Answer Key 1

MATH 10560: Calculus II **Practice Final 2** *December 15, 2007*

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 22 multiple choice questions worth 6 points each. You start with 18 points.

You may not use a calculator.

1. [•] b c d e
2. a b c [•] e
3. a b c d [•]
4. a b c d [•]
5. a b c d [•]
6. a b [•] d e
7. a b c [•] e
8. a [•] c d e
9. [•] b c d e
10. [•] b c d e
11. a [•] c d e
12. a b c [•] e
13. [•] b c d e
14. a [•] c d e
15. a b c [•] e
16. [•] b c d e
17. a b c d [•]
18. a [•] c d e
19. a b c [•] e
20. a b c d [•]
21. a b [•] d e
22. a b c d [•]

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19. [a] [b] [c] [d] [e]
20. [a] [b] [c] [d] [e]
21. [a] [b] [c] [d] [e]
22. [a] [b] [c] [d] [e]

1. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{1}{3^n}$.

(a) $2/\sqrt{3}$ (b) $\sqrt{2}/3$ (c) $\sqrt{3}$ (d) $3/\sqrt{2}$ (e) $3/2$

2. Evaluate the integral $\displaystyle\int_0^{\infty} \frac{1}{\sqrt{x}}\, dx$.

(a) $1/\sqrt{2}$ (b) 2 (c) 1 (d) *diverges* (e) 10

3. Find the Maclaurin series for $e^{-x/2}$.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{-1}{n!} x^{n/2}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2(n!)} x^n$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} x^n$

4. Suppose $g(x)$ is the inverse of a differentiable function $f(x)$ and let $G(x) = e^{g(x)}$. If $f(3) = 1$ and $f'(3) = 2$, find $G'(1)$.

(a) $3e^2$ (b) e^3 (c) 2 (d) $e^2/3$ (e) $e^3/2$

5. Find the area of the surface obtained by rotating the curve $x = 3\sin t$, $y = 3\cos t$, $0 \le t \le \pi/6$ about the x-axis.

(a) $3\sqrt{2}\pi$ (b) 3π (c) $3\sqrt{3}\pi/2$ (d) 12π (e) 9π

6. Evaluate the integral $\displaystyle\int_0^\infty \frac{1}{x^2+3x+2}\,dx$.

(a) $\ln(3/2)$ (b) $\ln(1/2)$ (c) $\ln(2)$ (d) $\ln(3)$ (e) $\ln(5/2)$

7. Evaluate the integral $\displaystyle\int_0^{\pi/2} x\cos^2(x)\,dx$.

(a) $\dfrac{\pi^2}{8}-\dfrac{1}{2}$ (b) 0 (c) $\dfrac{\pi^2}{4}-2$ (d) $\dfrac{\pi^2}{16}-\dfrac{1}{4}$ (e) $\dfrac{\pi^2}{8}$

8. Evaluate the intergal $\displaystyle\int_0^1 \frac{\ln(1-x)}{x}\,dx$ as a series.

(a) $1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+\cdots$

(b) $-1-\dfrac{1}{2^2}-\dfrac{1}{3^2}-\dfrac{1}{4^2}-\cdots$

(c) $1+\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{4}+\cdots$

(d) $-1+\dfrac{1}{2^2}-\dfrac{1}{3^2}+\dfrac{1}{4^2}-\cdots$

(e) $-\dfrac{1}{2}-\dfrac{1}{2\cdot 3}-\dfrac{1}{3\cdot 4}-\dfrac{1}{4\cdot 5}-\cdots$

9. Determine the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{4^n\sqrt{n}}(x-1)^{2n}$.

(a) $[-1,3]$ (b) $[0,2]$ (c) $(-\infty,\infty)$ (d) $(-2,4]$ (e) $(-3,5)$

10. Find the third degree Taylor polynomial for $f(x) = \tan(x)$ at $a = 0$.

(a) $x + \frac{1}{3}x^3$ (b) $1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$ (c) $x + \frac{1}{2}x^2 + \frac{1}{3}x^3$

(d) $x - x^2 + x^3$ (e) x

11. Find the sum of the series: $\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$.

(a) 3/2 (b) 3 (c) 2 (d) 5/2 (e) 1

12. Determine which of the following series are convergent.

(I) $\sum_{n=0}^{\infty}(-1)^n \frac{3^n}{n^3 + 3^{n+1}}$ (II) $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2+1}$ (III) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^{3/2}}$

(a) (I) & (III) (b) (II) (c) (I) & (II)

(d) (II) & (III) (e) (I), (II) & (III)

13. Find the Taylor series of $\sin(x)$ at $a = \pi/2$.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x - \pi/2)^{2n}$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(x - \pi/2)^{2n+1}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$ (d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}(x - \pi/2)^n$

14. Find the arc length of the curve $r = 4\cos^2(\theta/2)$, $0 \le \theta \le \pi$.

(a) 5 (b) 8 (c) $4\sqrt{2}$ (d) 10 (e) $2\sqrt{2}$

15. Evaluate the integral $\displaystyle\int_0^3 \frac{1}{\sqrt{36 - x^2}}\, dx$.

(a) $\pi/4$ (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/6$ (e) $\pi/12$

16. Use Simpson's Rule with $n = 4$ to approximate $\displaystyle\int_{-2}^2 \frac{1}{x^2 + 2}\, dx$.

(a) 4/3 (b) 8/3 (c) 5/3 (d) 7/3 (e) 2

17. Find the slope of the line tangent to the curve $r = 1/\theta$ at the point with $\theta = \pi$.

(a) $\pi/2$ (b) $\dfrac{1 - \pi}{1 + \pi}$ (c) $-1/\pi^2$ (d) $3/\pi$ (e) $-\pi$

18. Find the coordinates of the centroid of the region below the graph of $f(x) = 4 - x^2$ and above the x-axis. The area of the region is 32/3.

(a) $(0, 12/5)$ (b) $(0, 8/5)$ (c) $(0, 9/5)$ (d) $(0, 3)$ (e) $(0, 2)$

19. Compute the derivative of $f(x) = (x+1)^{\cos x}$.

(a) $\cos(x)(x+1)^{\cos x - 1}$

(b) $\sin(x)(x+1)^{\cos x - 1}$

(c) $x^{\sin x}$

(d) $(x+1)^{\cos x}\Big(\dfrac{\cos x}{x+1} - \sin(x)\,\ln(x+1)\Big)$

(e) $(x+1)^{\cos x}\ln(\cos x)$

20. Evaluate the integral $\displaystyle\int_0^1 3^{4t}\,dt$.

(a) $3^4 - 1$

(b) $4(3^4 - 1)\ln(3)$

(c) $\dfrac{3^5}{5}$

(d) $\dfrac{3^4 + 1}{4\ln(3)}$

(e) $\dfrac{3^4 - 1}{4\ln(3)}$

21. Compute the limit $\displaystyle\lim_{x\to 1}\Big(\frac{1}{\ln(x)} - \frac{1}{x-1}\Big)$.

(a) $1/e$

(b) 0

(c) 1/2

(d) *does not exist*

(e) 1

22. The function $y(x)$ satisfies the differential equation $y' = \dfrac{2x}{3y^2}$ with the initial condition $y(0) = -1$. Find $y(3)$.

(a) $\sqrt{19}$

(b) 8/3

(c) $\sqrt[3]{10}$

(d) 1

(e) 2