Statistical Design of Experiments Part I
Overview

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Quality in Japan

- After WWII, Japan restarted their economy by manufacturing inexpensive, low quality goods.
- By the mid-1970's, Japanese car makers, Toyota, Honda, and Nissan, began entering the US market with small, inexpensive cars, the Corolla, the Civic, and the Datsun.
- By the mid-1980's, people began to realize that these Japanese cars outlasted American cars by factors of 2 or more.
  - American cars were worn-out by 50k to 75k miles while many Japanese cars lasted over 100,000 miles!
- Why?

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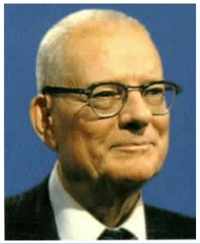
W. Edwards Deming

- W. Edwards Deming was an American who learned statistical technology from Walter Shewart of Bell Labs. He applied his learning first in the Department of Agriculture and later in WWII. After the war, he was sent to Japan to help the Japanese with their census. He stayed on, at the request of Japanese Union of Scientists and Engineers to help Japanese industry with statistical techniques. Because of his work in improving the quality of Japanese industry, the Japanese quality award is named the Deming Award.



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Genichi Taguchi

- Genichi Taguchi was initially trained as a textile engineer. After WWII he joined the Electrical Communications Laboratory (ECL) of the Nippon Telegraph and Telephone Corporation where he came under the influence of W. Edwards Deming. In 1954-1955 was visiting professor at the Indian Statistical Institute where he was introduced to the orthogonal arrays which became the foundation of his later work. He finished his doctorate in 1962 and became a professor of engineering at Aoyama Gakuin University, Tokyo where consulted with industry propagating what became known as Taguchi Methods.

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Taguchi Methods

- Off-line Quality Control
  - Use experimental design techniques to both improve a process and to reduce output variation.
    - Need to reduce a processes sensitivity to uncontrolled parameter variation.
  - The use a controllable parameter to re-center the design where is best fits the product.
- Example: Internal combustion engine cylinder and piston.

Orthogonal Array Methods

- Not new fromTaguchi
- Wide statistics literature on the subject.
- Taguchi make it accessible to engineers and propagated a limited set of methods that simplified the use of orthogonal arrays.
- Design of Experiments (DoE) is primarily covered in Section 5, Process Improvement of the NIST ESH.

Outline

1. Introduction
2. Design of Experiments Basics
3. Full Factorial Designs Simple Example
   A. 2^n Designs
   B. Single Factor
   C. 2 Factor Plots
4. Fractional Factorial Designs Arrays

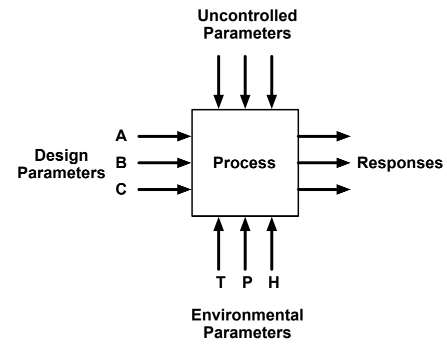
Why use Statistical Design of Experiments?

- Choosing Between Alternatives
- Selecting the Key Factors Affecting a Response
- Response Modeling to:
  - Hit a Target
  - Reduce Variability
  - Maximize or Minimize a Response
  - Make a Process Robust (i.e., the process gets the "right" results even though there are uncontrollable "noise" factors)
  - Seek Multiple Goals
- Regression Modeling

Design of Experiments

- Goal
  - Build a model of a process to efficiently control one or more responses.
  - Be able to adjust controllable parameters to obtain one or more desired responses.
  - Examples of parameters
    - Temperature (controlled or uncontrolled)
    - Pressure
    - Gas Mixture
    - Material
    - Voltage
  - Examples of response goals:
    - Yield/Defect Density
    - Variation in thickness of a layer
    - Composition of a layer
    - Speed

Process Schematic



How do we efficiently measure the response of a process to the various controllable inputs?

Process Response Model

- Two Factor Linear Model
  - $Y = \mu + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2$ + experimental error
- Three Factor Linear Model
  - $Y = \mu + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3 + \beta_{123} X_1 X_2 X_3 +$ experimental error
- μ is the mean value independent of factors.
- β_1, β_2, and β_3 are main effects.
- β_{12}, β_{13}, β_{23} are interaction terms.
- When experimental data are analyzed, the β_i terms are estimated and tested to determine if they are significantly different from 0.
- Interactions terms β_{ij} may also be estimated.
- Higher order terms, e.g., $\beta_{11} X_1^2$, are usually not included.

Assumptions

- Are the measurement systems capable for all of your responses?
- Is/Are your process/es stable?
- Are your responses likely to be approximated well by simple polynomial models?
- Are the residuals (the difference between the model predictions and the actual observations) well behaved?
  - Do they follow a normal distribution?

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Full Factorial Experiment 2^3

1. All possible combinations of the variables are used in the various runs.
   - A. Example: 2^3: Polysilicon Growth
     - i. Three Factors.
       - a. Temperature: T_1, T_2
       - b. Nitrogen flow: N_1, N_2
       - c. Silane Flow: S_1, S_2
     - ii. 8 Tests to test all combinations.
     - iii. What is to be optimized?
       - a. Defect density.

	Factors		
Test	1	2	3
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

NIST ESH 5.3.3

Full Factorial 3^2

1. All possible combinations of the variables are used in the various runs.
   - A. Example: 3^2: Polysilicon Growth
     - i. Two Factors.
       - a. Temperature: T_1, T_2, T_3
       - b. Silane Flow: S_1, S_2, S_3
     - ii. 9 Tests to test all combinations.
     - iii. What is to be optimized?
       - a. Defect density.

	Factors	
Test	1	2
1	1	1
2	2	1
3	3	1
4	1	2
5	2	2
6	3	2
7	1	3
8	2	3
9	3	3

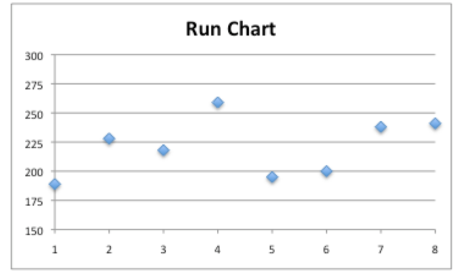
Experimental Design Questions

1. What is the responses that we are trying to improve?
   - A. There may be multiple responses which would complicate the experiment.
2. What factors may influence the responses?
   - A. What can you control?
3. At what levels should each factor be tested?
4. What other factors/variable may interfere with the results?
   - A. What can't you control?
     - i. Temperature for circuit
     - ii. Particular piece of equipment for manufacturing.
5. How many times can each test be run?
   - A. The more the better to assess variation.
6. In what order should the tests run?
   - A. For a large number of runs, randomize the order.
   - B. For a small number of runs, balance the order.

A 2^3 Experiment

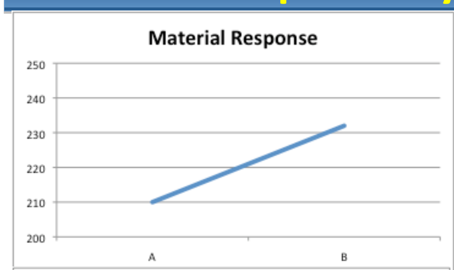
Test	A	B	C	Material	Pressure	Temperature	Result
1	-1	-1	-1	A	Low	Low	189
2	1	-1	-1	B	Low	Low	228
3	-1	1	-1	A	High	Low	218
4	1	1	-1	B	High	Low	259
5	-1	-1	1	A	Low	High	195
6	1	-1	1	B	Low	High	200
7	-1	1	1	A	High	High	238
8	1	1	1	B	High	High	241

Run Chart

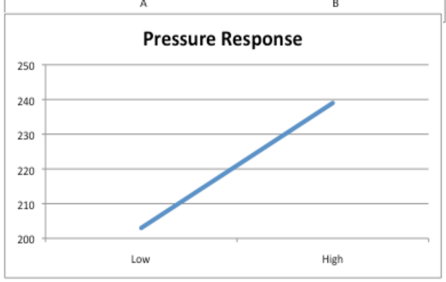


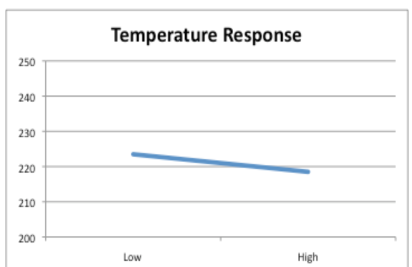
- A plot of the result in time order.
  - Is there any time dependence of the results?

Response by Parameter

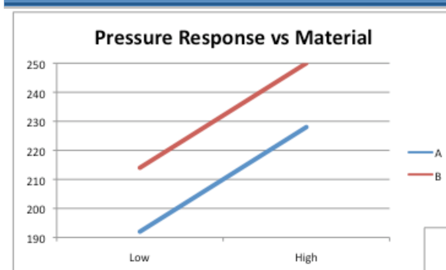


Each point averages the response for all the values of the other two variables.



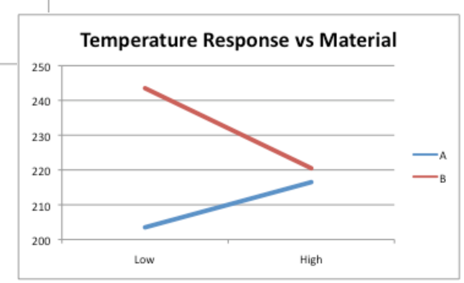


Pressure and Temperature Response vs Material



No Dependence

Each point averages the response of the two values of the other variable.



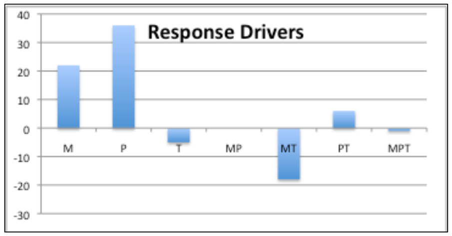
Large Dependence

Response Drivers

Test	A	B	C	M	P	T	MP	MT	PT	MPT	Result
1	-1	-1	-1	A	Low	Low	1	1	1	-1	189
2	1	-1	-1	B	Low	Low	-1	-1	1	1	228
3	-1	1	-1	A	High	Low	-1	1	-1	1	218
4	1	1	-1	B	High	Low	1	-1	-1	-1	259
5	-1	-1	1	A	Low	High	1	-1	-1	1	195
6	1	-1	1	B	Low	High	-1	1	-1	-1	200
7	-1	1	1	A	High	High	-1	-1	1	-1	238
8	1	1	1	B	High	High	1	1	1	1	241
Effect				22	36	-5	0	-18	6	-1	

MP = A * B
MT = A * C
PT = B * C
MPT = A * B * C

MP Response = (189-228-218+259+195-200-238+241)/4 = 0



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Fractional Factorial 2^{3-1}

- Not all parameter combinations are tested.
- Parameter values are balanced against every other parameter.
- Example: 2^{3-1}
  - Note balance for each parameter:
    - Each value of A has both values for B and C.
    - Each value of B has both values for A and C.
    - Each value of C has both values for A and B.

	Parameter		
Test	A	B	C
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Fractional Factorial 2^{7-4}

- Not all parameter combinations are tested.
- Parameter values are balanced against every other parameter.
  - i. e. the array is orthogonal.
- Example: 2^{7-4}
  - Note balance for each parameter:
    - Each value of A has both values for B, C, D, E, F, and G.
    - Each value of B has both values for A, C, D, E, F, and G.
    - Each value of C has both values for A, B, D, E, G and G.
    - Etc.

	Parameter						
	A	B	C	D	E	F	G
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2