Math 526 – Algebraic Geometry
Homework # 4
Due: Tuesday, October 1, 2013 8:30 am

Problem 1. *Let* $\phi : \mathbb{A}^n(k) \to \mathbb{A}^m(k)$ *be a morphism.*

a. *Show that* $\text{null } \phi^* := \{f \in k[y_1, \ldots, y_m] \mid \phi^* f = 0 \in k[x_1, \ldots, x_n]\}$ *is an ideal.*

b. *Show that* $\text{null } \phi^* = I(\phi(\mathbb{A}^n(k)))$.

Let $k = \mathbb{C}$, $\phi(t) = (2t+1, t^2)$, *and* $f(x_1, x_2) = a_1 x_1^2 + a_2 x_1 x_2 + a_3 x_2^2 + a_4 x_1 + a_5 x_2 + a_6$.

c. *Compute* $\phi^* f$.

d. *Find all* $(a_1, \ldots, a_6) \in \mathbb{C}^6$ *such that* $f \in \text{null } \phi^*$.

e. *Compute* $I(\phi(\mathbb{C}))$ *and compare with your answer from (d).*

Problem 2. *Let* $I = \langle t^2 + t - x, t^3 - y \rangle \subset \mathbb{C}[t, x, y]$ *and* $>$ *be the lexicographic ordering with* $t > x > y$.

a. *Compute a Gröbner basis for* I *with respect to* $>$.

b. *Use (a) to compute a Gröbner basis for* $I \cap \mathbb{C}[x, y]$.

Problem 3. *For* $f_1 = xw - yz$, $f_2 = x^2 z - y^3$, $f_3 = xz^2 - y^2 w$, *and* $f_4 = z^3 - yw^2$, *compute a generating set for* $Syz(f_1, f_2, f_3, f_4)$.