

Exam 1

September 14, 2017

This exam is in two parts on 9 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an \times through your answer to each problem.

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. _____

11. _____

12. _____

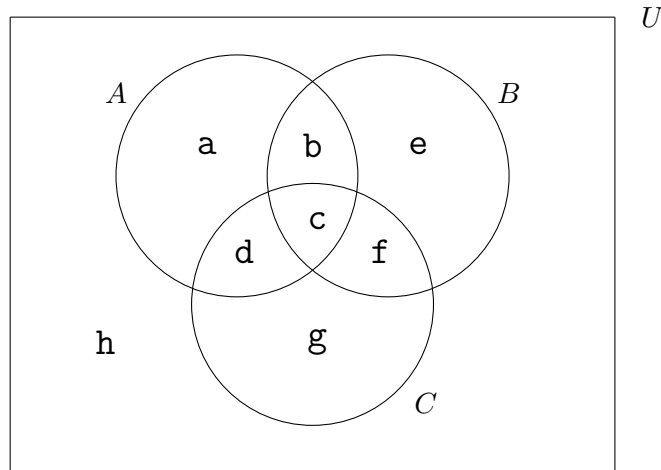
13. _____

14. _____

Tot. _____

Multiple Choice

1. (5 pts.) In the following Venn diagram, which of the following is equal to $(A \cup B) \cap C'$? (Note the "prime" there.)



- (a) $\{h\}$
- (b) $\{a, b, e\}$
- (c) $\{a, e\}$
- (d) $\{c, d, f\}$
- (e) $\{a, b, e, g, h\}$

2. (5 pts.) In a small school, the sixth grade class has 50 students. They have a history club and a reading club, and students are allowed to be in one, both or neither club. If 35 students are in the history club, 30 students are in the reading club and 28 students are in both clubs, how many students are in the history club but not the reading club? (A Venn diagram might be helpful.)

- (a) 2
- (b) 8
- (c) 10
- (d) 13
- (e) 7

3. (5 pts.) A euchre deck consists of 24 cards, namely the 9, 10, J, Q, K and A of each suit (clubs, diamonds, hearts and spades). In how many ways can a person choose one card from each suit?

- (a) $4 \cdot C(24, 4)$ (b) 6^4 (c) $C(24, 4)$
(d) $P(24, 4)$ (e) $4 \cdot 6^4$

4. (5 pts.) A license plate in a certain state features 3 letters (repetition allowed) followed by 3 digits (repetition not allowed). How many different license plates are possible?

- (a) $26^3 \cdot P(10, 3)$ (b) $P(26, 3) \cdot P(10, 3)$ (c) $26^3 \cdot C(10, 3)$
(d) $C(26, 3) \cdot P(10, 3)$ (e) $26^3 \cdot 10^3$

5. (5 pts.) Three couples line up for a picture. In how many ways can this be done if the members of each couple must stand next to each other?

- (a) 14 (b) 120 (c) 720 (d) 48 (e) 8

6. (5 pts.) In St. Patrick's College, all first year students are required to take 1 Math and 3 Philosophy courses. They can pick any course they like from among 4 Math and 5 Philosophy courses. How many different ways can a student pick his courses?

- (a) $P(4, 1) + P(5, 3)$ (b) $4! \cdot 5!$ (c) $C(4, 1) + C(5, 3)$
(d) $C(4, 1) \cdot C(5, 3)$ (e) $P(4, 1) \cdot P(5, 3)$

7. (5 pts.) Recall that there are 52 cards in a standard deck, 13 from each suit (clubs, diamonds, hearts and spades). A Poker hand consists of 5 cards.

How many Poker hands have all 5 cards from the same suit?

- (a) $4 \cdot C(13, 5)$ (b) $C(13, 5) + C(4, 1)$ (c) $C(13, 5)$
(d) $4 \cdot P(13, 5)$ (e) $P(13, 5)$

8. (5 pts.) John, Ryan, Emily and Anna want to play a game of Poker. How many ways are there to deal 5 cards to each of the players?

- (a) $4 \cdot P(52, 5)$ (b) $4 \cdot C(52, 5)$
(c) $C(52, 5) \cdot C(47, 5) \cdot C(42, 5) \cdot C(37, 5)$ (d) $P(52, 5) \cdot P(47, 5) \cdot P(42, 5) \cdot P(37, 5)$
(e) $C(52, 5) \cdot C(47, 5) \cdot C(42, 5)$

9. (5 pts.) Suppose I roll a six sided die 5 times and record the resulting sequence of numbers. In how many ways can I get exactly three sixes? (Don't forget that there are five rolls.)

(a) $C(5, 3) \cdot 6^3$

(b) $C(5, 3) \cdot 5^2$

(c) $C(5, 3)$

(d) $P(5, 3)$

(e) 6^3

10. (5 pts.) A certain class has 11 students. The instructor has in mind four projects: a history project, a math project, a biology project and a philosophy project. She plans to divide the class into four groups: three to do the history project, three to do the math project, three to do the biology project and two to do the philosophy project. In how many ways can she divide the class into these groups?

(a) $\binom{11}{3,3,3,2}/3!$

(b) $\binom{11}{3,3,3,2} \cdot 3!$

(c) $\frac{11!}{(3!)^4 \cdot 4!}$

(d) $\binom{11}{3,3,3,2}/4!$

(e) $\binom{11}{3,3,3,2}$

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (15 pts.) A club has 6 men and 7 women. For each of the following questions, give a numerical answer (e.g. if the answer should be $C(4,2)$, write 6.) These questions should be assumed to be independent of each other.

(a) In how many ways can they choose a president and a vice president?

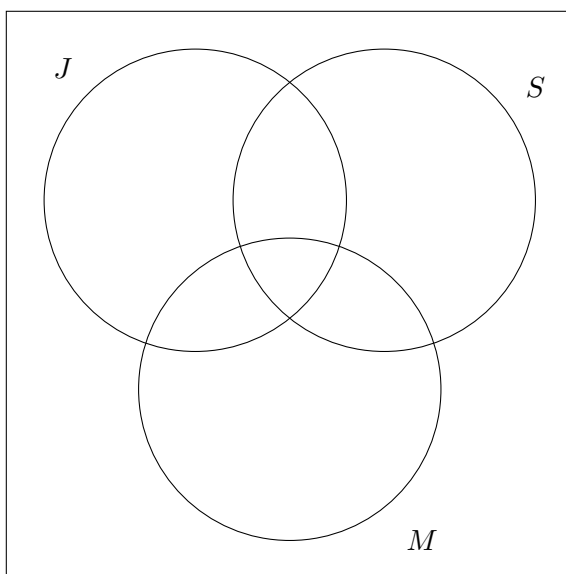
(b) In how many ways can they choose an executive committee of 3 people?

(c) In how many ways can they choose a dance committee consisting of 2 men and 2 women?

12. (10 pts.) A certain college has 1100 students. We have the following information about the clubs that they belong to.

- 400 belong to the juggling club.
- 600 belong to the science club.
- 500 belong to the Mock Trial club
- 110 belong to both the juggling club and the science club.
- 250 belong to both the science club and the Mock Trial club.
- 150 belong to both the juggling club and the Mock Trial club.
- 60 belong to all three clubs.

Fill in **all** regions of the following Venn diagram, where J represents the juggling club, S represents the science club and M represents the Mock Trial club.



13. (15 pts.) I have a standard coin that comes up heads or tails each time I toss it. Suppose I toss the coin 12 times and note down the sequence of heads and tails that shows up.

Note: In the following three parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k).

(a) How many different sequences of heads and tails are possible?

(b) How many of the sequences have at least 9 and no more than 11 heads?

(c) In how many ways can I get a total of 5 heads with the first and last toss being heads?

14. (10 pts.)

A bag contains 9 colored marbles, of which 5 are red and 4 are blue marbles. I plan to pick 3 marbles from the bag.

Note: In the following three parts, it is not necessary to give a numerical answer, i.e. you may express your answers using the notation for permutations ($P(n, k)$), combinations ($C(n, k)$), factorials ($n!$) and powers (a^k).

(a) What is the total number of ways 3 marbles can be selected?

(b) If I pick 3 marbles, in how many ways can I get all red marbles or all blue marbles?

(c) In how many ways can I get at least one red and at least one blue marble among the 3 that I select?