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Title: Almost maximal growth of the Hilbert function.

The Hilbert function of a graded algebra $R/I = \bigoplus_{i\in\mathbb{Z}}[R/I]_i$ is the function $h_{R/I}(t) = \dim[R/I]_t$. For large t, this agrees with the Hilbert polynomial, which in general conveys a great deal of geometric information (e.g. for curves it gives the degree and the arithmetic genus). What information, especially geometric, can we get from lower degrees? And what numerical functions actually occur as Hilbert functions? An elegant answer to the latter question was provided by Macaulay (1927) in terms of the growth of the function from any degree to the next. A partial answer to the former question was given by Gotzmann (1978), who described what happens when the growth is the maximum possible allowed by Macaulay (called "maximal growth"). An interesting special case is when the algebra is the Artinian reduction, A, of the coordinate ring of a finite set of points in projective space. When the points lie in $\mathbb{P}^2$, Davis (1985) gave a beautiful consequence of A having maximal growth (although he did not state it in terms of maximal growth). When the points lie in a larger projective space, Bigatti, Geramita and I (1994) extended the result of Davis. Now with Luca Chiantini, we asked what can be said if the growth is one less than maximal, which we call "almost maximal growth." Now the results of Gotzmann, of Davis, and of Bigatti-Geramita-Migliore go out the window. Although their conclusions no longer hold now, we give results in Gotzmann's setting and in the Davis/Bigatti-Geramita-Migliore setting.