# Math 526 – Algebraic Geometry
# Homework # 8
# Due: Thursday, December 5, 2013 8:30 am

**Problem 1.** *Let* $X = \{(x, a, b) \mid ax^2 + bx + 1 = 2ax + b = 0\} \subset \mathbb{C}^3$ *and* $\pi(x, a, b) = (a, b)$.

a. *Compute the constructible set* $\pi(X)$ *and the algebraic set* $\overline{\pi(X)}$.
b. *Describe* $X^c \subset \mathbb{P}^1 \times \mathbb{C}^2$, *the closure of* $X$ *in* $\mathbb{P}^1 \times \mathbb{C}^2$.
c. *Compute the algebraic set* $\gamma(X^c)$ *where* $\gamma : \mathbb{P}^1 \times \mathbb{C}^2 \to \mathbb{C}^2$ *is the extension of* $\pi$.

**Problem 2.** *Consider the homogeneous ideal* $I = \langle xw - yz, x^2z - y^3, xz^2 - y^2w, z^3 - yw^2\rangle$ *contained in* $\mathbb{C}[x, y, z, w]$.

a. *Compute* $HF_I$ *and* $HP_I$.
b. *Given* $I = \sqrt{I}$, *compute* $\dim \mathcal{V}(I)$ *and* $\deg \mathcal{V}(I)$.

**Problem 3.** *Verify Bézout's Theorem holds in* $\mathbb{P}^2(\mathbb{C})$ *for the following:*

a. $F_1 = x^2 + y^2 - z^2$ *and* $F_2 = x^2 - y^2 - z^2$
b. $F_1 = xz^2 - y^3$ *and* $F_2 = xz^2 - xy^2 - y^3$