Answer Key 1

MATH 10560: Calculus II **Practice Exam 3.1** *November 27, 2007*

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 6 points each and 4 partial credit problems worth 10 points each. You start with 12 points. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may not use a calculator.

1. [a] [b] [c] [•] [e]
2. [a] [b] [c] [d] [•]
3. [a] [b] [c] [•] [e]
4. [a] [b] [c] [d] [•]
5. [•] [b] [c] [d] [e]
6. [a] [b] [c] [d] [•]
7. [a] [b] [c] [d] [•]
8. [•] [b] [c] [d] [e]

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1. Determine which of the following statements applies to the series $\sum_{n=1}^{\infty} \frac{(-3)^{2n}}{n^n}$.

(a) Diverges by the root test.

(b) The root and alternating series tests are inconclusive.

(c) Diverges by the alternating series test.

(d) Converges by the root test.

(e) Converges by the alternating series test.

2. Find the sum of the series $\sum_{n=0}^{\infty} \frac{2}{5^{n+1}}$.

(a) 5/4 (b) 2 (c) 5/2 (d) 2/5 (e) 1/2

3. Use the Binomial Series to expand $\frac{1}{(1-x^2)^{1/3}}$ as a power series.

(a) $1 - \frac{1}{3}x + \frac{2}{3}x^2 - \frac{2}{9}x^3 + \cdots$

(b) $1 + x^{2/3} + x^{4/3} + x^2 + \cdots$

(c) $1 - \frac{1}{3}x^2 + \frac{1}{9}x^4 + \frac{7}{81}x^6 + \cdots$

(d) $1 + \frac{1}{3}x^2 + \frac{2}{9}x^4 + \frac{14}{81}x^6 + \cdots$

(e) $1 + \frac{1}{3}x^2 + \frac{1}{9}x^4 + \frac{7}{27}x^6 + \cdots$

4. A sequence is defined recursively by $a_1 = 1$ and $a_{n+1} = \frac{1}{4}(a_n + 5)$ for $n \geq 1$. Assuming the sequence is increasing and bounded above, find the limit $\lim_{n\to\infty} a_n$.

(a) 9/5 (b) 7/4 (c) 3/2 (d) 2 (e) 5/3

5. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$.

(a) $[-1, 5)$ (b) $[-1, 1]$ (c) $[1, 3]$ (d) $(1, 3)$ (e) $(-1, 5]$

6. Use power series to compute $\lim_{x\to 0} \frac{\ln(1-x^6)+x^6}{\sin(x^4)-x^4}$.

(a) 1/2 (b) 1 (c) 1/6 (d) 0 (e) 3

7. Determine which of the following statements are *true*.

(I) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.

(II) The Ratio Test cannot be used to determine whether $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.

(III) If $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(a) (I) and (III) (b) (I), (II), and (III) (c) (I) and (II)

(d) None (e) (II) and (III)

8. Determine which *one* of the following series converges.

(a) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$ (b) $\sum_{n=1}^{\infty} \frac{n-1}{n^2+1}$ (c) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

(d) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ (e) $\sum_{n=1}^{\infty} (-1)^n$

9. *a)* Use the integral test to show that the series $\sum_{n=2}^{\infty} \frac{n}{(n^2-1)^2} = \frac{2}{3^2} + \frac{3}{8^2} + \frac{4}{15^2} + \cdots$ converges.
b) Write down a partial sum (do not try to simplify it) that approximates the series to within 1/40. Be sure to justify your answer.

10. Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n+1}}{n^2+n-3}$ converges absolutely, converges conditionally, or diverges.

11. Find a power series representation of $f(x) = \frac{x}{1+2x^2}$ and determine its radius of convergence.

12. *a)* Derive the Taylor series for $f(x) = \ln(x)$ centered at $a = 1$. (Use the basic formula for Taylor series.)
b) Estimate the maximum error of approximating $\ln(x)$ by its third-degree Taylor polynomial on the interval $[0.5, 1.5]$.