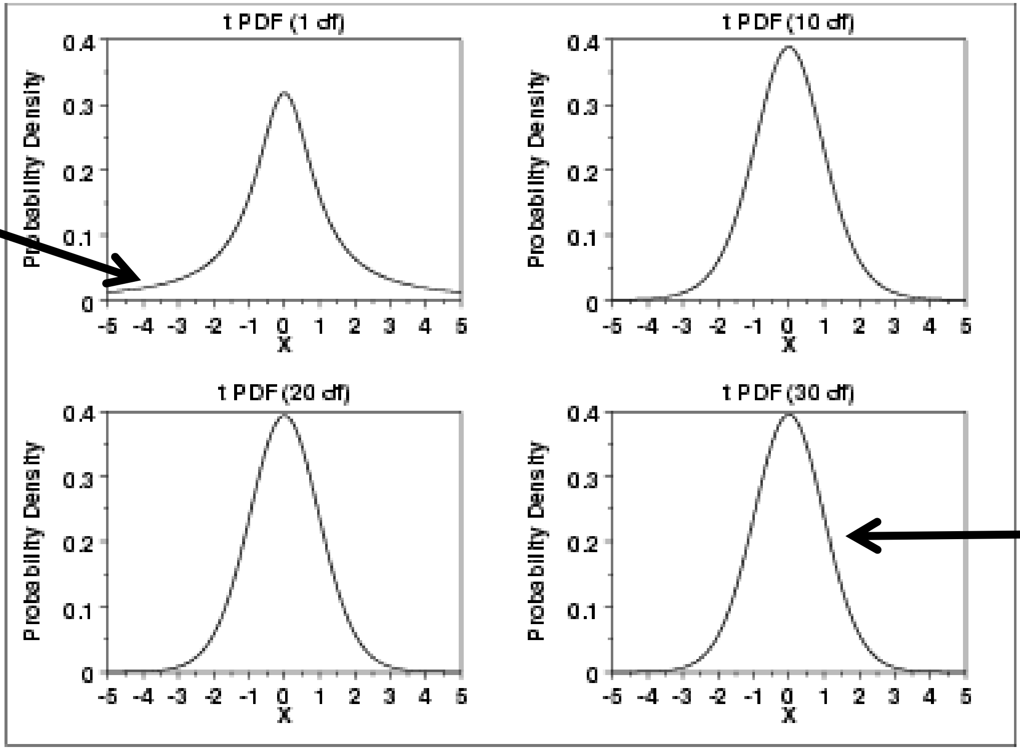
The Beta function is:

$$\mathrm{B}(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt$$

- The t-distribution is equal to the Cauchy distribution for $\nu$ = 1.
- The t-distribution approaches the normal distribution for large $\nu$.

# t Probability Density Function

Large tails for $\nu = 1$




Approaches Normal Distribution for large $\nu$

# Confidence Limits for the Mean

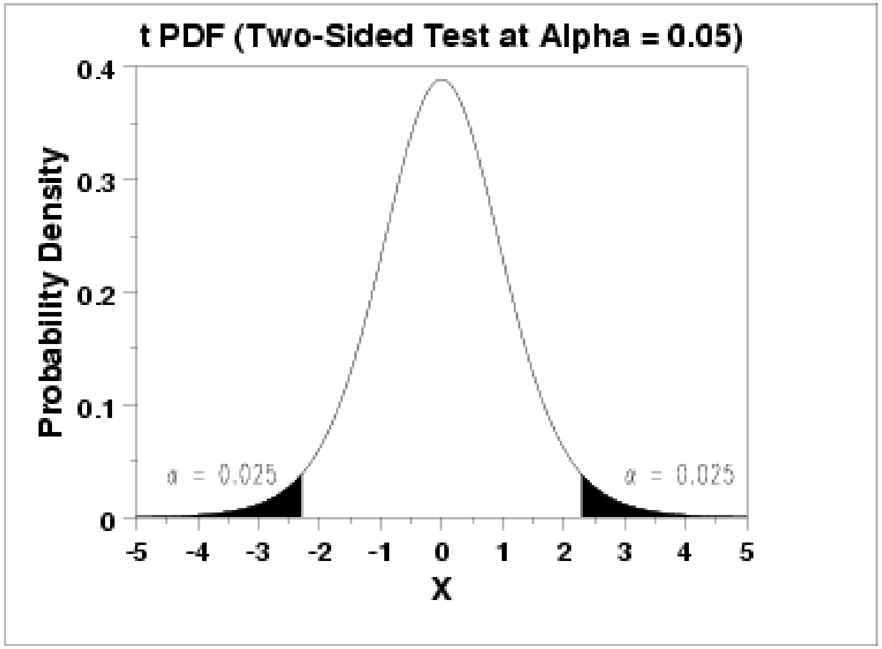
- By definition

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

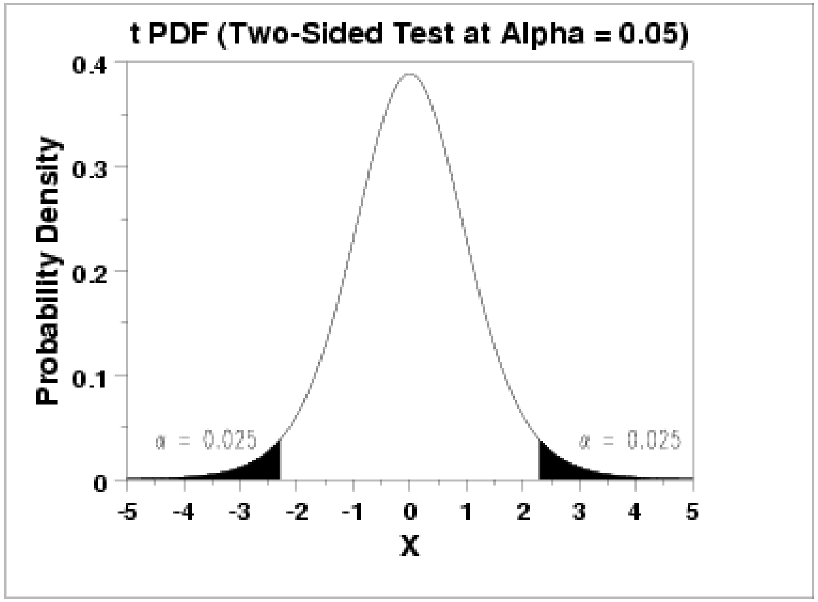
- So

$$\mu = \bar{x} - t_{\alpha,\nu}\frac{s}{\sqrt{n}}$$

- The probably value of $\mu$ is distributed around $\bar{x}$.



# t-distribution Table Instructions




Given a specified value for $\alpha$ :

1. For a two-sided test, find the column corresponding to $1-\alpha/2$ and reject the null hypothesis if the absolute value of the test statistic is greater than the value of $t_{1-\alpha/2,\nu}$ in the table below.
2. For an upper, one-sided test, find the column corresponding to $1-\alpha$ and reject the null hypothesis if the test statistic is greater than the table value.
3. For a lower, one-sided test, find the column corresponding to $1-\alpha$ and reject the null hypothesis if the test statistic is less than the negative of the table value.

# t-distribution table

Probability less than the critical value ($t_{1-\alpha,\nu}$)

| $\nu$ | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 |
|---|---|---|---|---|---|---|
| 1. | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.313 |
| 2. | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 3. | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 |
| 4. | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 |
| 5. | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 |
| 6. | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 |
| 7. | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.782 |
| 8. | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.499 |
| 9. | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.296 |
| 10. | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.143 |
| 11. | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.024 |
| 12. | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.929 |
| 13. | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 |
| 14. | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 |
| 15. | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 |
| 16. | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 |
| 17. | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |
| 18. | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 |
| 19. | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 |
| 20. | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 |
| 21. | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 |
| 22. | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 |
| 23. | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 |
| 24. | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 |
| 25. | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 |
| 26. | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 |
| 27. | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 |
| 28. | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 |
| 29. | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 |
| 30. | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 |

Probability less than the critical value ($t_{1-\alpha,\nu}$)

| $\nu$ | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 |
|---|---|---|---|---|---|---|
| 31. | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 3.375 |
| 32. | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 |
| 33. | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 3.356 |
| 34. | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 |
| 35. | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 3.340 |
| 36. | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 |
| 37. | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 3.326 |
| 38. | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 |
| 39. | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 3.313 |
| 40. | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 |
| 41. | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 3.301 |
| 42. | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 |
| 43. | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 3.291 |
| 44. | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 |
| 45. | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 3.281 |
| 46. | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 |
| 47. | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 3.273 |
| 48. | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 |
| 49. | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 3.265 |
| 50. | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 |
| 51. | 1.298 | 1.675 | 2.008 | 2.402 | 2.676 | 3.258 |
| 52. | 1.298 | 1.675 | 2.007 | 2.400 | 2.674 | 3.255 |
| 53. | 1.298 | 1.674 | 2.006 | 2.399 | 2.672 | 3.251 |
| 54. | 1.297 | 1.674 | 2.005 | 2.397 | 2.670 | 3.248 |
| 55. | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 | 3.245 |
| 56. | 1.297 | 1.673 | 2.003 | 2.395 | 2.667 | 3.242 |
| 57. | 1.297 | 1.672 | 2.002 | 2.394 | 2.665 | 3.239 |
| 58. | 1.296 | 1.672 | 2.002 | 2.392 | 2.663 | 3.237 |
| 59. | 1.296 | 1.671 | 2.001 | 2.391 | 2.662 | 3.234 |
| 60. | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 |

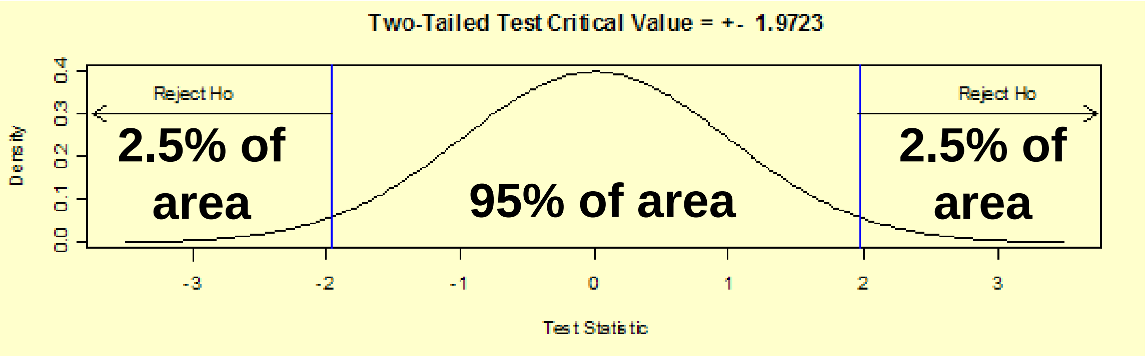
# t-distribution table

| | Probability less than the critical value ($t_{1-\alpha,\nu}$) | | | | | |
|---|---|---|---|---|---|---|
| $\nu$ | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 |
| 61. | 1.296 | 1.670 | 2.000 | 2.389 | 2.659 | 3.229 |
| 62. | 1.295 | 1.670 | 1.999 | 2.388 | 2.657 | 3.227 |
| 63. | 1.295 | 1.669 | 1.998 | 2.387 | 2.656 | 3.225 |
| 64. | 1.295 | 1.669 | 1.998 | 2.386 | 2.655 | 3.223 |
| 65. | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 | 3.220 |
| 66. | 1.295 | 1.668 | 1.997 | 2.384 | 2.652 | 3.218 |
| 67. | 1.294 | 1.668 | 1.996 | 2.383 | 2.651 | 3.216 |
| 68. | 1.294 | 1.668 | 1.995 | 2.382 | 2.650 | 3.214 |
| 69. | 1.294 | 1.667 | 1.995 | 2.382 | 2.649 | 3.213 |
| 70. | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 |
| 71. | 1.294 | 1.667 | 1.994 | 2.380 | 2.647 | 3.209 |
| 72. | 1.293 | 1.666 | 1.993 | 2.379 | 2.646 | 3.207 |
| 73. | 1.293 | 1.666 | 1.993 | 2.379 | 2.645 | 3.206 |
| 74. | 1.293 | 1.666 | 1.993 | 2.378 | 2.644 | 3.204 |
| 75. | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 | 3.202 |
| 76. | 1.293 | 1.665 | 1.992 | 2.376 | 2.642 | 3.201 |
| 77. | 1.293 | 1.665 | 1.991 | 2.376 | 2.641 | 3.199 |
| 78. | 1.292 | 1.665 | 1.991 | 2.375 | 2.640 | 3.198 |
| 79. | 1.292 | 1.664 | 1.990 | 2.374 | 2.640 | 3.197 |
| 80. | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 |

| | Probability less than the critical value ($t_{1-\alpha,\nu}$) | | | | | |
|---|---|---|---|---|---|---|
| $\nu$ | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 |
| 81. | 1.292 | 1.664 | 1.990 | 2.373 | 2.638 | 3.194 |
| 82. | 1.292 | 1.664 | 1.989 | 2.373 | 2.637 | 3.193 |
| 83. | 1.292 | 1.663 | 1.989 | 2.372 | 2.636 | 3.191 |
| 84. | 1.292 | 1.663 | 1.989 | 2.372 | 2.636 | 3.190 |
| 85. | 1.292 | 1.663 | 1.988 | 2.371 | 2.635 | 3.189 |
| 86. | 1.291 | 1.663 | 1.988 | 2.370 | 2.634 | 3.188 |
| 87. | 1.291 | 1.663 | 1.988 | 2.370 | 2.634 | 3.187 |
| 88. | 1.291 | 1.662 | 1.987 | 2.369 | 2.633 | 3.185 |
| 89. | 1.291 | 1.662 | 1.987 | 2.369 | 2.632 | 3.184 |
| 90. | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 |
| 91. | 1.291 | 1.662 | 1.986 | 2.368 | 2.631 | 3.182 |
| 92. | 1.291 | 1.662 | 1.986 | 2.368 | 2.630 | 3.181 |
| 93. | 1.291 | 1.661 | 1.986 | 2.367 | 2.630 | 3.180 |
| 94. | 1.291 | 1.661 | 1.986 | 2.367 | 2.629 | 3.179 |
| 95. | 1.291 | 1.661 | 1.985 | 2.366 | 2.629 | 3.178 |
| 96. | 1.290 | 1.661 | 1.985 | 2.366 | 2.628 | 3.177 |
| 97. | 1.290 | 1.661 | 1.985 | 2.365 | 2.627 | 3.176 |
| 98. | 1.290 | 1.661 | 1.984 | 2.365 | 2.627 | 3.175 |
| 99. | 1.290 | 1.660 | 1.984 | 2.365 | 2.626 | 3.175 |
| 100. | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 |

# Confidence Limits of the Mean

- Confidence Limits are a two sided test.
  - i.e. the real mean can be greater than or less then the estimate.
- Example:
  - n = 195 m = 9.2615 s = 0.0228
  - 95% confidence interval
  - $\alpha = 0.05$
  - $t_{1-\alpha/2,194} = 1.9723$
  - Lower Limit = m – t * s / sqrt(n) = 9.2615 – 1.9723 * 0.0228 / sqrt(195) = 9.2582
  - Upper Limit = m + t * s / sqrt(n) = 9.2615 + 1.9723 * 0.0228 / sqrt(195) = 9.2647



# t distribution in Excel

- **Values of t can be obtained using the TINV function in Excel.**
- **TINV(probability, degrees of freedom)**
  - **probability = $\alpha$ for a <u>two sided</u> distribution**
    - **e.g. instead of 1-$\alpha$/2 = 0.975 in table, use $\alpha$ = 0.05**
  - **degrees of freedom = $\nu$ = n-1**
- **For the previous example:**

**=tinv(0.05, 194) returns 1.9723**

| | |
|---|---|
| alpha | 5% |
| N | 195 |
| N - 1 = d of f | 194 |
| = TINV(alpha, N-1) | 1.9723 |

- **For a one-sided distribution, use 2* $\alpha$**
- **Hint: Before using tinv, try duplicating an example in the NIST ESH.**

# Are two Means Possibly Equal

- We have two estimates of the mean, $m_1$ and $m_2$ with $m_1 > m_2$.
- We have two estimates of the standard deviation, $s_1$ and $s_2$.
- We have two sample sizes, $n_1$ and $n_2$.
- This is a one sided test.
- Null Hypothesis: $\mu_1 = \mu_2$.
- Test Statistic

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2 / n_1 + s_2^2 / n_2}}$$

  - Similar to $t = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

# Are two Means Possibly Equal

- **Reject the Null Hypothesis if:**

$$T > t_{1-\alpha,\nu}$$

  - **where $t_{1-\alpha,\nu}$ is the critical value of the t-distribution with $\nu$ degrees of freedom where**

$$\nu = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}$$

# Equal Variances

- **If equal variances are assumed:**

$$T = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{1/n_1 + 1/n_2}}$$

  – **where $s_p$ is the pooled estimate of the standard deviation:**

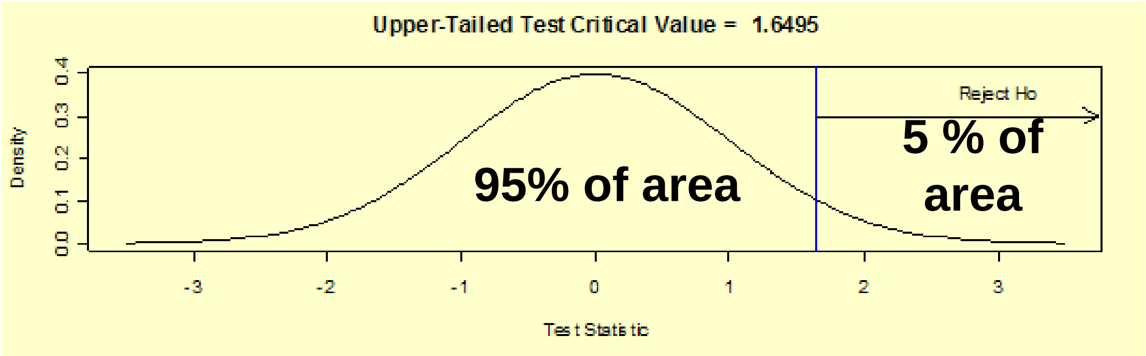
$$s_p = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

- **and**

$$\nu = n_1 + n_2 - 1$$

# Example

- Mileage Data from US and Japanese cars in 1990s
  - $n_1 = 79$ $m_1 = 30.48$ $s_1 = 6.108$
  - $n_2 = 249$ $m_2 = 20.14$ $s_2 = 6.415$
- Assuming variances are equal
  - $T = 12.621$
  - $s_p = 6.343$
  - $\nu = 326$
- For 95% confidence, $\alpha = 0.05$
- $t_{0.95, \nu=326} = 1.6495$
- Since T > t, the hypothesis that the means are equal is rejected!



# Large n

- **What happens to the confidence limits as n gets large?**

$$\mu = \lim_{x\to\infty}\left(\bar{x} - t_{\alpha,\nu}\frac{s}{\sqrt{n}}\right) = \bar{x}$$

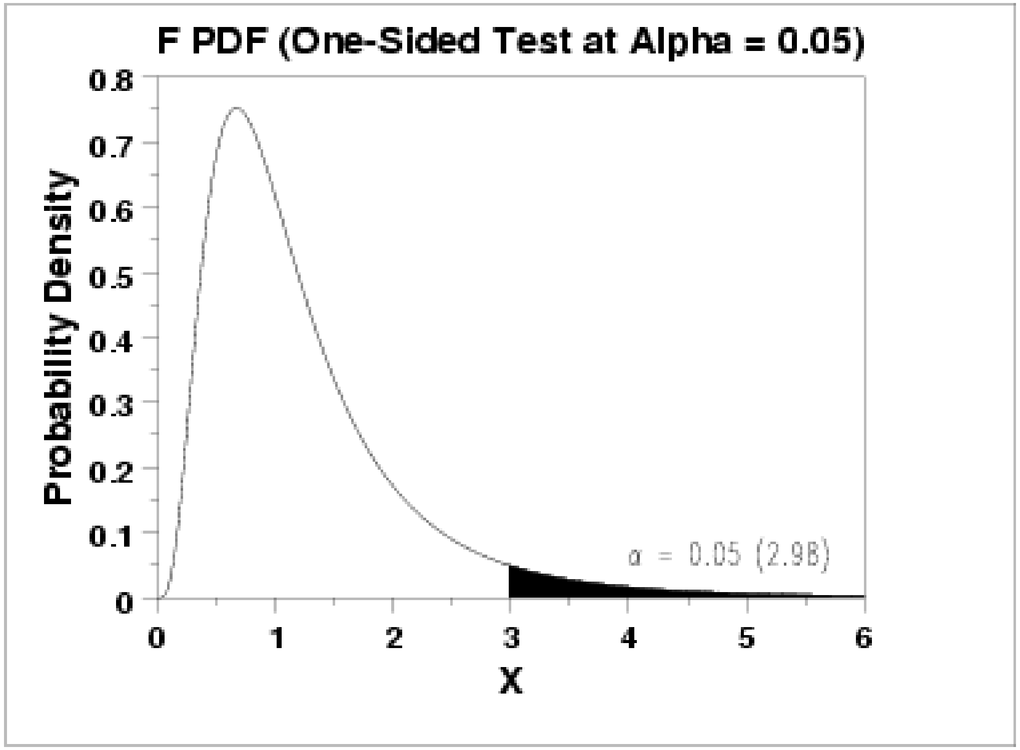
# Are Two Variances Equal?

- **Null Hypothesis, H0:** $\sigma_1^2 = \sigma_2^2$
- **Alternative Hypotheses, Ha:**
  - $\sigma_1^2 < \sigma_2^2$ **for a lower one-tailed test**
  - $\sigma_1^2 > \sigma_2^2$ **for an upper one-tailed test**
  - $\sigma_1^2 \neq \sigma_2^2$ **for a two-tailed test**
  - **Test Statistic: $F = s_1^2/s_2^2$**
  - **Where $s_1^2$ and $s_2^2$ are the sample variances with sample sizes of $N_1$ and $N_2$ respectively**
  - **Significance Level: $\alpha$**

# Are Two Variances Equal? (cont.)

- The Hypothesis that the two variances, $\sigma$ 12, and $\sigma$ 22, are equal is rejected if:
  - $F > F_{\alpha, N_1-1, N_2-1}$ for an upper one-tailed test
  - $F < F_{1-\alpha, N_1-1, N_2-1}$ for a lower one-tailed test
  - $F > F_{\alpha, N_1-1, N_2-1}$ or $F < F_{1-\alpha, N_1-1, N_2-1}$ for a two-tailed test
- where $F_{\alpha, N_1-1, N_2-1}$ is the critical value of the F distribution with $N_1$-1 and $N_2$-1 degrees of freedom and a significance level of $\alpha$ .

# F Distribution



# F Distribution

Upper critical values of the F distribution
for $\nu_1$ numerator degrees of freedom and $\nu_2$ denominator degrees of freedom

5% significance level

$F_{.05}(\nu_1, \nu_2)$

| $\nu_2$ \ $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 161.448 | 199.500 | 215.707 | 224.583 | 230.162 | 233.986 | 236.768 | 238.882 | 240.543 | 241.882 |
| 2 | 18.513 | 19.000 | 19.164 | 19.247 | 19.296 | 19.330 | 19.353 | 19.371 | 19.385 | 19.396 |
| 3 | 10.128 | 9.552 | 9.277 | 9.117 | 9.013 | 8.941 | 8.887 | 8.845 | 8.812 | 8.786 |
| 4 | 7.709 | 6.944 | 6.591 | 6.388 | 6.256 | 6.163 | 6.094 | 6.041 | 5.999 | 5.964 |
| 5 | 6.608 | 5.786 | 5.409 | 5.192 | 5.050 | 4.950 | 4.876 | 4.818 | 4.772 | 4.735 |
| 6 | 5.987 | 5.143 | 4.757 | 4.534 | 4.387 | 4.284 | 4.207 | 4.147 | 4.099 | 4.060 |
| 7 | 5.591 | 4.737 | 4.347 | 4.120 | 3.972 | 3.866 | 3.787 | 3.726 | 3.677 | 3.637 |
| 8 | 5.318 | 4.459 | 4.066 | 3.838 | 3.687 | 3.581 | 3.500 | 3.438 | 3.388 | 3.347 |
| 9 | 5.117 | 4.256 | 3.863 | 3.633 | 3.482 | 3.374 | 3.293 | 3.230 | 3.179 | 3.137 |
| 10 | 4.965 | 4.103 | 3.708 | 3.478 | 3.326 | 3.217 | 3.135 | 3.072 | 3.020 | 2.978 |
| 11 | 4.844 | 3.982 | 3.587 | 3.357 | 3.204 | 3.095 | 3.012 | 2.948 | 2.896 | 2.854 |
| 12 | 4.747 | 3.885 | 3.490 | 3.259 | 3.106 | 2.996 | 2.913 | 2.849 | 2.796 | 2.753 |
| 13 | 4.667 | 3.806 | 3.411 | 3.179 | 3.025 | 2.915 | 2.832 | 2.767 | 2.714 | 2.671 |
| 14 | 4.600 | 3.739 | 3.344 | 3.112 | 2.958 | 2.848 | 2.764 | 2.699 | 2.646 | 2.602 |
| 15 | 4.543 | 3.682 | 3.287 | 3.056 | 2.901 | 2.790 | 2.707 | 2.641 | 2.588 | 2.544 |
| 16 | 4.494 | 3.634 | 3.239 | 3.007 | 2.852 | 2.741 | 2.657 | 2.591 | 2.538 | 2.494 |
| 17 | 4.451 | 3.592 | 3.197 | 2.965 | 2.810 | 2.699 | 2.614 | 2.548 | 2.494 | 2.450 |
| 18 | 4.414 | 3.555 | 3.160 | 2.928 | 2.773 | 2.661 | 2.577 | 2.510 | 2.456 | 2.412 |
| 19 | 4.381 | 3.522 | 3.127 | 2.895 | 2.740 | 2.628 | 2.544 | 2.477 | 2.423 | 2.378 |
| 20 | 4.351 | 3.493 | 3.098 | 2.866 | 2.711 | 2.599 | 2.514 | 2.447 | 2.393 | 2.348 |
| 21 | 4.325 | 3.467 | 3.072 | 2.840 | 2.685 | 2.573 | 2.488 | 2.420 | 2.366 | 2.321 |
| 22 | 4.301 | 3.443 | 3.049 | 2.817 | 2.661 | 2.549 | 2.464 | 2.397 | 2.342 | 2.297 |
| 23 | 4.279 | 3.422 | 3.028 | 2.796 | 2.640 | 2.528 | 2.442 | 2.375 | 2.320 | 2.275 |
| 24 | 4.260 | 3.403 | 3.009 | 2.776 | 2.621 | 2.508 | 2.423 | 2.355 | 2.300 | 2.255 |
| 25 | 4.242 | 3.385 | 2.991 | 2.759 | 2.603 | 2.490 | 2.405 | 2.337 | 2.282 | 2.236 |

# Using Excel for F Dist

- Use finv( $\alpha$ , N-1, N-1) function in Excel.
  - Use exampel in NIST ESH to check usage.

| | |
|---|---|
| Alpha | 0.05 |
| N1 | 240 |
| N2 | 240 |
| | |
| FINV(a/2, N1-1, N2-2)= | 1.289384 |
| FINV(1-a/2, N1-1, N2-1)= | 0.775564 |

# Ceramic Data Example

```
BATCH 1:
    NUMBER OF OBSERVATIONS        =      240
    MEAN                          =   688.9987
    STANDARD DEVIATION            =   65.54909

BATCH 2:
    NUMBER OF OBSERVATIONS        =      240
    MEAN                          =   611.1559
    STANDARD DEVIATION            =   61.85425
```

**Is 65.5 significantly different from 61.9?**

$H_0$: $\sigma_1^2 = \sigma_2^2$

$H_a$: $\sigma_1^2 \neq \sigma_2^2$

Test statistic: $F = 1.123037$

Numerator degrees of freedom: $N_1 - 1 = 239$

Denominator degrees of freedom: $N_2 - 1 = 239$

Significance level: $\alpha = 0.05$

Critical values: $F(1-\alpha/2, N_1-1, N_2-1) = 0.7756$

$F(\alpha/2, N_1-1, N_2-1) = 1.2894$

Rejection region: Reject $H_0$ if $F < 0.7756$ or $F > 1.2894$

**From Excel**

| | |
|---|---|
| Alpha | 0.05 |
| N1 | 240 |
| N2 | 240 |
| | |
| FINV(a/2, N1-1, N2-2)= | 1.289384 |
| FINV(1-a/2, N1-1, N2-1)= | 0.775564 |