Math 526 – Algebraic Geometry
Homework # 7
Due: Tuesday, November 26, 2013 8:30 am

Problem 1. *Let* $I = \langle x^2, xy \rangle \subset \mathbb{C}[x, y]$.

a. *For each* $\ell \geq 1$, *show that* $J_\ell = \langle x^2, xy, y^\ell \rangle$ *is primary and* $I = \langle x \rangle \cap J_\ell$. *This shows that an ideal can have infinitely many different primary decompositions.*
b. *Compute the standard monomials of* I, $\langle x \rangle$ *and* J_ℓ *with respect to any graded monomial ordering.*

Problem 2. *For* $I = \langle xy - xz, x^2 + y^2 - 2z^2 \rangle \subset \mathbb{C}[x, y, z]$, *compute* $\mathcal{V}(I) \subset \mathbb{P}^2(\mathbb{C})$. *Then, for each of the following affine patches* U *in* $\mathbb{P}^2(\mathbb{C})$, *compute* $\mathcal{V}(I) \cap U$*:*

a. $U = \{x \neq 0\}$,
b. $U = \{y \neq 0\}$,
c. $U = \{z \neq 0\}$,
d. $U = \{x + 3y - 2z \neq 0\}$.

Problem 3. *Construct the triple Segre product, that is, given* ℓ, m, *and* n, *determine* P *and morphism* ϕ *so that*

$$\phi : \mathbb{P}^\ell(k) \times \mathbb{P}^m(k) \times \mathbb{P}^n(k) \longhookrightarrow \mathbb{P}^P(k).$$