Math 526 – Algebraic Geometry
Homework # 5
Due: Thursday, October 31, 2013 8:30 am

Problem 1. *If* $V \subset \mathbb{A}^n(k)$ *and* $W \subset \mathbb{A}^m(k)$ *are both irreducible, prove that* $V \times W = \{(v, w) \mid v \in V, w \in W\} \subset \mathbb{A}^n(k) \times \mathbb{A}^m(k)$ *is irreducible.*

Problem 2. *Let* $V_1 = \mathcal{V}(x_1, x_2)$*,* $V_2 = \mathcal{V}(x_2, x_3)$*, and* $V_3 = \mathcal{V}(x_1, x_3)$*.*

a. *Compute* $I(V_1) \cdot I(V_2) \cdot I(V_3)$*.*
b. *Compute* $I(V_1) \cap I(V_2) \cap I(V_3)$*.*
c. *Show that* $I(V_1) \cdot I(V_2) \cdot I(V_3) \subsetneq I(V_1) \cap I(V_2) \cap I(V_3)$*.*

Problem 3. *Suppose that* $I = \langle f_1, f_2 \rangle \subset \mathbb{C}[x, y]$ *such that* f_1 *is linear and* f_2 *is an irreducible quadratic polynomial. If* $g \in \sqrt{I}$*, show that* $g^2 \in I$*.*