

*Math 80750, Spring 2009*

*Course Information*

*Metric Geometry and Applications to Perelman's recent work*

**Instructor:** Jianguo Cao, 279 Hayes-Healy, 631-8847 (office)

**Time to meet:** M W F 10:40a-11:30a

**classroom assigned:** Hayes Healy Center 117

**Course Description:** This is a graduate course on metric differential geometry with an emphasis on spaces with curvature bounded from below. If time permits, we will discuss metric geometric portion of Perelman's work on Thurston's Geometrization Conjecture. Among other things, we will discuss the collapsing theory developed by Cheeger, Gromov and Perelman.

Our course will be divided into two parts.

(1) *Comparing geometry via differential inequalities*

We will present a new version of comparison geometry via various differential inequalities. Such a new approach provides straight forward proofs of many known comparison theorems. Using differential inequalities, we discuss the comparison theorems for volume, diameter and topology with curvature bounds.

(2) *A new critical point theory on spaces with curvature bounded from below.*

We discuss the critical point theory of distance functions on Riemannian and Kähler manifolds with curvature bounded below. Topics will include the Gromov's Betti number theorem, Grove-Shiohama sphere theorem, Cheeger-Gromoll-Perelman soul theory, Grove's proof of diffeomorphism part of soul theorem, Cheeger-Gromov-Fukaya collapsing theory, Perelman's collapsing theorem for the geometrization of 3-manifolds.

**Reference books:**

1. D. Burago; Yuri Burago and S. Ivanov: "A course in metric geometry. Graduate Studies in Mathematics," volume 33. American Mathematical Society, Providence, RI, 2001. xiv+415 pp. ISBN: 0-8218-2129-6
2. Jianguo Cao, Hongyan Tang and Youde Wang "Lectures on modern metric differential geometry", to be published jointly by AMS and International Press.