

Exam I

February 9, 2012

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Place an \times through your answer to each problem.

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Multiple Choice

1. (5 pts.) Let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c, d, e\}$, and $B = \{c, d, e, f, g\}$. Find $(A \cap B)' \cap A$.

- (a) $\{h\}$ (b) $\{a, b, f, g, h\}$ (c) $\{a, b\}$
(d) \emptyset (e) $\{c, d, e\}$

2. (5 pts.) Let

$$\begin{aligned}U &= \{\text{all people}\} \\A &= \{\text{people who have been in Argentina}\} \\B &= \{\text{people who have been in Bolivia}\}\end{aligned}$$

Which of the following represents in words the set $(A \cap B)'$? [Hint: First decide what $A \cap B$ means, then decide what its complement is.]

- (a) People who have not been in **either** Argentina **or** Bolivia.
(b) People who have **either** been in Argentina but not Bolivia, **or** in Bolivia but not Argentina.
(c) People who have been in Argentina and Bolivia.
(d) People who have never been in South America.
(e) People who have not been in **both** Argentina **and** Bolivia.

5. (5 pts.) A club consisting of ten men and twelve women decide to make a brochure to attract new members. On the cover of the brochure, they want to have a picture of two men and two women from the club. How many pictures are possible (taking into account the order in which they line up for the picture)? [Hint: there are at least two ways to do this, so if you don't see your answer in terms of C 's and P 's, compute the numerical value just in case it matches one of these.]

(a) $[C(10, 2) \cdot C(12, 2)]! = 2, 970$

(b) $C(10, 2) \cdot C(12, 2) \cdot 4! = 71, 280$

(c) $P(10, 2) \cdot P(12, 2) = 11, 880$

(d) $[C(10, 2) + C(12, 2)] \cdot 4! = 2, 664$

(e) $P(10, 2) \cdot P(12, 2) \cdot 4! = 285, 120$

6. (5 pts.) Suppose you know that $C(n, 3) = 56$. Find n . [Hint: one quick way to do this is with Pascal's triangle.]

(a) 10

(b) 7

(c) 9

(d) 8

(e) 11

7. (5 pts.) To order a pizza, you have to first choose a sauce and then choose toppings. There are three kinds of sauces (red, white and green) and six kinds of toppings (mushroom, pepperoni, sausage, green pepper, artichoke and seaweed). You must choose one of the three sauces, and **at least one** topping. How many different pizzas can be created?

- (a) 191 (b) 189 (c) 378 (d) 383 (e) 18

8. (5 pts.) Suppose that an experiment consists of tossing a coin 10 times and recording the sequence of heads and tails. How many different outcomes have exactly 2 heads and 8 tails?

- (a) 80,640 (b) 1024 (c) 90 (d) 45 (e) 16

9. (5 pts.) Calculate the value of $C(201, 199)$.

(a) 40,400

(b) 402

(c) 39,999

(d) 200

(e) 20,100

10. (5 pts.) There are 100 Senators in the U.S. Senate, two from each of the 50 states. A committee of six Senators is to be formed, such that no two are from the same state. In how many ways can this be done?

(a) $C(100, 6) - 2^6$

(b) $C(50, 6) \cdot 2^6$

(c) $C(50, 6) \cdot 6^2$

(d) $\frac{C(100, 6)}{50}$

(e) $C(50, 6) \cdot 2^{50}$

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) All of the 500 students at State University are required to participate in at least one sport (varsity or intramural). Suppose 180 play volleyball, 200 play basketball, 250 play softball, 50 play volleyball and softball but not basketball, 75 play volleyball and basketball but not softball, 40 play only volleyball, 80 play only basketball.

(a) Draw and label a Venn Diagram representing the above information. Be sure to put a number in all relevant regions of your diagram.

(b) How many play only softball?

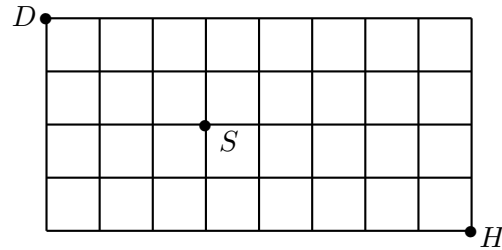
(c) How many do not play any of the three sports?

12. (10 pts.) A family has nine cats and three dogs. They want to choose four out of these twelve pets to take with them on a vacation.

(a) In how many ways can this be done?

(b) In how many ways can this be done if they have to take at least one dog?

13. (10 pts.) All three parts of this problem refer to the following city map. For this problem, you may leave your answer in terms of mixtures of combinations and permutations (i.e. $C(n, r)$ and $P(n, r)$ for appropriate n and r) if you like.



(a) On Saturday, Dave starts at the point marked D and needs to get home, marked H . If he has to do it in as few blocks as possible (12), in how many ways can it be done? (Ignore the “ S ” at this point.)

(b) On Sunday, Dave is at point D and wants to watch the Superbowl at Sam’s house, at point S . However, he needs to go home first to get his glasses. If he wants to make the trip from D to H and then from H to S in as few total blocks as possible (19), in how many ways can it be done.

(c) On Monday, Dave is at point D and wants to go home. However, he owes Sam \$100 from losing a bet on the Superbowl, so he wants to choose a 12-block route that does **not** pass through point S . In how many ways can this be done?

14. (10 pts.) An urn contains 12 numbered balls, of which 5 are red and 7 are blue. Bob has to select 4 of these balls. Please give numerical answers.

(a) In how many ways can this be done, independent of the color of the balls?

(b) How many samples contain only blue balls?

(c) How many samples contain at least 3 red balls?

15. (10 pts.) Six married couples are going to be in a group picture, all lined up in a row.

(a) In how many ways can the 12 people line up? You can give your answer using either C , P or factorial notation, or you can give a numerical answer.

(b) In how many ways can they line up if everyone has to be standing next to their spouse? You can give your answer using either C , P or factorial notation, or you can give a numerical answer.

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