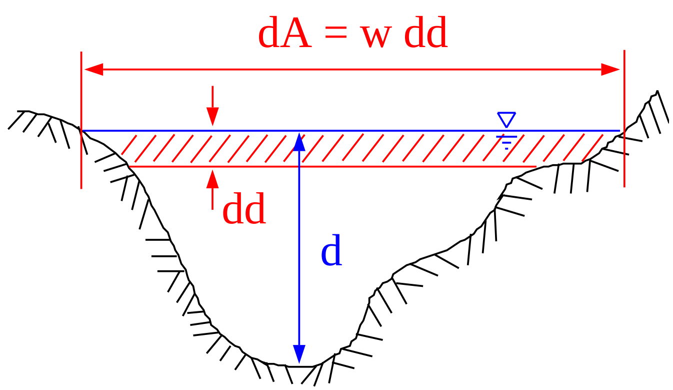
3.2 CRITICAL DEPTH IN NONRECTANGULAR CHANNELS AND OCCURRENCE OF CRITICAL DEPTH

Critical Depth in Non-Rectangular Channels

- Consider an irregular channel:



- Specific energy is defined as:

$$E = d + \frac{\tilde{u}^2}{2g} \quad (3.2.1)$$

where $\tilde{u}$ = cross-sectionally averaged velocity (3.2.2)

- Noting that

$$\tilde{u} = \frac{Q}{A} \tag{3.2.3}$$

- Substituting

$$E = d + \frac{Q^2}{2gA^2} \tag{3.2.4}$$

- Find the minimum energy for a given Q; note that $A = A(d)$

$$\frac{\partial E}{\partial d} = 1 - \frac{Q^2}{2g}\left(\frac{2}{A^3}\frac{dA}{dd}\right) = 0 \tag{3.2.5}$$

- Set equal to zero and solve for the critical value of $d(d_c)$.

- We note that $dA = wdd \Rightarrow \frac{dA}{dd} = w$, where w = width of the water surface.

- Thus

$$1 - \frac{Q^2}{2g}\left(\frac{2}{A^3}w\right)\Bigg|_{d = d_c} = 0 \tag{3.2.6}$$

- Finally, we establish the relationship between Q, g, A and w at critical flow:

$$\frac{Q^2}{g} = \left(\frac{A^3}{w}\right)_{d = d_c} \tag{3.2.7}$$

- For a given cross section, the right hand side is a function of d only. Noting that

$$Q = A_c \tilde{u}_c \tag{3.2.8}$$

- Substituting

$$\frac{A_c^2 \tilde{u}_c^2}{g} = \frac{A_c^3}{w_c} \tag{3.2.9}$$

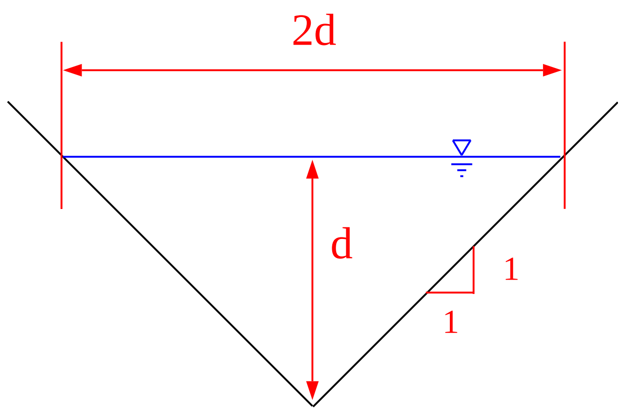
$$\tilde{u}_c^2 = \frac{gA_c}{w_c} \tag{3.2.10}$$

- Thus

$$\tilde{u}_c = \sqrt{\frac{gA_c}{w_c}} \tag{3.2.11}$$

Example

- Water flows uniformly at a steady rate of Q = 14 cfs in a very long triangular flume which has side slopes 1:1. The bottom of the flume is on a slope of 0.006 and bottom roughness is n = 0.012. Determine if the flow is subcritical or supercritical.



- Compute uniform flow using the steady state uniform flow formula developed:

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2} \tag{3.2.12}$$

- For this case:

$$A = \frac{1}{2}(2d)(d) = d^2 \tag{3.2.13}$$

$$P_w = 2\sqrt{2d} = 2.83d \tag{3.2.14}$$

$$R_h = \frac{A}{P_w} = \frac{d^2}{2.83d} = 0.354d \tag{3.2.15}$$

- Substituting in all values, we can compute the normal flow depth:

$$14 = \frac{1.49}{0.012}d_0(0.354d_0)^{2/3}(0.006)^{1/2} \tag{3.2.16}$$

- We solve to find the normal flow depth:

$$d_0 = 1.495 \tag{3.2.17}$$

- Now we must compute the critical depth at which

$$\frac{Q^2}{g} = \frac{A^3}{w} \tag{3.2.18}$$

- Substituting in values

$$\frac{14^2}{32.2} = \frac{(d_c^2)^3}{2d_c} \tag{3.2.19}$$

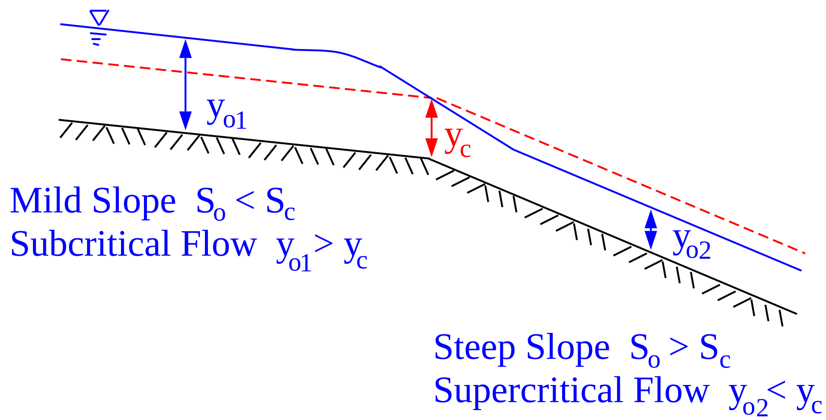
- Solving

$$d_c = 1.65 \text{ ft} \tag{3.2.20}$$

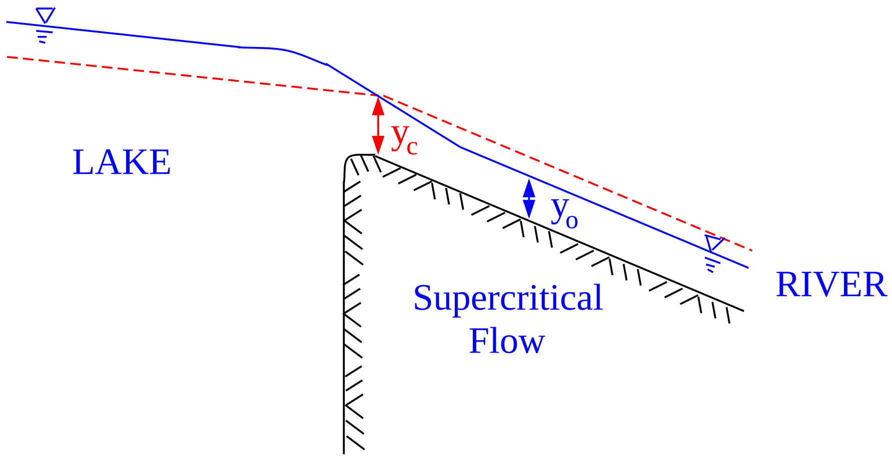
- Since $d_c > d_0$, the flow is supercritical

Occurrence of Critical Depth

- Flow changing from subcritical to supercritical or vice-versa requires that depth must pass through the critical depth.
  - Subcritical to supercritical → ***Control Section***
  - Supercritical to subcritical → ***Hydraulic Jump***
- At a break in slope, the depth can pass through the critical depth. This point in the stream is referred to as a control section since the depth at the break controls the depth upstream.



- Similarly, for a lake issuing into a river;



- By measuring the depth at a control section, you can compute a reasonably accurate value of Q:

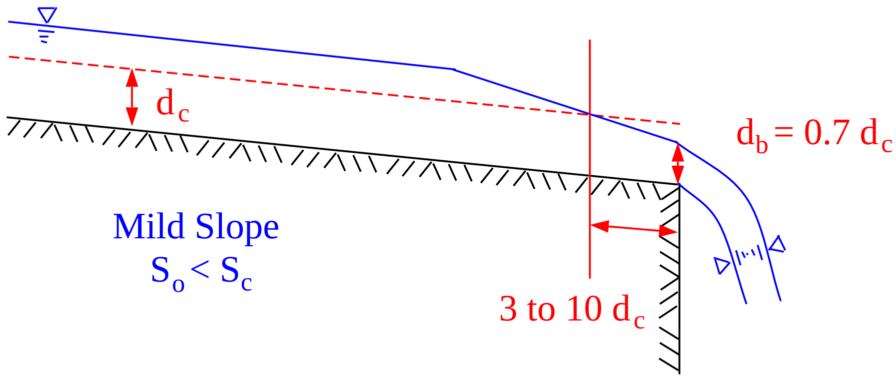
$$d_c = \left(\frac{q^2}{g}\right)^{1/3} \tag{3.2.21}$$

$$q^2 = g d_c^3 \tag{3.2.22}$$

- Thus at a control section

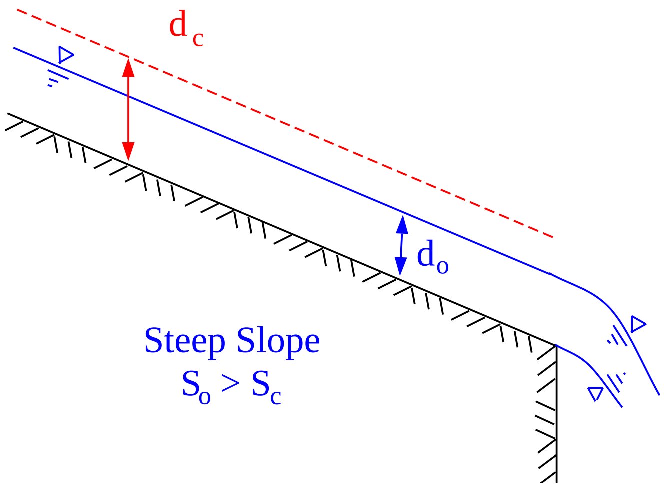
$$q = \sqrt{g d_c^3} \tag{3.2.23}$$

- Critical flow at a free outfall with subcritical flow in the channel prior to the outfall:



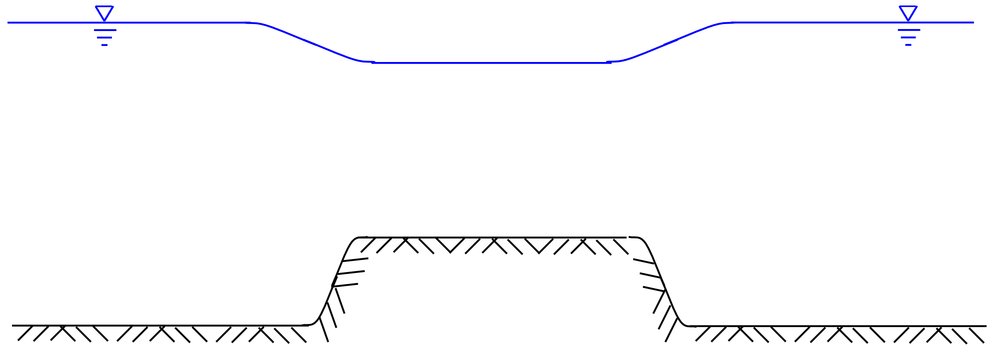
- If streamlines at the outfall were not curved, then the critical depth would occur there.
- However, due to the curvature of the streamlines (caused by gravity at the outfall), results in the depth at the brink less than critical.

- If the flow is supercritical, there is no drop in the curve:



- Note that in a supercritical flow, no information can be transmitted upstream and therefore any portion of the river upstream of the drop is not affected by the drop-off.

- Critical flow can also occur if the bottom of the channel is humped or if the sidewalls are moved in to form a contracted section.



  - Critical depth may but will not always occur.
  - Head loss through such a contraction is in general small and may usually be neglected without introducing a sizable error.

Summary of Critical Depth

- For any value of specific energy, E, and flow, Q, one of the following is true:
  - There are two possible depths, called alternate depths.
  - There is one possible depth called the critical depth.
  - There are no physically realistic depths for the given energy and channel.
- Critical depth, d_c, plays a critical role in the analysis of flow in open channels as it divides the flow regime into supercritical and subcritical flow.
- Note that critical depth is a measure of the energy state for the channel and it is independent of the channel slope or roughness.
- The actual depth for a steady uniform flow, d_0, may or may not be at the critical depth.
- At the critical depth:
  - E is a minimum for a given discharge, Q.
  - Q is a maximum for a given E.
  - The velocity head is equal to one-half the hydraulic depth.
  - The Froude number equals 1.
  - The flow velocity equals the wave celerity.