UNIVERSITY OF NOTRE DAME
Department of Civil Engineering
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CE 60130 Finite Elements in Engineering **January 24^{th}, 2012**
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Homework Set #1

Problem 1

Consider the following ordinary differential equation (ODE) of $u(x)$:

$$\frac{d^2u}{dx^2} + x\frac{du}{dx} + \frac{u}{6} = -3 \qquad \text{for the domain } 0 \leq x \leq 10$$

with boundary conditions

$$u|_{x=0} = 5$$
$$\left.\frac{du}{dx}\right|_{x=10} = -\frac{\pi}{5}$$

Solve this ODE using the method of weigthed residuals on the domain $0 \leq x \leq 10$ using polynomial basis functions. Recall that the approximate solution follows

$$u_{app} = u_B + \sum_{i=1}^{N} \alpha_i \phi_i$$

where u_B is the boundary function that satisfies the boundary conditions, α_i are the unknown coefficients which must be determined and ϕ_i are the known basis functions that form a complete set.

a) Formulate the boundary function, u_B, such that the boundary conditions are satisfied.

Hint: The boundary function must satisfy both a functional value and a derivative value, consider $u_B = a_1 + a_2 x$ and solve for a_1 and a_2.

b) Determine the form of the basis functions, ϕ_i, such that the admissibility and completeness criteria are strictly satisfied.

c) Develop formulae for the entries in the system of simultaneous equations and the right hand side vector to solve using the Collocation, Least Squares and Galerkin methods.

Hint: Setting up the general system of equations that will need to be solved can be quite extensive analytically, requiring integration of complicated expressions. You may want to take advantage of a symbolic math manipulator program such as Mathematica or Maple.

d) Using Matlab, or other suitable language, develop a code to solve each system (Collocation, Least Squares, Galerkin) for 2, 10, 20 and 100 term expansions. Plot the solution for each expansion set. Determine the error for each solution by comparing with the point based analytical solution provided on the course website (Problem_1_exact_soln.txt). Compute the L_2 error for each expansion and plot these errors on a log–log plot with the number of terms as the horizontal axis. In addition plot the pointwise error over the entire domain using a logarithmic scale for the y-axis.

Hint: The L_2 error can be calculated as the Root–Mean–Square–Error, as follows

$$\varepsilon = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - f_i)^2}$$

where N is the number of points in the point–wise comparison, f_i is the exact value at the i^{th} point and y_i is the value given by your model at the i^{th} point. In addition a log–log plot can be achieved in Matlab using the command 'loglog' instead of 'plot'. Simlarly a plot with a logrithmic vertical axis can be made using 'semilogy' instead of 'plot'.

e) Comment on each method used. Discuss properties such as convergence, accuracy, computational cost. Inter-compare the solutions from each method.