Math 526 – Algebraic Geometry
Homework # 1
Due: Thursday, August 29, 2013 8:30 am

This homework is a review of some concepts from abstract and linear algebra.

Problem 1. *Write the following polynomials as a product of irreducible polynomials in*

    a. $\mathbb{Q}[x]$;
    b. $\mathbb{R}[x]$;
    c. $\mathbb{C}[x]$.

- $g(x) = x^8 - 1$.
- $f(x) = x^3 + x - 10$.
- $h(x) = x^3 - x^2 - 2x + 2$.

Problem 2. *Construct a parameterization of the line in $\mathbb{C}^3$ defined by the equations*

$$
\begin{aligned}
3x + 5y - 2z &= 1 \\
x - 3y + 6z &= -2
\end{aligned}
$$

Problem 3. *Consider the following parameterically defined curves in $\mathbb{R}^2$:*

- $x(t) = t^5 - 4t^3$, $y(t) = t^2$;

- $x(t) = \sin t$, $y(t) = \cos t$;

- $x(t) = t - 2\sin t$, $y(t) = 1 - 2\cos t$;

- $x(t) = \dfrac{1 - t^2}{1 + t^2}$, $y(t) = \dfrac{2t}{1 + t^2}$.

    a. *Sketch each curve in $\mathbb{R}^2$.*

    b. *For each curve, if possible, find a polynomial $g \in \mathbb{R}[x, y]$ such that $g(x(t), y(t)) \equiv 0$. If this is not possible, give a brief explanation why.*