Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Fall 2017

Name:______________________________

Instructors: Basit & Migliore

Exam 1

September 14, 2017

This exam is in two parts on 9 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an $\times$ through your answer to each problem.

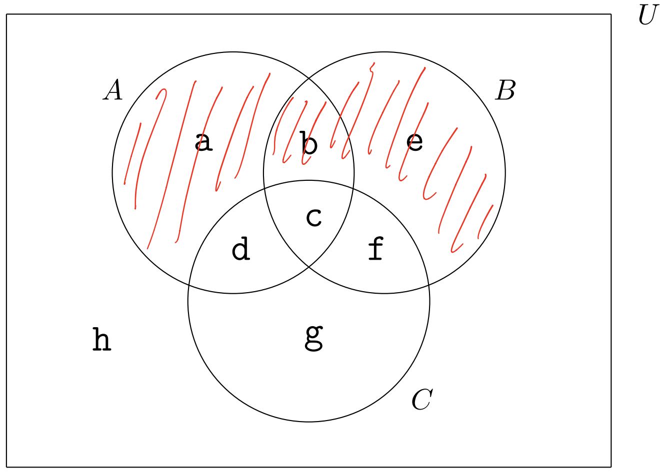
1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Note there are three versions of the exam, with the same questions but the answers permuted. The above answers correspond to <u>this</u> version, but not necessarily to yours!

MC.	______
11.	______
12.	______
13.	______
14.	______
Tot.	______

Multiple Choice

1. (5 pts.) In the following Venn diagram, which of the following is equal to $(A \cup B) \cap C'$? (Note the "prime" there.)



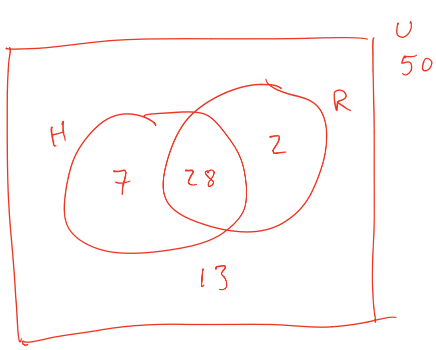
(a) $\{h\}$ ~~(b)~~ $\{a, b, e\}$ (c) $\{a, e\}$

(d) $\{c, d, f\}$ (e) $\{a, b, e, g, h\}$

In either A or B <u>and</u> not in C

2. (5 pts.) In a small school, the sixth grade class has 50 students. They have a history club and a reading club, and students are allowed to be in one, both or neither club. If 35 students are in the history club, 30 students are in the reading club and 28 students are in both clubs, how many students are in the history club <u>but not</u> the reading club? (A Venn diagram might be helpful.)

(a) 2 (b) 8 (c) 10 (d) 13 ~~(e)~~ 7



3. (5 pts.) A euchre deck consists of 24 cards, namely the 9, 10, J, Q, K and A of each suit (clubs, diamonds, hearts and spades). In how many ways can a person choose one card from each suit?

(a) $4 \cdot C(24, 4)$ (b) 6^4 (c) $C(24, 4)$

(d) $P(24, 4)$ (e) $4 \cdot 6^4$

There are 6 cards in each suit. So you have 6 choices for the club.
For each choice of the club you have 6 choices for the diamond. And so on.
So the answer is $6 \cdot 6 \cdot 6 \cdot 6 = 6^4$.

4. (5 pts.) A license plate in a certain state features 3 letters (repetition allowed) followed by 3 digits (repetition not allowed). How many different license plates are possible?

(a) $26^3 \cdot P(10, 3)$ (b) $P(26, 3) \cdot P(10, 3)$ (c) $26^3 \cdot C(10, 3)$

(d) $C(26, 3) \cdot P(10, 3)$ (e) $26^3 \cdot 10^3$

For each of the letters you have 26 choices.
Then you have 10 choices for the 1st digit, 9 for the 2nd and 8 for the 3rd (order matters). So it's $26^3 \cdot 10 \cdot 9 \cdot 8 = 26^3 P(10,3)$

5. (5 pts.) Three couples line up for a picture. In how many ways can this be done if the members of each couple must stand next to each other?

(a) 14 (b) 120 (c) 720 ~~(d)~~ 48 (e) 8

First figure out in how many orders the couples can line up as a unit. This is $3 \cdot 2 \cdot 1 = P(3,3) = 6$. Now for each couple there are 2 ways they could line up. So it's $6 \cdot 2^3 = 48$.

6. (5 pts.) In St. Patrick's College, all first year students are required to take 1 Math and 3 Philosophy courses. They can pick any course they like from among 4 Math and 5 Philosophy courses. How many different ways can a student pick his courses?

(a) $P(4,1) + P(5,3)$ (b) $4! \cdot 5!$ (c) $C(4,1) + C(5,3)$

~~(d)~~ $C(4,1) \cdot C(5,3)$ (e) $P(4,1) \cdot P(5,3)$

There are $C(4,1)$ ways to choose the math, and for each there are $C(5,3)$ ways to choose the philosophy.

7. (5 pts.) Recall that there are 52 cards in a standard deck, 13 from each suit (clubs, diamonds, hearts and spades). A Poker hand consists of 5 cards.

How many Poker hands have all 5 cards from the same suit?

(a) $4 \cdot C(13, 5)$ (b) $C(13, 5) + C(4, 1)$ (c) $C(13, 5)$

(d) $4 \cdot P(13, 5)$ (e) $P(13, 5)$

You could have five clubs or five diamonds or five hearts or five spades. For each suit there are $C(13,5)$ ways to choose five of them.

So $4 \cdot C(13,5)$

8. (5 pts.) John, Ryan, Emily and Anna want to play a game of Poker. How many ways are there to deal 5 cards to each of the players?

(a) $4 \cdot P(52, 5)$ (b) $4 \cdot C(52, 5)$

(c) $C(52, 5) \cdot C(47, 5) \cdot C(42, 5) \cdot C(37, 5)$ (d) $P(52, 5) \cdot P(47, 5) \cdot P(42, 5) \cdot P(37, 5)$

(e) $C(52, 5) \cdot C(47, 5) \cdot C(42, 5)$

There are $C(52,5)$ ways to give John five cards, then $C(47,5)$ ways to give five to Ryan, then $C(42,5)$ ways to give five to Emily, then $C(37,5)$ ways to give five to Anna.

9. (5 pts.) Suppose I roll a six sided die 5 times and record the resulting sequence of numbers. In how many ways can I get exactly three sixes? (Dont forget that there are five rolls.)

(a) $C(5,3) \cdot 6^3$ ~~(b)~~ $C(5,3) \cdot 5^2$ (c) $C(5,3)$

(d) $P(5,3)$ (e) 6^3

First choose which of the five rolls will be sixes. There are $C(5,3)$ ways to do this. Then the other two rolls have to be accounted for with anything but a six. There are $5 \cdot 5 = 5^2$ ways to do this.

10. (5 pts.) A certain class has 11 students. The instructor has in mind four projects: a history project, a math project, a biology project and a philosophy project. She plans to divide the class into four groups: three to do the history project, three to do the math project, three to do the biology project and two to do the philosophy project. In how many ways can she divide the class into these groups?

(a) $\binom{11}{3,3,3,2}/3!$ (b) $\binom{11}{3,3,3,2} \cdot 3!$ (c) $\frac{11!}{(3!)^4 \cdot 4!}$

(d) $\binom{11}{3,3,3,2}/4!$ (e) $\binom{11}{3,3,3,2}$

You want to partition your set of 11 students into distinguishable subsets, ie it's an ordered partition. So it's

$$\binom{11}{3\ 3\ 3\ 2}$$

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (15 pts.) A club has 6 men and 7 women. For each of the following questions, give a numerical answer (e.g. if the answer should be $C(4,2)$, write 6.) These questions should be assumed to be independent of each other.

(a) In how many ways can they choose a president and a vice president?

There are 13 members, so the answer is $\underset{\text{pres}}{\underline{13}} \cdot \underset{\text{VP}}{\underline{12}} = 156 = P(13,2)$

(order is important since the president is different from the VP.)

(b) In how many ways can they choose an executive committee of 3 people?

They want to choose 3 of the 13 people so we have

$C(13,3) = \frac{13 \cdot 12 \cdot 11}{3 \cdot 2} = 286.$

(c) In how many ways can they choose a dance committee consisting of 2 men and 2 women?

There are 6 men and 7 women. For each choice of 2 of the 6 men there are $C(7,2)$ ways to choose 2 of the 7 women.

$C(6,2) \cdot C(7,2) = 15 \cdot 21 = 315$

12. (10 pts.) A certain college has 1100 students. We have the following information about the clubs that they belong to.

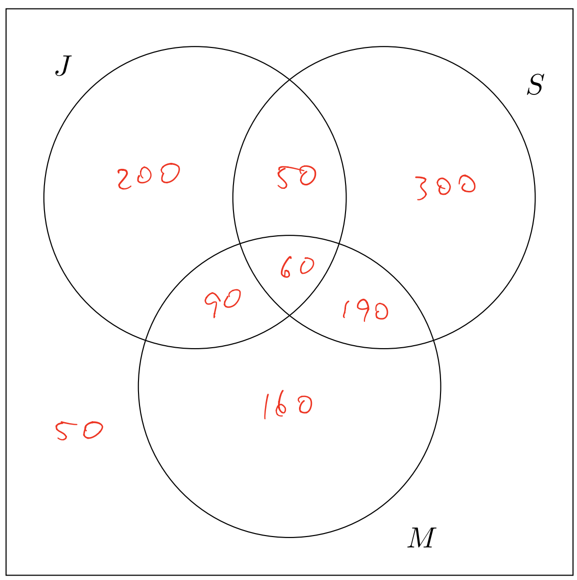
3rd
- 400 belong to the juggling club.
- 600 belong to the science club.
- 500 belong to the Mock Trial club

2nd
- 110 belong to both the juggling club and the science club.
- 250 belong to both the science club and the Mock Trial club.
- 150 belong to both the juggling club and the Mock Trial club.

1st
- 60 belong to all three clubs.

Fill in **all** regions of the following Venn diagram, where J represents the juggling club, S represents the science club and M represents the Mock Trial club.

4th



13. (15 pts.) I have a standard coin that comes up heads or tails each time I toss it. Suppose I toss the coin 12 times and note down the sequence of heads and tails that shows up.

Note: In the following three parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k).

(a) How many different sequences of heads and tails are possible?

There are 12 tosses, each with 2 possible outcomes, so the answer is 2^{12}.

(b) How many of the sequences have at least 9 and no more than 11 heads?

Of the 12 tosses we want either 9, 10 or 11 heads.

$C(12,9) + C(12,10) + C(12,11)$

(c) In how many ways can I get a total of 5 heads with the first and last toss being heads?

H _ _ _ _ _ _ _ _ _ _ H

The 1st and last are determined but of the middle 10 we have to choose which are H and which are T.

We need 3 more H out of the 10 tosses, and the rest are automatically T.

So the answer is $C(10,3)$.

14. (10 pts.)

A bag contains 9 colored marbles, of which 5 are red and 4 are blue marbles. I plan to pick 3 marbles from the bag.

Note: In the following three parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k).

(a) What is the total number of ways 3 marbles can be selected?

There are 9 marbles and we choose 3, so $C(9,3)$.

(b) If I pick 3 marbles, in how many ways can I get all red marbles or all blue marbles?

all red: $C(5,3)$

all blue: $C(4,3)$

So $C(5,3) + C(4,3)$

(c) In how many ways can I get at least one red and at least one blue marble among the 3 that I select?

This means either 2 red and 1 blue or 1 red and 2 blue

$$\left.\begin{array}{l} 2R\ 1B - C(5,2)\cdot C(4,1) \\ 1R\ 2B - C(5,1)\cdot C(4,2) \end{array}\right\} \text{ so } C(5,2)\cdot C(4,1) + C(5,1)\cdot C(4,2)$$