Math 526 – Algebraic Geometry
Homework # 6
Due: Tuesday, November 12, 2013 8:30 am

Problem 1. *For the following* $f \in \mathbb{Q}[x, y, z]$ *and* $I \subset \mathbb{Q}[x, y, z]$*:*

a. *determine if* $f \in \sqrt{I}$*;*

b. *if* $f \in \sqrt{I}$*, for each* $m \geq 1$*, compute the normal form of* f^m *with respect to* I *using the graded lexicographic ordering defined by* $x > y > z$*;*

c. *if* $f \in \sqrt{I}$*, use (b) to compute the minimum* $m \geq 1$ *such that* $f^m \in I$*.*

- $f = x + y$, $I = \langle x^3, x^2y + xy^2, y^3 \rangle$
- $f = x^2 + 3xz$, $I = \langle x + z, x^2y, x - z^2 \rangle$

Problem 2. *Use the Positivstellensatz to prove* $\{(x, y) \in \mathbb{R}^2 \mid y+x^2+2 = 0, x+3 \geq y^2\} = \emptyset$.

Problem 3. *For* $f_1 = x^2 - 2$ *and* $f_2 = y^8 - z^8$*, compute the prime decomposition for*

(1) $\langle f_1, f_2 \rangle \subset \mathbb{Q}[x, y, z]$;

(2) $\langle f_1, f_2 \rangle \subset \mathbb{R}[x, y, z]$;

(3) $\langle f_1, f_2 \rangle \subset \mathbb{C}[x, y, z]$.

Problem 4. *Let* $I = \langle xz - y^2, x^3 - yz \rangle \subset \mathbb{Q}[x, y, z]$.

a. *With respect to the lexicographic ordering* $t > x > y > z$*, compute a Gröbner basis for* $t \cdot I + (1 - t) \cdot \langle x \rangle$ *and for* $t \cdot I + (1 - t) \cdot \langle y \rangle$*. (By hand or via software)*

b. *Use (a) compute a Gröbner basis for* $I \cap \langle x \rangle$ *and* $I \cap \langle y \rangle$ *with respect to the lexicographic ordering* $x > y > z$*.*

c. *Use (b) to compute* $I : \langle x \rangle$ *and* $I : \langle y \rangle$*.*

d. *Compute a prime decomposition of* I*.*