Teaching Statement

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1. Philosophy

In my view, the task of the teacher is to lead the students on a clear and efficient path through the given subject. The tour should be informative, but also entertaining. The route should be easy to navigate, allowing those students who lag behind the group a chance to catch up, yet novel enough to give those wanting to move ahead of the pace the opportunity to explore intriguing side roads.

> "The stringent precision attainable for mathematical thought has led many authors to a mode of writing which must give the reader an impression being shut up in brightly illuminated cell where every detail sticks out with same dazzling clarity but without relief. I prefer the open landscape under a clear sky with its depth of perspective, where the wealth of sharply defined nearby detail gradually fades away towards the horizon."
> $\sim$ Hermann Weyl

More often than not, the mathematics instructor is leading a tour parallel to that led by the author of the accompanying textbook. The quote above was taken from the preface to Weyl's book *The Classical Groups: Their Invariants and Representations (London, H. Milford, Oxford University Press, 1939)* and it is equally valid today. My teaching style is deeply inspired by these remarks and I endeavor to expose my students to both points of view. Without perspective, it will be difficult for them to communicate with other people who use mathematics. Without knowledge of the detailed structure of the subject, it will be difficult for them to use mathematics productively. While the style of the textbook is static, I have the advantage as the instructor of flexibility in communication. I strive to adapt to the needs of the students throughout the course while continually seeking the maintain balance between perspective and detail in my communication of mathematics.

2. Teaching Experience

One of my favorite teaching moments to date came when I was teaching Honors Calculus II at the University of Notre Dame in the spring of 2010. The first half of this proof-based course concerns sequences and series. I wanted the students to appreciate the theory we were developing, the tests for convergence, analytic estimates, etc. I began our second lecture on the topic by examining numerically the partial sums of the harmonic series using a TI-89 projected to the side of the board via a document camera. We computed a table of the first 10 partial sums exactly and I asked the students what they thought about the convergence. "If it converges, then we are interested in computing the sum, i.e., the limit of the partial sums, right? So let's compute the partial sum of a large number of terms in case it does. How many terms do you want to add up?" I said. I chided them for being wimps when they asked for the 500th or 1000th partial sum. "Let's try the $n = 320,000$th partial sum!" A carefully chosen one. We set the calculator to work and, in the remaining 40 minutes, proved in two different ways that the harmonic series diverged, developed an analytic estimate for the n^{th} partial sum, and worked out the ball park decimal value by hand at the black-board using calculus, all while the calculator sat there displaying the output BUSY. At the end of the class, and after more than 40 minutes of computation, the TI-89 finally popped out a number in the vicinity of 11, within 1 of our estimate, and I seized the opportunity to drive home the point about just how slowly the harmonic series diverged. Student comments in evaluations basically said that presentation evoked a paradigm shift in their thinking, prompting them to stand tall on their knowledge and place their calculator crutch firmly into a box labeled: Tools. That was one of the most enjoyable lectures I have ever given and I can't wait to do it again.

Both as a visiting assistant professor at the University of Notre Dame and as a graduate student at the University of Arizona, I have served as the instructor of record for a variety of courses ranging from a proof-based honors calculus course to vector calculus with applications, various versions of calculus, and elementary algebra. I have also worked as an assistant for graduate courses in differential geometry

and topology and complex analysis, and designed curriculum for an entrance workshop for first year graduate students. Throughout the 2008-2009 academic year I was the principal lecturer in a seminar leading graduate students, post-docs, and other participants, on a tour of mathematical physics from classical mechanics to the standard model. In the spring of 2011, I will be giving a short course on quantization to exceptional undergraduates as part of the Notre Dame Mathematics Institute. Each class level presents its own set of pedagogical challenges and I have enjoyed tackling those challenges time and time again.

My efforts in teaching have not been confined to the classroom. I have also given a number of public lectures on mathematics to various audiences. I welcome the challenge of communicating scientific ideas to broad audiences. I am particularly proud of a presentation titled *Mathematics and Astronomy: Kepler's Laws of Planetary Motion.* This was developed as part of my honors calculus II course as a preamble to our final lectures using calculus to deduce the Kepler's Laws from Newton's theory of gravity. Then I was asked to give a version of the presentation to an audience of teachers and local high school students participating in the Notre Dame Math Circle Institute during the summer of 2010. What makes the presentation special, is that it is designed to use the immersion environment of the digital planetarium at the University of Notre Dame. I motivate, define, and illustrate the coordinate system on the celestial sphere used for astronomical observation of the planets, and guide the audience through Kepler's empirical reasoning which led to his celebrated laws.

3. Mentoring Experience

I have had the privilege of directing the undergraduate research and senior theses of three honors mathematics students over the past five years, two at the University of Notre Dame, and one at the University of Arizona. Two have continued on to graduate study in mathematics and one is pursuing a graduate degree in education. Work with such students is much more than regular meetings to discuss readings, it is an opportunity to expose them to the fascinating world of my profession and help them shape their post-collegiate future.

- Vivian Healey. BA 2010 Mathematics and Theology, University of Notre Dame. Pursuing a Ph.D. in Mathematics at Brown University. Her senior thesis, *Penrose and Ammann Tilings: Structure, Dynamics, and Noncommutative Geometry*, was primarily original research. She gave invited talks at MathFest, and the Young Mathematicians Conference on this work and received award for outstanding senior thesis in the honors college at Notre Dame.
- Sarah Pastorek. BS 2010 Mathematics, University of Notre Dame. Pursuing a masters degree in eduction at Creighton University. Her senior thesis, *Weyl's Asymptotic Law*, was primarily an exposition and evolved out of a reading course with me on functional analysis during the spring of her junior year. It also connected with an REU experience at the University of Tennessee Knoxville on spectral geometry.
- Christopher McMurdie. BS 2007 Mathematics and Music, University of Arizona, MS Mathematics 2009, University of Paris Sud. Pursuing a Ph.D. in Mathematics at Washington University. His senior thesis, *The Non-Commutative Geometry of Penrose Tilings* was primarily an exposition, but this project was carried out while I was a graduate student and overseen by my dissertation director. He followed a series of exercises that I designed, while researching background material, to construct a proof of Gelfand's theorem on commutative C*-algebras. He then worked to understand the set of all Penrose tilings as an interesting example of a noncommutative space.

More about my experience in the classroom and as a mentor can be found in my teaching portfolio at `http://www.nd.edu/~jcaine1`, including sample course materials such as my java applets, and some of my thoughts on pedagogy. I thoroughly enjoy teaching, in part because of the deeply satisfying experience of stimulating interested minds, and in part because I enjoy paying forward my debt to the teachers who inspired me. Above all, I encourage my students to keep a curious mind, ask questions, and demand explanation.