LECTURE 23

SETS OF SIMULTANEOUS FIRST ORDER O.D.E.'S

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = \begin{bmatrix} f_1(y_1, y_2, \ldots, y_n, t) \\ f_2(y_1, y_2, \ldots, y_n, t) \\ \cdot \\ \cdot \\ \cdot \\ f_n(y_1, y_2, \ldots, y_n, t) \end{bmatrix}$$

- Using vector notation:

$$\frac{d\underline{y}}{dt} = \underline{f}(\underline{y}, t)$$

- All procedures discussed for single o.d.e.'s apply
- However for both R.K. type methods and multi-step methods we must complete the computation for the entire vector before moving on to the next step of the procedure due to the coupling inherent to the system

- Notes:
  - It is especially important to use efficient methods when dealing with the large systems of o.d.e.'s associated with the differentially time dependent equations which result from application of F.D./F.E. methods to discretize spatial variation.
  - If the o.d.e.'s are linear (both for single and multiple equations), no iterative procedure is required for closed methods!

Boundary Value Problems

- Boundary value problems must be 2nd order o.d.e.'s or higher
- Example:

$$\frac{d^2y}{dx^2} + Ay = B$$

with b.c.'s $y(0) = 0$ and $y(L) = 0$

- Consider 2 types of methods
  - Matrix formulation methods
  - Shooting methods

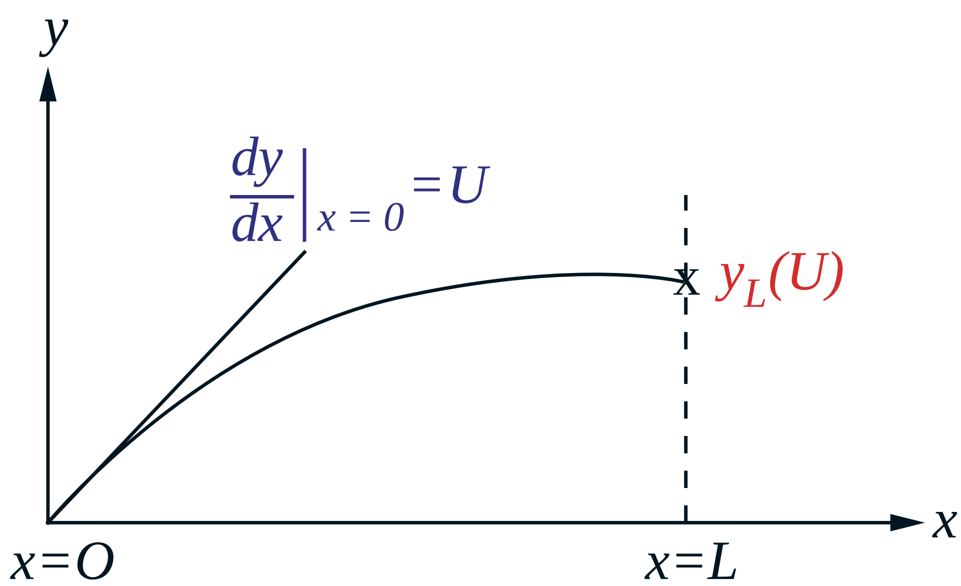
Matrix Methods

- Generate a matrix by discretizing the domain and replacing differential equations and/or boundary conditions by discrete algebraic equations by using F.D. approximations for the derivatives (see Lecture 15)

Shooting Methods

- Convert the b.v.p. into an i.v.p. and use standard i.v.p. methods
  - e.g. Instead of solving $\frac{d^2y}{dx^2} + Ay = B, \quad y(0) = 0$ and $y(L) = 0$
  - Solve $\frac{d^2y}{dx^2} + Ay = B, \quad y(0) = 0$ and $\frac{dy}{dx}(0) = U$.

    where $\boldsymbol{U}$ = an unknown value
  - Hence the i.c. U must be selected such that the boundary value $y(L) = 0$ is reproduced
  - Therefore guess U $\Rightarrow$ use i.v.p. solution method $\rightarrow$ final value will be $y_L(U)$
  - However we wish to find U such that $y_L(U) = y(L) = 0$



y
$\left.\frac{dy}{dx}\right|_{x=0} = U$
x
$y_L(U)$
x
x=O
x=L

- Therefore we must solve the problem $y_L(U) = 0 \rightarrow$ This is now a root solving problem $\rightarrow$ use the secant method (discrete derivative form of the Newton-Raphson algorithm to find the solution)

  - Provide 2 estimates for the root of equation $y_L(U) = 0 \Rightarrow \begin{cases} U_o \\ U_{oo} \end{cases}$

  - Find solutions for $y_L(U_o)$ and $y_L(U_{oo})$

  - Now a new estimate for U can be obtained

$$U_1 = U_o - \frac{y_L(U_o)}{[y_L(U_o) - y_L(U_{oo})]/(U_o - U_{oo})}$$

  - Continue this procedure until convergence

- Note that shooting methods are only practical in one dimensional applications and are <u>not</u> practical in 2 and 3 dimensional b.v.p.'s.