

Practice Exam I

September 14, 2017

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given.

The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an \times through your answer to each problem.

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. _____

11. _____

12. _____

13. _____

14. _____

15. _____

Tot. _____

Multiple Choice

1. (5 pts.) Let $U = \{1, 2, \dots, 10\}$, $A = \{2, 4, 6\}$, $B = \{\text{odd numbers between 2 and 10}\}$. Which of the following statements is **false**?

- (a) $n(B') = 6$ (b) $A \cup B = U$ (c) $A \cap B' = A$
(d) A and B are disjoint (e) $A \subseteq B'$

2. (5 pts.) Sammy's Pizzeria has 20 different pizza toppings to choose from. They currently have a deal where if you order a large pizza, you get up to 4 toppings free. How many pizzas that qualify for this deal can be made? [Note: A pizza with no toppings is also a possibility.]

- (a) $C(20, 4)$
(b) $P(20, 4)$
(c) $P(20, 0) + P(20, 1) + P(20, 2) + P(20, 3) + P(20, 4)$
(d) $C(20, 0) \times C(20, 1) \times C(20, 2) \times C(20, 3) \times C(20, 4)$
(e) $C(20, 0) + C(20, 1) + C(20, 2) + C(20, 3) + C(20, 4)$

5. (5 pts.) Carrie is going on a trip. She has 20 books she would like to bring; 12 are mysteries and 8 are Science Fiction. In how many ways can Carrie choose 3 mysteries and 2 Science Fiction books to take on her trip?

(a) $P(20, 5)$

(b) $C(20, 5)$

(c) $P(12, 3) \cdot P(8, 2)$

(d) $C(12, 3) \cdot C(8, 2)$

(e) $C(12, 3) + C(8, 2)$

6. (5 pts.) The Finite Math Team and the Calculus Team are two Bookstore Basketball teams. Both have seven players, and they will play against each other in the first round. Both have to choose five members to start the game. How many possible choices are there for the five-against-five start of the game? In this problem we don't care which starters play which position. We just want to know who is starting.

(a) $P(7, 5) \cdot P(7, 5)$

(b) $(7!)(7!)$

(c) $\binom{7}{5} + \binom{7}{5}$

(d) $\binom{7}{5} \cdot \binom{7}{5}$

(e) $P(7, 5) + P(7, 5)$

7. (5 pts.) Euchre is a card game played with only 24 cards, namely the ace, king, queen, jack, 10 and 9 of each of the four suits. Each player is given five cards. In how many ways can a player receive five cards that are all from the same suit? (Remember that there are four suits, six cards in each suit, and it doesn't matter in what order the player receives the cards.)

(a) $4 \cdot \binom{6}{5}$

(b) $4 \cdot P(6, 5)$

(c) $\binom{24}{5}$

(d) $\binom{24}{5} \cdot \binom{6}{5}$

(e) $6 \cdot \binom{24}{5}$

8. (5 pts.) The Philosophy Club of Notre Dame has 7 members. The annual Philosophers United convention will take place next month, and the ND club needs to send a delegation. The delegation can be any choice of **at least two** members of the club. How many possible delegations are there?

(a) 126

(b) 5040 (= 7!)

(c) 120

(d) 125

(e) 128

9. (5 pts.) An urn contains 13 numbered marbles, of which 7 are red and 6 are white. A sample of 5 is to be selected. How many possible samples are there where **at least** 4 are red?

(a) $\binom{7}{4} \cdot \binom{7}{5}$

(b) $\binom{7}{4} \cdot \binom{6}{1} + \binom{7}{5}$

(c) $\binom{7}{4} \cdot \binom{6}{1} \cdot \binom{7}{5}$

(d) $\binom{13}{4} + \binom{13}{5}$

(e) $\binom{7}{4} + \binom{7}{5}$

10. (5 pts.) A club has 15 members. In how many ways can they divide up into three (unordered) groups of five?

(a) $\frac{15!}{5!5!5!}$

(b) $\frac{1}{3!} \binom{15}{5} \binom{15}{5} \binom{15}{5}$

(c) $P(15, 5)^3$

(d) $\binom{15}{5} \binom{15}{5} \binom{15}{5}$

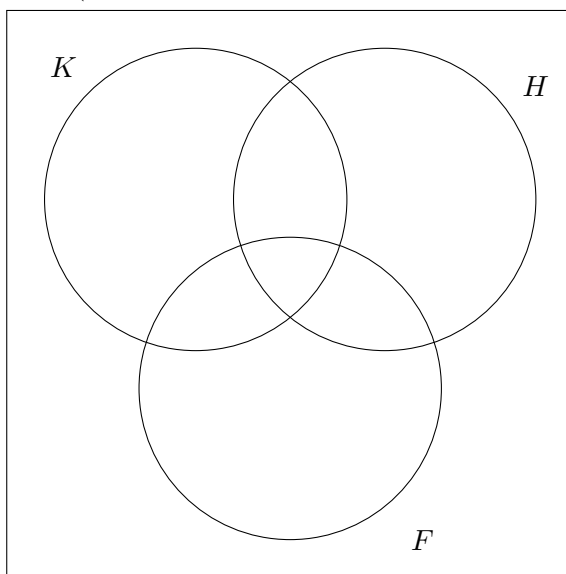
(e) $\frac{1}{3!} \cdot \frac{15!}{5!5!5!}$

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The 75 members of the juggling club are given a survey. Here are the results.
- 10 club members like juggling fire, hammers, and knives
 - 10 club members only like juggling knives and not the other two objects
 - 15 club members are afraid of fire, so they don't like juggling fire, but do like juggling hammers and knives
 - 20 club members like juggling hammers and fire
 - 40 club members like juggling knives
 - 50 club members like juggling hammers
 - 30 club members like juggling fire

(a) If $U = \{\text{club members}\}$, $K = \{\text{club members who like juggling knives}\}$, $H = \{\text{club members who like juggling hammers}\}$, and $F = \{\text{club members who like juggling fire}\}$, fill in the Venn diagram with the results of the survey (you should have a number in every space).



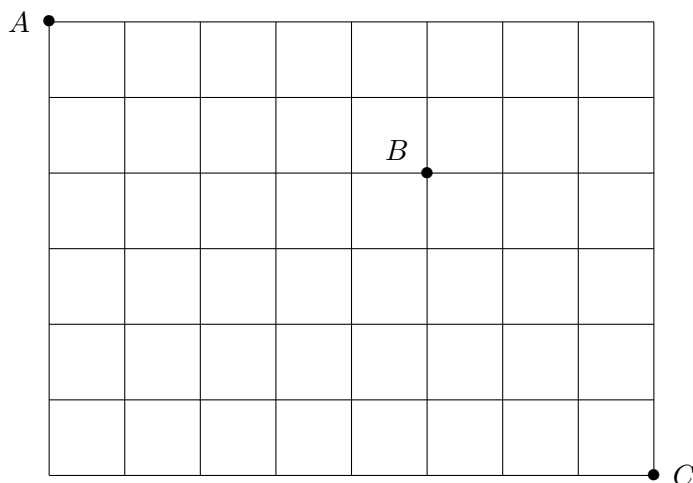
(b) How many club members (if any) do not like juggling any of the three objects?

12. (10 pts.) For this problem you do not need to simplify your answers; you may leave permutations, combinations, and factorials (i.e. $P(n, k)$, $C(n, k)$, $\binom{n}{k}$, or $n!$) or products or sums of numbers, permutations, combinations, or factorials in your answers if you so choose.

- (a) The juggling club needs to elect officers for the coming year. The club wants to elect a president, a vice president, a secretary and a treasurer. Assuming any of the 75 members of the club can hold any position and no member can hold more than one position, how many ways are there to choose the four officers?
- (b) The juggling club also wants to have a club photo of all 75 members taken. The newly elected officers will stand in the front row, the second row will contain 21 members, and the third and fourth rows will contain 25 members each. How many different photographs can be taken? (Note: the four officers have already been chosen when the picture is taken, and these four have to be in the front row.)

13. (10 pts.) For this problem you do not need to simplify your answers; you may leave permutations, combinations, and factorials (i.e. $P(n, k)$, $C(n, k)$, $\binom{n}{k}$, or $n!$) or products or sums of numbers, permutations, combinations, or factorials in your answers if you so choose.

The following is part of the city map of Anytown, USA.



(a) If one only travels east (i.e. to the right) or south (i.e. down), how many paths are there from A to C ?

(b) How many paths from A to C (again only traveling east or south) **avoid** passing through B ?

14. (10 pts.) For this problem you do not need to simplify your answers; you may leave permutations, combinations, and factorials (i.e. $P(n, k)$, $C(n, k)$, $\binom{n}{k}$, or $n!$) or products or sums of numbers, permutations, combinations, or factorials in your answers if you so choose.

A contingent of 8 Russians and 10 Americans meet for high-level talks.

(a) When the meeting starts, each Russian shakes hands with each American. How many handshakes are there?

(b) During the break, each Russians slaps each other Russian on the back, and each American slaps each other American on the back, because they both think that the talks are going better for them than for the other guys. How many backslaps are there?

(c) At the end of the meeting, each participant hugs each other participant, regardless of nationality. How many hugs are there?

15. (10 pts.) For all parts of this problem, using factorials is preferred to actual numbers. Doug has 6 math books, 5 psychology books and 4 biology books.

(a) In how many ways can the 15 books be lined up on the shelf without regard to subject?

(b) In how many ways can the 15 books be lined up on the shelf if all books on the same subject have to be together? (Note that there is no specified order for the subjects; e.g. math-psychology-biology is different from psychology-biology-math.)