

Review Sheet for Exam 2

Standard disclaimer: The following represents a sincere effort to help you prepare for our exam. It is not guaranteed to be perfect. There might well be minor errors or (especially) omissions. These will not, however, absolve you of the responsibility to be fully prepared for the exam. If you suspect a problem with this review sheet, please bring it to my attention (bounty points are possible).

Time and place: the exam will take place Thursday, Nov 16 in class. It will cover the material from chapters 2 and 3 of Shifrin.

Ground Rules: the exam is closed book and no calculators are allowed. All you'll need are sharp pencils and a good eraser. No pens please!

Format: Like exam 1, exam 2 will ask you to state some definitions and named theorems, and it will include some computational/proof problems of varying length. Unlike exam 1, there won't be much in the way of true/false questions. Instead I'll ask you for/about a few specific examples and proofs that we've gone over in class and homework. Specifically, you should know how to prove the following (the parentheses indicate where to find these things in my summary notes).

- Linear maps are continuous (6.6) (including the proof that $\|A\mathbf{v}\| \leq \|A\| \|\mathbf{v}\|$).
- The multiplication function $(x, y) \mapsto xy$ is continuous.
- Limits commute with continuous functions (6.8).
- Differentiability implies continuity (7.4).
- Differentiability implies directional derivatives exist (7.5).
- C^1 implies differentiable (7.7). Here it's enough to do as I did in class and prove it only for $f : \mathbf{R}^2 \rightarrow \mathbf{R}$.

I'd also like you to be familiar with the following specific functions and their significance for our class.

- $(x, y) \mapsto \frac{xy}{x^2+y^2}$ (except that $(0, 0) \mapsto 0$).
- $(x, y) \mapsto \begin{cases} 0 & \text{if } 0 < |y| < x^2 \\ 1 & \text{otherwise} \end{cases}$
- $(x, y) \mapsto \frac{x^2-y^2}{x^2+y^2}$ (except that $(0, 0) \mapsto 0$).

As always:

- Make sure you thoroughly understand the solutions to all the homework problems. Even if you got full credit on a given problem, it's worth comparing your solution with mine and/or trying to write out the solution from scratch. If you haven't already done the warmup problems, you can use them to practice.
- Get a good night's sleep.