August 31, 2008

Addendum to "*Bonus Vetus* OLS"

(Baier and Bergstrand, or B-B, 2007)

In "*Bonus Vetus* OLS," the MR (linear) approximations are derived under the assumption of symmetric bilateral trade costs (SBTC), or $t_{ij} = t_{ji}$ (as in A-vW, 2003). As an approximation method, the comparative-static trade effects of trade-cost changes should naturally be biased relative to the "true" model (the A-vW system), as we show in the paper.

However, it is possible that in a world with *asymmetric* bilateral trade costs (or ABTC, $t_{ij} \neq t_{ji}$) that, on average, the bias of the comparative-static trade effects of trade-cost changes using the approximation method are considerably smaller than the bias of the comparative statics using the A-vW approach (since the A-vW approach assumes SBTC). In fact, systematic Monte Carlo evidence of this is provided in Table 2 in Bergstrand, Egger and Larch (October 2007 working paper at www.nd.edu/~jbergstr/working_papers.html). This provides another motivation for the approximation method in our paper.

In this addendum, we demonstrate why, in a world with ABTC, the MR approximation terms provide good approximations to the BV approximation terms generated under SBTC in B-B. In other words, we solve here for MR approximations allowing ABTC.

We start with A-vW's (2003) 2N equations (10) and (11) for MR under asymmetry:

$$\Pi_i^{1-\sigma} = \sum_{j=1}^{N} \left(t_{ij} / P_j\right)^{1-\sigma} \theta_j \qquad \textbf{(A-vW, 10)}$$

$$P_j^{1-\sigma} = \sum_{i=1}^{N} \left(t_{ij} / \Pi_i\right)^{1-\sigma} \theta_i \qquad \textbf{(A-vW, 11)}$$

Applying the first-order log-linear Taylor-series expansion in B-B, we can derive

$$(1-\sigma)\ln\widetilde{\Pi}_i = -(1-\sigma)\sum_{j=1}^{N}\theta_j \ln\widetilde{P}_j + (1-\sigma)\sum_{j=1}^{N}\theta_j \ln\widetilde{t}_{ij} \qquad \textbf{(1)}$$

and

$$\left(1-\sigma\right)\ln\widetilde{P}_j = -\left(1-\sigma\right)\sum_{i=1}^{N}\theta_i \ln\widetilde{\Pi}_i + \left(1-\sigma\right)\sum_{i=1}^{N}\theta_i \ln\widetilde{t}_{ij} \tag{2}$$

where notation is defined in B-B. We can easily divide both equations by (1-σ) to eliminate that term. Henceforth, for ease of notation, we will use the terms $P_i \equiv \left(1-\sigma\right)\ln\widetilde{P}_i$, $\Pi_i \equiv \left(1-\sigma\right)\ln\widetilde{\Pi}_i$, and $t_{ij} \equiv \left(1-\sigma\right)\ln\widetilde{t}_{ij}$. NOTE: We have re-defined P_i, Π_i and t_{ij} (vis-a-vis A-vW).

Because of the complexity of allowing ABTC and the matrix inversion needed, we assume a three-country world (i = 1, 2, 3). It will be useful now to specify, in this case, what B-B would derive for the MR approximation in the case of SBTC. For illustration, P_2 and P_3 would be:

$$P_2 = \theta_1 t_{12} + \theta_2 t_{22} + \theta_3 t_{32} = \theta_1 t_{21} + \theta_2 t_{22} + \theta_3 t_{23} \tag{3a}$$

$$P_3 = \theta_1 t_{13} + \theta_2 t_{23} + \theta_3 t_{33} = \theta_1 t_{31} + \theta_2 t_{32} + \theta_3 t_{33} \tag{3b}$$

Defining $t_2 \equiv \theta_1 t_{12} + \theta_2 t_{22} + \theta_3 t_{32}, t_2^* \equiv \theta_1 t_{21} + \theta_2 t_{22} + \theta_3 t_{23}, t_3 \equiv \theta_1 t_{13} + \theta_2 t_{23} + \theta_3 t_{33}, t_3^* \equiv \theta_1 t_{31} + \theta_2 t_{32} + \theta_3 t_{33}$ for simplicity, we have:

$$P_2 = t_2 = t_2^* \tag{4a}$$

$$P_3 = t_3 = t_3^* \tag{4b}$$

Analogously, we define $t_1 \equiv \theta_1 t_{11} + \theta_2 t_{21} + \theta_3 t_{31}\ and\ t_1^* \equiv \theta_1 t_{11} + \theta_2 t_{12} + \theta_3 t_{13}$.

In a three-country world, the system of 2N equations simplifies to:

$$\begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & 1 & 0 & 0 \\ \theta_1 & \theta_2 & \theta_3 & 0 & 1 & 0 \\ \theta_1 & \theta_2 & \theta_3 & 0 & 0 & 1 \\ 1 & 0 & 0 & \theta_1 & \theta_2 & \theta_3 \\ 0 & 1 & 0 & \theta_1 & \theta_2 & \theta_3 \\ 0 & 0 & 1 & \theta_1 & \theta_2 & \theta_3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{bmatrix} \begin{bmatrix} t_1^* \\ t_2^* \\ t_3^* \\ t_1 \\ t_2 \\ t_3 \end{bmatrix} \tag{5}$$

To solve the system of equations for P_1, P_2, P_3, Π_1, Π_2 and Π_3, we need to invert the LHS 6x6 matrix.

However, the determinant of this matrix is zero, implying a redundant equation (because $\theta_1 + \theta_2 + \theta_3 = 1$ is imposed). We set P_1 (=1) as the numeraire and also eliminate one equation, $\theta_1 P_1 + \theta_2 P_2 + \theta_3 P_3 + \Pi_1 = t_1^*$. This leaves the following system of 5 equations in 5 unknowns (P_2, P_3, Π_1, Π_2, Π_3):

(6)

$$\begin{bmatrix} \theta_2 & \theta_3 & 0 & 1 & 0 \\ \theta_2 & \theta_3 & 0 & 0 & 1 \\ 0 & 0 & \theta_1 & \theta_2 & \theta_3 \\ 1 & 0 & \theta_1 & \theta_2 & \theta_3 \\ 0 & 1 & \theta_1 & \theta_2 & \theta_3 \end{bmatrix} \begin{bmatrix} P_2 \\ P_3 \\ \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{bmatrix} = \begin{bmatrix} t_2^* \\ t_3^* \\ t_1 \\ t_2 \\ t_3 \end{bmatrix}$$

Since the determinant of the LHS 5x5 matrix is nonzero (and equal to θ_1), we can invert this matrix (matrix algebraic derivations of inversion available on request). Consequently, we find:

$$\begin{bmatrix} P_2 \\ P_3 \\ \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ -\theta_2/\theta_1 & -\theta_3/\theta_1 & 2-\theta_1 & \theta_2(\theta_2+\theta_3)/\theta_1 & \theta_3(\theta_2+\theta_3)/\theta_1 \\ 1 & 0 & \theta_2+\theta_3 & -\theta_2 & -\theta_3 \\ 0 & 1 & \theta_2+\theta_3 & -\theta_2 & -\theta_3 \end{bmatrix} \begin{bmatrix} t_2^* \\ t_3^* \\ t_1 \\ t_2 \\ t_3 \end{bmatrix} \quad \textbf{(7)}$$

We can now examine the factors influencing each price term; since good 1 is the numeraire, we ignore P_1 and $\Pi_{1.}$ We have:

$$P_2 = t_2 - t_1 = \sum_{i=1}^{3} \theta_i t_{i2} - \sum_{i=1}^{3} \theta_i t_{i1} \quad \textbf{(8a)}$$

$$P_3 = t_3 - t_1 = \sum_{i=1}^{3} \theta_i t_{i3} - \sum_{i=1}^{3} \theta_i t_{i1} \quad \textbf{(8b)}$$

$$\Pi_2 = t_2^* + \theta_2(t_1 - t_2) + \theta_3(t_1 - t_3) = \sum_{j=1}^{3} \theta_j t_{2j} + \sum_{i=1}^{3} \theta_i t_{i1} - \sum_{i=1}^{N} \sum_{j=1}^{N} \theta_i \theta_j t_{ij} \quad \textbf{(8c)}$$

$$\Pi_3 = t_3^* + \theta_2(t_1 - t_2) + \theta_3(t_1 - t_3) = \sum_{j=1}^{3} \theta_j t_{3j} + \sum_{i=1}^{3} \theta_i t_{i1} - \sum_{i=1}^{N} \sum_{j=1}^{N} \theta_i \theta_j t_{ij} \quad \textbf{(8d)}$$

It is clear that this will generalize to an N-country setting, such that for any countries *i* and *j* (not equal to1, the numeraire):

$$P_j = \sum_{i=1}^{N} \theta_i t_{ij} - \sum_{i=1}^{N} \theta_i t_{i1} \text{ , } j = 2,\ldots,N \tag{9a}$$

$$\Pi_i = \sum_{j=1}^{N} \theta_j t_{ij} + \sum_{i=1}^{N} \theta_i t_{i1} - \sum_{i=1}^{N}\sum_{j=1}^{N} \theta_i \theta_j t_{ij} \text{ ,} i = 2,\ldots,N \tag{9b}$$

First, note that, in a setting with N countries, the second RHS term in equation (9a) will be constant across all P_j (j = 2, ..., N); it reflects that country 1 is the numeraire. Hence, variation in P_j will be driven by the first RHS term in (9a), which is precisely the (log-linear) approximation used in B-B for the exporting country *i*. This is confirmed by examining equations (8a) and (8b).

Second, note that, in a setting with N countries, the second and third RHS terms in equation (9b) will be constant across all Π_i (i = 2, ..., N). The second RHS term reflects that country 1 is the numeraire; the third RHS term reflects that an "inward" price is the numeraire. Hence, variation in Π_i will be driven by the first RHS term in (9b), which is precisely the (log-linear) approximation used in B-B for importing country *j*. This is confirmed by examining equations (8c) and 8d).

Finally, we can rewrite the above equations using the *original* notation:

$$\ln \Pi_i = \sum_{j=1}^{N} \theta_j \ln t_{ij} + \sum_{i=1}^{N} \theta_i \ln t_{i1} - \sum_{i=1}^{N}\sum_{j=1}^{N} \theta_i \theta_j \ln t_{ij} \text{ ,} i = 2,\ldots,N \tag{10a}$$

$$\ln P_j = \sum_{i=1}^{N} \theta_i \ln t_{ij} - \sum_{i=1}^{N} \theta_i \ln t_{i1} \text{, } j = 2,\ldots,N \tag{10b}$$

where t_{ij} need not equal t_{ji}. Thus, the MR approximations derived in B-B under SBTC can also be derived under ABTC (and they are the same). Thus, the MR approximations derived in B-B under SBTC also work under ABTC, as the Monte Carlo results in Table 2 of Bergstrand, Egger and Larch (2007) confirm.