Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Fall 2017

Name: SOLUTIONS

Instructors: Basit & Migliore

# Practice Exam II

**October 10, 2017**

This exam is in two parts on 10 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

**You must record on this page your answers to the multiple choice problems.**

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Place an × through your answer to each problem.

| | | | | | |
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## Multiple Choice

**1.** (5 pts.) An experiment consists of rolling two 6-sided dice and recording the pair of numbers rolled. Consider the following events

$$\begin{aligned} E &= \text{event at least one of the numbers is a 6} \\ F &= \text{event both numbers are odd} \\ G &= \text{event at least one of the numbers is a 3} \\ H &= \text{event the numbers add to 8} \end{aligned}$$

Which of the following is a mutually exclusive pair of events?

~~(a)~~ $E$ and $F$ (b) $E$ and $G$ (c) $G$ and $H$

(d) $F$ and $G$ (e) $E$ and $H$

a) E and F are mutually exclusive. If both #'s are odd (i.e F happens) then the #'s can not be 6 (i.e E can't happen)

b) $(6,3) \in E \cap G$ c) $(3,5) \in G \cap H$ d) $(3,3) \in F \cap G$

e) $(6,2) \in E \cap H$

So the other options are not mutually exclusive

**2.** (5 pts.) An experiment has sample space $S = \{a, b, c, d, e\}$. The following table gives the probabilities for these outcomes, except $P(c)$ is unknown.

| Outcome | Probability |
|---|---|
| $a$ | 0.20 |
| $b$ | 0.10 |
| $c$ | $x$ |
| $d$ | 0.25 |
| $e$ | 0.30 |

If we define the event $E = \{a, b, c\}$, what is $P(E)$?

(a) 0.70 (b) 0.15 (c) 0.55

~~(d)~~ 0.45 (e) 0.85

$P(S) = P(\{a,b,c,d,e\}) = P(a) + P(b) + P(c) + P(d) + P(e)$

i.e $0.2 + 0.1 + x + 0.25 + 0.3 = 1$

$x = 0.15$

$P(E) = P(a) + P(b) + P(c) = 0.2 + 0.1 + 0.15 = 0.45$

**3.** (5 pts.) A coin is flipped 12 times and the sequence of heads and tails is recorded. If $E =$ the event that exactly 6 tails are obtained, what is $P(E)$?

(a) $C(12,6) \cdot 2^{12}$ (b) $\frac{1}{2}$ (c) $\frac{6}{2^{12}}$

(d) $C(12,6)$ ~~(e)~~ $\frac{C(12,6)}{2^{12}}$

Equally likely Probabilities.

$P(E) = \frac{n(E)}{n(S)} = \frac{C(12,6)}{2^{12}}$ → because choose which 6 of 12 are tails. The remaining 6 are heads

Each toss has 2 options. So multiplication rule

$\underbrace{2 \times 2 \times \cdots \times 2}_{12 \text{ times}} = 2^{12}$

**4.** (5 pts.) Of a group of 100 students, 60 like downhill skiing, 40 like cross country skiing, and 80 like at least one of the two. A student is chosen at random. If it is known that the student chosen likes cross country skiiing, what is the probability he or she likes downhill skiing?

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ ~~(c)~~ $\frac{1}{2}$

(d) $\frac{1}{5}$ (e) $\frac{1}{3}$

$D = \{$student likes downhill skiing$\}$ $P(D) = 0.6$

$C = \{$ - - —— cross country ··· $\}$ $P(C) = 0.4$

Given $P(C \cup D) = 0.8$

$P(C \cup D) = P(C) + P(D) - P(C \cap D)$

so $P(C \cap D) = 0.6 + 0.4 - 0.8 = 0.2$

$P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{0.2}{0.4} = 1/2$

**5.** (5 pts.) Three cards are drawn from a standard deck (without replacement). What is the probability that the first card is red and the second and third cards are black? (All answer choices are rounded to three decimal places.)

(a) 0.382 ~~(b)~~ 0.127 (c) 0.125

(d) 0.120 (e) 1.510

$P(R1 \cap B2 \cap B3) = \frac{26}{52} \times \frac{26}{51} \times \frac{25}{50}$ → 25 remaining black out of 50 remaining total

$\simeq 0.127$

26 red out of 52

26 black out of 51 remaining

**6.** (5 pts.) A club has 10 members, of which 5 are Math majors and 4 are English majors. A student is chosen at random and asked to state her major(s). Let $M$ be the event that the student is a Math major and $E$ be the event that the student is an English major. How many students must be majoring in *both* Math and English in order for $M$ and $E$ to be independent events? (Hint: What is $P(M)$? What is $P(E)$?)

(a) 0 ~~(b)~~ 2 (c) 1 (d) 3 (e) 4

$P(M) = 0.5$

$P(E) = 0.4$

For $M, E$ to be independent, $P(M \cap E) = P(M)P(E)$

so $P(M \cap E)$ should be equal to $0.5 \times 0.4$
$= 0.2$

$P(M \cap E) = 0.2 = \frac{n(M \cap E)}{10}$

$\Rightarrow n(M \cap E) = 2$

**7.** (5 pts.) Bob and Mary are having a basketball free-throw shooting contest. Bob gets a basket on 20% of his attempts, and Mary gets a basket on 30% of her attempts. (They're not very good free-throw shooters!) If they each make two attempts (for a total of four attempts between them) what is the probability that all four attempts are **misses**? Assume that the attempts are independent of each other. Round to the nearest 0.1 percent.

~~(a)~~ 31.4% (b) 68.6% (c) 0.4% (d) 56% (e) 6%

Let $M1$ be the event that the first throw misses

$M2$ — — — second — —

etc

say Bob goes first, Mary second.

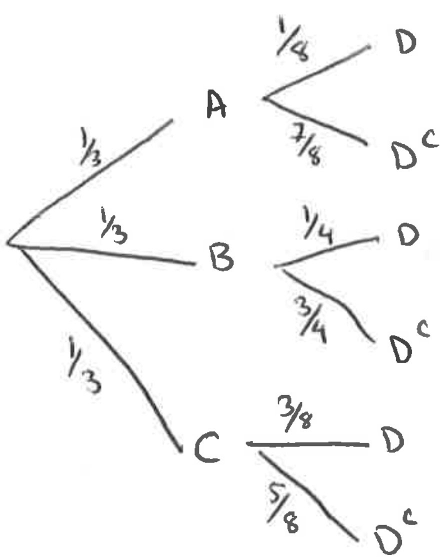
$$P(M1 \cap M2 \cap M3 \cap M4) = 0.8 \times 0.8 \times 0.7 \times 0.7$$

$$\approx 0.314 = 31.4\%$$

**8.** (5 pts.) A store sells three brands of batteries: Ajax, Batterymundo and Cheap-O. Not being of very high quality, $\frac{1}{8}$ of the Ajax batteries are defective, $\frac{1}{4}$ of the Batterymundo batteries are defective, and $\frac{3}{8}$ of the Cheap-O batteries are defective. The store sells equal numbers of all three batteries. Bob chooses a battery at random from the store. If it turns out to be defective, what is the probability that it was a Cheap-O battery? (Answers below are given to three decimal places.)

(a) $\frac{1}{8} = 0.125$ (b) $\frac{3}{8} = 0.375$ (c) $\frac{1}{3} = 0.333$

(d) $\frac{1}{4} = 0.250$ ~~(e)~~ $\frac{1}{2} = 0.500$



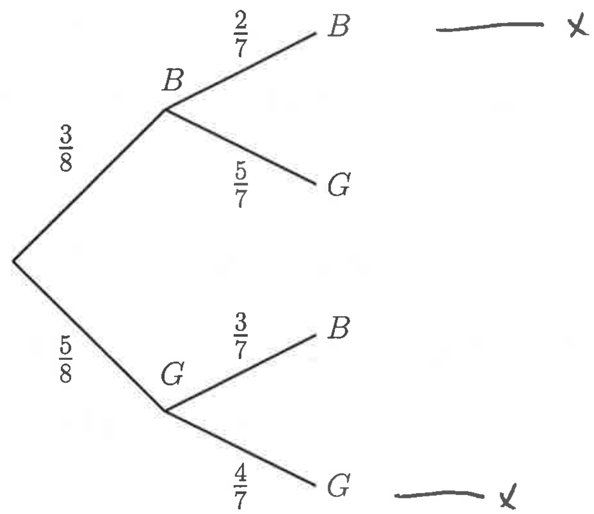

We want

$$P(C|D) = \frac{P(C \cap D)}{P(D)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{3} \times \frac{3}{8} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{8}}$$

$$= \frac{1}{2}$$

**9.** (5 pts.) My sock drawer contains 3 blue socks and 5 green socks (socks, not pairs of socks). I reach in without turning on the light and grab a sock. Then I reach in again and grab a second sock. If the socks match color, what is the probability that they are blue? Hint: The following tree diagram represents this situation.




(a) $\frac{13}{28}$  (b) $\frac{5}{14}$  (c) $\frac{4}{7}$  ~~(d)~~ $\frac{3}{13}$  (e) $\frac{3}{28}$

$$P(B|M) = \frac{P(B\cap M)}{P(M)} = \frac{\frac{3}{8}\times\frac{2}{7}}{\frac{3}{8}\times\frac{2}{7}+\frac{5}{8}\times\frac{4}{7}} = \frac{\frac{6}{56}}{\frac{6}{56}+\frac{20}{56}} = \frac{6}{26} = \frac{3}{13}$$

**10.** (5 pts.) My sock drawer contains 3 blue socks and 5 green socks. I reach in without turning on the light and grab a sock. Then I reach in again and grab a second sock. What is the probability that they **do NOT match**? Hint: If you like you can use the above tree diagram.)

(a) $\frac{5}{8}$  ~~(b)~~ $\frac{15}{28}$  (c) $\frac{3}{8}$  (d) $\frac{33}{56}$  (e) $\frac{15}{56}$

$$P(M^c) = 1 - P(M) = 1 - \left(\frac{6}{56}+\frac{20}{56}\right) \leftarrow \text{from 9}$$

$$= \frac{30}{56} = \frac{15}{28}$$

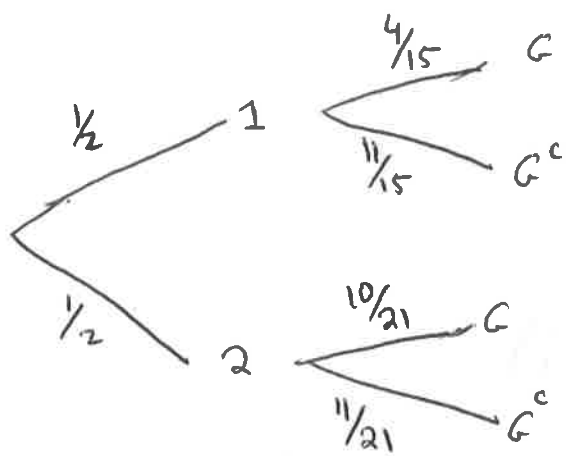
## Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

**11.** (10 pts.) Andrew plays the following game. First he flips a coin with a 1 painted on one side and a 2 painted on the other. The number on the coin determines how many marbles he gets to draw (without replacement) from an urn containing 11 blue marbles and 4 green marbles (he draws the marbles in succession, not at the same time). Once he's drawn the allowed number of marbles, he wins if at least one of the marbles drawn is green.

(a) Draw a tree diagram to represent the game. All branches of the diagram should be labeled with probabilities.

$G = \{\text{at least one green}\}$




$$P(G \mid 2) = 1 - P(G^c \mid 2)$$
$$= 1 - \left(\frac{11}{15} \times \frac{10}{14}\right) \text{ both blue}$$
$$= 10/21$$

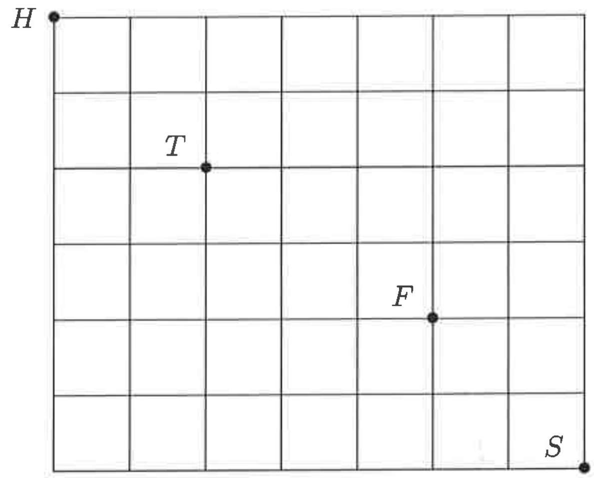
(b) What is the probability of winning? (You do not need to simplify your answer.)

$$P(\text{win}) = P(G) = \frac{1}{2} \times \frac{4}{15} + \frac{1}{2} \times \frac{10}{21}$$
$$= \frac{13}{35}$$

(c) What is the probability that Andrew wins given that he gets a 2 on the coin flip? (You do not need to simplify your answer.)

P(win | 2) = 10/21 (See part (a) above)

**12.** (10 pts.) For this problem you do not need to simplify your answers; you may leave permutations, combinations, and factorials (i.e. $P(n,k)$, $C(n,k)$, $\binom{n}{k}$, or $n!$) or products, sums, or quotients of numbers, permutations, combinations, or factorials in your answers if you so choose. The following is a map of Carl's neighborhood.




Carl wants to walk from his house at H to the store at S without backtracking (he travels only south and east). He chooses a route at random.

(a) What is the probability Carl passes his friend Fred's house at F?

$n(S) = \binom{13}{7}$, $n(F) = \binom{9}{5}\binom{4}{2}$

$P(F) = \binom{9}{5}\binom{4}{2} / \binom{13}{7}$

(b) If Carl's friend Taylor lives at T, what is the probability Carl passes both Fred's house and Taylor's house?

$$P(F \cap T) = \frac{\binom{4}{2}\binom{5}{2}\binom{4}{2}}{\binom{13}{7}}$$

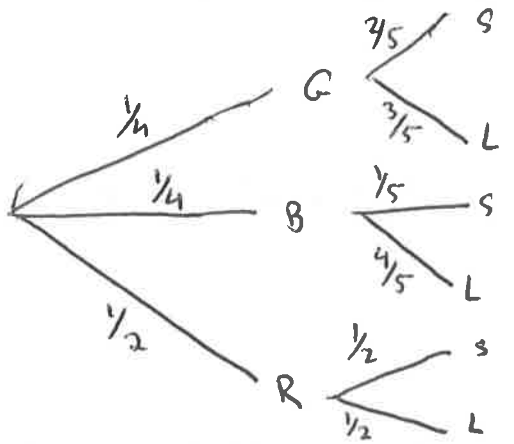
(c) What is the probability Carl passes at most one of his friends' houses?

$$\begin{aligned} P(\text{at most one}) &= 1 - P(\text{both}) \\ &= 1 - P(F \cap T) \\ &= 1 - \frac{\binom{4}{2}\binom{5}{2}\binom{4}{2}}{\binom{13}{7}} \end{aligned}$$

**13.** (10 pts.) A college bookstore sells three kinds of T-shirts: green, blue and red. For each kind, they sell small and large T-shirts. On a given day, they have 200 T-shirts in stock, all mixed together in a large bin and divided as follows:

| Color | Number Small | Number Large | |
|---|---|---|---|
| Green | 20 | 30 | = 50 |
| Blue | 10 | 40 | = 50 |
| Red | 50 | 50 | = 100 |

(a) Draw a tree diagram to represent this situation with the branches labelled as probabilities. Use $G$, $B$ and $R$ for the first set of branches and $S$ and $L$ for the second set of branches. (E.g. the 50 greens would correspond to the probability $P(G) = \frac{50}{200} = 25\%$ and the 30 large greens would correspond to the conditional probability $P(L|G) = \frac{30}{50} = 60\%$).




(b) During the night a thief breaks in and randomly steals one T-shirt from the bin. What is the probability that he stole a red T-shirt? Explain your answer.

$$P(R) = \frac{100}{200} = \frac{1}{2}$$

(c) Before he gets into the light he tries it on and realizes that it is a large T-shirt. With this additional information, what is the probability that he stole a red T-shirt? Explain your answer. You do not need to simplify your answer.

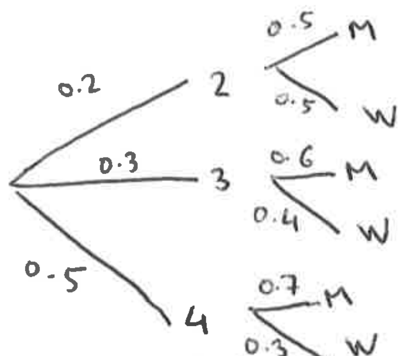
$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{4}{5} + \frac{1}{4} \times \frac{3}{5}} = 5/12$$

**14.** (10 pts.) In the country of Freedonia, a poll is taken concerning the number of hats owned by its citizens, and the responses range from 2 to 4. The following table gives the percentages for each number of hats, and for each number of hats it gives the breakdown between men and women.

| **Number** | **Percentage of Population** | **Men** | **Women** |
|---|---|---|---|
| 2 | 20% | 50% | 50% |
| 3 | 30% | 60% | 40% |
| 4 | 50% | 70% | 30% |

(For example, 20% of the population are people with 3 hats, and among people with 3 hats, 60% are men and 40% are women.) A person is chosen at random.

(a) What is the probability that he/she has four hats?




$P(4 \text{ hats}) = 0.5$

(b) What is the probability that it's a man with four hats?

$P(M \cap (4 \text{ hats})) = 0.5 \times 0.7$

$= 0.35$

(c) Given that it is a man, what is the probability that he has four hats?

$P(4 \text{ hats} \mid M) = \dfrac{P((4 \text{ hats}) \cap M)}{P(M)}$

$= \dfrac{0.35}{0.5 \times 0.7 + 0.3 \times 0.6 + 0.2 \times 0.5} = 0.555\ldots$

**15.** (10 pts.) Let $A$, $B$ and $C$ be events and assume the following probabilities:
$P(A) = 0.2$, $P(B) = 0.3$, $P(C) = 0.5$, $P(A \cap B) = 0.06$, $P(A \cap C) = 0$, $P(B \cap C) = 0.5$.

(a) Of the three events $A, B, C$, which two are independent? Explain.

$P(A)P(B) = 0.2 \times 0.3 = 0.06 = P(A \cap B) \longrightarrow A, B$ indept

$P(A)P(C) = 0.2 \times 0.3 = 0.1 \neq P(A \cap C) \longrightarrow A, C$ dept

$P(B)P(C) = 0.3 \times 0.5 = 0.15 \neq P(B \cap C) \longrightarrow B, C$ dept

(b) Of the three events $A, B, C$, which two are mutually exclusive? Explain.

A and C are mutually exclusive
because $P(A \cap C) = 0$ (definition of m.e.)

(c) What is the conditional probability $P(B|C)$?

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0.5}{0.5} = 1$$

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