A Hybrid Control Strategy based on the Equilibria of the Continuous Dynamics

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Timed Petri nets in Hybrid Systems: Stability and Supervisory Control*

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Abstract

In this paper, timed Petri nets are used to model and control hybrid systems. Petri nets are used instead of finite automata primarily because of the advantages they offer in dealing with concurrency and complexity issues. A brief overview of existing results on hybrid systems that are based on Petri nets is first presented. A class of timed Petri nets named programmable timed Petri nets is then used to model hybrid systems. Based on this model stability and supervisory control of hybrid systems are addressed and efficient algorithms are introduced. In particular, uniform ultimate boundness of hybrid systems is addressed by considering collections of linear time invariant plants with a switching logic generated by a Petri net and tests of sufficient conditions are derived. Supervisory control addresses the transfer of the continuous state to a region of the state space under safety specifications on the discrete and continuous dynamics; the discrete specifications are satisfied using Petri net place invariants while the continuous specification are imposed allowing switching only in invariant sets.

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1 Introduction

In hybrid systems the behavior of interest is governed by interacting continuous and discrete dynamic processes. There are several reasons for using hybrid models to represent dynamic behavior of interest. Reducing complexity was and still is an important reason for dealing with hybrid systems. For example, in order to avoid dealing directly with a set of nonlinear equations one may choose to work with sets of simpler equations (e.g. linear), and switch among these simpler models; this is a rather common approach in modeling physical phenomena. In control, switching among simpler dynamical systems has been used successfully in practice for many decades. Recent efforts in hybrid systems research along these lines typically concentrate on the analysis of the dynamic behaviors and aim to design controllers with guaranteed stability and performance. The advent of digital machines has made hybrid systems very common indeed. Whenever a digital device interacts with the continuous world, the behavior involves hybrid phenomena that need to be analyzed and understood. Whenever the behavior of a computer program depends on values of continuous variables within that program (e.g. continuous time clocks) one needs hybrid system methodologies to guarantee correctness of the program and the safe operation of the hybrid system; in fact the verification of such digital computer programs has been one of the main goals of several serious research efforts in hybrid systems literature. The investigation of hybrid systems is creating a new and fascinating discipline bridging control engineering, mathematics and computer science; further information on hybrid systems may be found in references (Grossman *et al.*, 1993; Antsaklis *et al.*, 1995; Alur *et al.*, 1996*a*; Antsaklis *et al.*, 1997; Antsaklis *et al.*, 1998; Morse, 1997; Antsaklis and Nerode, 1998).

Hybrid control systems typically arise from the interaction of discrete planning algorithms and continuous processes, and as such, they frequently arise in the computer aided control of continuous processes in manufacturing, communication networks, and industrial process control, for example. The study of hybrid control systems is essential in designing sequential supervisory controllers for continuous systems, and it is central in designing intelligent control systems with a high degree of autonomy.

This paper considers systems that arise when computers are used to supervise or synchronize the actions of subsystems described by continuous dynamics (that involve continuous variables.) Examples of such systems arise in chemical process control, command and control networks, power distribution networks, as well as distributed manufacturing systems. The size and complexity of such systems often requires that the system use a number of distinct operational modes. Consequently, these systems can be viewed as *supervised* systems, in which a high-level discrete (-event) supervisor is used to coordinate the actions of various subsystems so that overall system safety is not compromised. By *safety*, we mean that pre-specified limits or tolerances on the subsystem states are not violated. So, these systems can be viewed as a *hybrid* mixture of systems with continuous dynamics (continuous variables) supervised by a switching law generated by a (discrete-event) supervisor described by discrete dynamics (discrete variables). As such, our system is properly viewed as a *hybrid dynamical system*, since it mixes two distinctly different types of dynamical processes: continuous and discrete.

In recent years, a variety of models have been introduced for hybrid systems. These models generally model the continuous part of the system by a set of ordinary differential equations and model the discrete part of the system by a discrete-event system. The discrete-event model which has been most widely used in the past is the finite automaton. Finite automata provide a particularly convenient method for hybrid system modeling. In (Stiver *et al.*, 1996*c*), the use of finite automata allowed an extension of the logical supervisory control framework to hybrid systems. A timed automata structure known as the hybrid automata was developed in (Alur and Dill, 1994; Alur *et al.*, 1996*b*) and permitted the extension of symbolic model checking methods to the verification of real time systems (Henzinger *et al.*, 1995).

In spite of this success, however, there are some significant limitations to using finite automata in the modeling, analysis and synthesis of hybrid control systems. The principle limitation concerns the complexity of such automata when used to design control supervisors, and particularly when used to model concurrent processes. Concurrent systems are systems in which there is a variety of subsystems working at the same time. The problem here is that the state space for a finite automaton representing the various discrete operational states that the network of systems can function in will grow in an exponential manner with the number of processes. In other words, many of the techniques developed in (Stiver *et al.*, 1996*c*) and (Henzinger *et al.*, 1995) will not scale well with problem size. This fact is made particularly evident when we begin examining a number of the negative results associated with the decidability of problems frame as finite automata. It appears that decidability results for hybrid automata (Puri and Varaiya, 1994) restrict the class of decidable systems to a rather specialized set. Recent attempts to transform linear systems to this form (Pappas and Sastry, 1997) seem to suggest that these forms are too restrictive for general hybrid systems. We therefore see that while providing an initial tool for the design and analysis of hybrid systems, automata based methods for hybrid modeling have an intrinsic limitation when dealing with highly concurrent processes.

In order to deal with highly concurrent processes, therefore, it is necessary to use discrete-event system models which are better suited to model system concurrency. One such model is the ordinary Petri net (Reisig, 1985). Petri nets can be viewed as a generalization of the finite automaton. A finite automaton is generally represented as a finite directed graph consisting of vertices and arcs between these vertices. The current state of the automaton is represented by the *marking* of the directed graph. For finite automata the marking rules are relatively simple. The Petri net can also be viewed as a directed graph in which there are two different types of vertices; places and transitions. Unlike the automaton, marking rules for Petri nets are more complex and allow the system modeller to synchronize the actions of various parts within the system in a way which is not easily accomplished using a finite automaton. Petri nets provide an excellent tool for easily capturing the inherent concurrency of a complex system as well as providing the means of modeling concurrency and conflict within the system. In general, a Petri net representation for a concurrent process will be more compact (fewer vertices) than its associated automaton representation and with the use of partial order semantics it is now possible to search the Petri net's state space in a much more efficient manner than is possible using automata models of the same system. Furthermore recent results in the supervisory control of discrete-event systems using ordinary Petri nets (Moody, 1997) have made possible to design supervisors in an efficient and transparent manner; and this methodology is used in this paper.

In this paper, we use ordinary Petri net models of hybrid systems and we study stability and supervision of such systems. It is shown that Petri nets represent a very powerful tool in the analysis and design of hybrid control systems. In particular, two approaches that use timed Petri nets are presented to address problems important in the safe supervision of hybrid systems. In particular, this paper is organized as follows: In Section 2, a brief discussion of existing Petri net approaches to hybrid control systems reported in the literature are presented. In Section 3, programmable timed Petri nets are introduced; they are used to model and study hybrid control systems later in the paper. In sections 4 and 5 we discuss in detail two Petri net approaches to hybrid control; the first emphasizes stability while the second emphasizes the supervisory control of hybrid systems. Note that related work has appeared in (Lemmon and Bett, 1996; Koutsoukos and Antsaklis, 1997).

2 Petri Nets in Hybrid Control Systems

Petri nets have been used extensively as tools for modeling, analysis and synthesis of discrete event systems. As it was discussed in the introduction, Petri net modeling formalisms offer advantages over finite automata, particularly when the issues of model complexity and concurrency of processes are of concern. Such advantages are also present in hybrid systems when using of Petri nets and later in this paper, in Section 4 and in Section 5, Petri net models are used successfully in the study of hybrid control system stability and supervision. In this section a brief overview of several results on hybrid systems that are based on Petri nets is presented, with emphasis on results that are more closely related to the ones presented later in this paper.

(Peleties and DeCarlo, 1994) presented a model based on the work by (Ramadge, 1990) on the periodicity of symbolic observations of piecewise smooth discrete-time systems. In their work, the continuous plant is approximated by a Petri net and a partition of the state space is defined with each region corresponding to a place of the Petri net; a transition between two places exists depending on conditions derived from the continuous dynamics. The analysis is based on the construction of a suitable projection from the state space of the continuous dynamics to the space of the symbolic dynamics. A Petri net supervisor controls the behavior of the plant. The supervisor consists of two Petri nets, which communicate with each other and the Petri net representation of the plant through an event based interface. The first Petri net acts as a marker identifying which of the subsystems is currently activated and the other carries out the actual control/supervision task by selecting the next system structure to be activated. The control objective is to drive the continuous state from between any two regions of the state space as these are defined by the partition. The proposed methodology requires the identification of properties which are invariant to the system evolution. Under the assumption that the regions formed by the partition of the state space satisfy these invariant properties, it is shown that any region is reachable from any region through some switching sequence and the advantages of carrying out the analysis using symbolic dynamics are examined.

In (Lunze *et al.*, 1997) a Petri net model is used as discrete event representation of the continuous variable system. The continuous-variable system can be represented by a state-space model $\dot{x} = f(x(t), u(t)),\ y = g(x(t), u(t)),\ x(0) = x_0$. The interface consists of two parts, which are called the *quantiser* and the *injection*. The output events are generated by a quantiser, which can be represented by a partition of the state space or the output space. The qualitative description of the state defines a partition of the state space into regions which are called *qualitative states*. The injection associates a control input with each control event. The supervisor represents a mapping of the output event sequence into the input event sequence and therefore, the supervisor determines the input event in dependence upon the current output event only. The control aim is to reach a prescribed qualitative state. The system with the quantiser and the injection is modeled using a class of Petri nets, namely finite state machines. Each place represents a qualitative state and each transition is associated with a Boolean expression. The transition is enabled if the predecessor place is marked and the Bollean expression holds. A transition exists only if there exists an input which can force the continuous state to cross the hypersurface between two qualitative states. An algorithm for supervisory control is proposed where the Petri net structure is used to find a set of input sequences that drive the system to the target qualitative state. The Petri net capture the nondeterministic properties of the discrete-event approximation of the plant; the supervisory control algorithm uses information from the state space model to reduce the nondeterminism intrinsic to the model.

Several other approaches to modeling of hybrid systems that use Petri nets have also been reported in the literature. Among these, *hybrid Petri nets* were proposed by (Le Bail *et al.*, 1991); they are combinations of ordinary and continuous Petri nets. (Pettersson and Lennartson, 1995) used Bond graphs to verify systems described by hybrid Petri nets and compared hybrid Petri net and switched Bond graph modeling using a process example. *High-level hybrid Petri nets* proposed by (Giua and Usai, 1996) are characterized by the use of structured individual tokens (e.g. colors) in the discrete part of the net, and provide a rich modeling formalism which takes advantage of the modular structure of Petri nets. *High level Petri nets including a set of differential equations* were proposed by (Vibert *et al.*, 1997) to model batch processes taking into account fluctuations of continuous variables. *Hybrid flow nets*, an extension of the hybrid Petri nets have been proposed by (Flaus and Alla, 1997); they can be seen as a continuous nonlinear extension of Petri nets and are analyzed using a generalization of the theory of structural invariants. (Demongodin and Koussoulas, 1998) considered a new extension of Petri nets, called *differential Petri nets.* Through the introduction of the differential place, the differential transition, and suitable evolution rules, it is possible to model concurrently discrete-event processes and continuous-time dynamic processes, represented by systems of linear ordinary differential equations.

In the next sections timed Petri nets are used to model hybrid control systems and study stability and supervision.

3 Programmable Timed Petri Nets

This section introduces hybrid system model in which timed Petri nets (Sifakis, 1977), generate the switching logic of the system. In particular, we introduce a *programmable timed Petri net* (PTPN) (Lemmon *et al.*, 1998) which is a timed Petri net whose places, transtions, and arcs are all labeled with formulae representing constraints and reset conditions on the rates and times generated by a set of continuous-time systems called *clocks*. The model can be seen as an extension of the Alur-Dill hybrid system model (Alur and Dill, 1994; Alur *et al.*, 1996*b*).

An ordinary Petri net is a directed graph in which there are two types of nodes; places and transitions. Graphically, we represent the places by open circles and the transitions by bars. Petri nets are often characterized by the 4-tuple, (P, T, I, O) where P is the set of *places*, T is the set of *transitions*, $I \subset P \times T$ is a set of input arcs (from places to transitions), and $O \subset T \times P$ is a set of output arcs (from transitions to places). We denote the *preset* of a transition $t \in T$ as $\bullet t$ and define it as the set of places, $p \in P$ such that $(p, t) \in I$. In a dual manner, we introduce the postset of a transition $t \in T$ as $t\bullet$ and define it as the set of places, $p \in P$ such that $(t, p) \in O$.

The dynamics of ordinary Petri nets are characterized by the way in which the network marking evolves. The marking $\mu : P \to Z$ is a mapping from the places onto non-negative integers. The marking $\mu(p)$ of place p denotes the number of *tokens* in the places (represented graphically by small filled circles). We say that the transition t is *enabled* if $\mu(p) > 0$ for all $p \in \bullet t$. An enabled transition may *fire*. We introduce a firing function $q : T \to \{0, 1\}$ such that $q(t) = 1$ if t is firing and is zero otherwise. If $\mu(p)$ and $\mu'(p)$ denote the marking of place p before and after the firing of enabled transition t, then

$$\mu'(p) = \begin{cases} \mu(p) + 1 & \text{if } p \in t \bullet / \bullet t \\ \mu(p) - 1 & \text{if } p \in \bullet t / t\bullet \\ \mu(p) & \text{otherwise} \end{cases} \tag{1}$$

In ordinary Petri nets, places and transitions represent abstractions of the system "states" and "actions"; note that the actions take a finite amount of time to fire (complete). It is therefore necessary to work with timed Petri nets (Sifakis, 1977). In a timed Petri net the firing vector and marking vectors become functions of a global time τ. We denote the timed firing vector as q_τ. It indicates which transitions are in the act of "firing" at time τ. The timed marking vector is denoted as μ_τ. Just as in ordinary Petri nets, we will say that a transition t is enabled at time τ if $\mu_\tau(p) > 0$ for all $p \in \bullet t$. An enabled transition is free to fire. For the timed Petri net, however, the firing of a transition occurs over a time interval $[\tau_0, \tau_f]$. The length of this interval is called the transition's *holding time*. A transition t starts to fire at time τ_0 is said to be *committed* and its firing function $q_{\tau_0}(t)$ is set to unity. During the time that the transition is committed, the network's marking vector is not changed. It is only when the firing is completed at time τ_f that the marking vector is changed according to equation 1 given above. At the time the transition has completed firing, we also reset the firing function q_{τ_f} to zero.

The duration of the firing interval (holding time) can be characterized in a variety of ways. Common approaches assume that the holding time is either a fixed constant or a random variable. In some applications, there is a growing realization that these holding times can be treated as control variables. These times can be controlled by introducing "local" timers which cause transitions to fire when specified conditions *programmed* by the system designer are satisfied. This approach was used for concurrent state machines in (Alur and Dill, 1994). Essentially, this approach characterizes the holding times by logical propositions defined over the times generated by a set of *local clocks*. Petri nets whose holding times are defined in this way will be referred to as *programmable timed Petri Nets* (PTPN).

Let $\mathcal{N} = (P, T, I, O)$ be an ordinary Petri net. We introduce a set, $\mathcal{X}$, of N *local clocks* where the ith clock $\mathcal{X}_i$ is denoted by the triple $(\dot{x}_i, x_{i0}, \tau_{i0})$. $x_{i0} \in \Re^n$ is a real vector representing the clock's offset. τ_{i0} is an initial time (measured with respect to the global clock) indicating when the local clock was started. $\dot{x}_i : \Re^n \rightarrow \Re^n$ is a Lipschitz continuous automorphism over $\Re^n$ characterizing the local clock's rate. Assume that the clock rate $\dot{x}_i$ is denoted by the automorphism f. The *local time* generated by the ith clock will be denoted as x_i which is a continuous function over $\Re^n$ that is the solution to the initial value problem,

$$\frac{dx_i}{dt} = f(x_i) \tag{2}$$

$$x_i(\tau_{i0}) = x_{i0} \tag{3}$$

for $\tau > \tau_{i0}$. We therefore see that the local timers are vector dynamical equations. The local time of the ith timer at global time τ is denoted as $x_i(\tau)$ and the timer's rate is denoted as $\dot{x}_i(\tau)$. We say that the *state* of the ith timer is the ordered pair $z_i(\tau) = (x_i(\tau), \dot{x}_i(\tau))$. The ensemble of all local clock states will simply be denoted as $z(\tau)$.

The interval $[\tau_0, \tau_f]$ over which a transition t will be firing is going to be characterized by formulae in a propositional logic whose atomic formulae are equations over the local times or clock rates of $\mathcal{X}$.

Definition 1 An textitatomic formula, p, takes one of the following forms;

1. It can be a *time constraint* of the form $h(x_i) = 0$ or $h(x_i) < 0$ where $h : \Re^n \rightarrow \Re$ is a real valued function. This formula means that the clock time x_i satisfies the equation.
2. The atomic formula p can be a *rate constraint* of the form $\dot{x}_i = f$ which means that the ith clock's rate $\dot{x}_i$ is equal to the vector field $f : \Re^n \rightarrow \Re^n$.
3. Finally, p can be a *reset equation* of the form $x_i(\tau) = \bar{x}_0$ which says that the ith clock's local time at global time τ is set to the vector $\bar{x}_0$.

Definition 2 We define a *well-formed formula* or *wff* as any expression generated by a finite number of application of the following rules;

1. Any atomic formula is a wff,
2. If p and q are wff's, then $p \wedge q$ is a wff.
3. If p is a wff, then $\tilde{}p$ is a wff

The set of all wffs formed in this manner will be denoted as $\mathcal{P}$.

The *syntax* for well formed formulas is defined with respect to an underlying Petri net structure of the form $\mathcal{N} = (P, T, I, O)$ and a set of local clocks $\mathcal{X}$. The local clock state z at time τ is said to *satisfy* a formula $p \in \mathcal{P}$ if p is "true" for the given clock state, $z(\tau)$. The satisfaction of p by $z(\tau)$ is denoted as $\mathcal{N} \models p[z(\tau)]$. The truth of the atomic formula is understood in the usual sense. We say that an atomic formla, $p \in \mathcal{P}$ is satisfied by $z(\tau)$ if and only if the evaluation of that formula is true. We say that $\mathcal{N} \models \tilde{}p[z(\tau)]$ if and only if $z(\tau)$ does not satisfy p. We say that $\mathcal{N} \models p \wedge q[z(\tau)]$ if and only if $\mathcal{N} \models p[z(\tau)]$ and $\mathcal{N} \models q[z(\tau)]$.

Consider an ordinary Petri net, $\mathcal{N} = (P, T, I, O)$ and a set of logical timers, $\mathcal{X}$. A *programmable timed Petri net* (PTPN) is denoted by the ordered tuple, $(\mathcal{N}, \mathcal{X}, \ell_P, \ell_T, \ell_I, \ell_O)$ where the functions $\ell_P : P \to \mathcal{P}$, $\ell_T : T \to \mathcal{P}$, $\ell_I : I \to \mathcal{P}$, and $\ell_O : O \to \mathcal{P}$ label the places, transition, input arcs, and output arcs (respectively) of the Petri net $\mathcal{N}$ with wffs in $\mathcal{P}$.

4 Switched Dynamical Systems

This section examines the qualitative behaviour of switched dynamical systems. A switched system is a continuous-time system whose structure changes in a discontinuous manner as the system state evolves into switching sets. More formally, such systems are often represented by equations of the following form

$$\dot{x} = f_{i(t)}(x(t), w(t)) \tag{4}$$

$$i(t) = q(x(t), i(t^-)) \tag{5}$$

where $x : \Re \to \Re^n$ and $i : \Re \to Z$ denote the continuous and discrete states of the system, respectively. The signal $w : \Re \to \Re^m$ is an exogenous disturbance. We say that $w \in \mathcal{BL}_\infty$ if $\operatorname{ess\,sup}_\tau \|w(\tau)\| \leq 1$. The continuous dynamics are controlled by a finite collection of N control strategies

$$\mathcal{D} = \{f_1, f_2, \cdots f_N\} \tag{6}$$

where $f_i : \Re^n \times \Re^m \to \Re^n$ for $i \in \{1, \ldots, N\}$ are locally Lipschitz continuous functions. The discrete state of the system is controlled by a *successor* function $q : \Re^n \times Z \to Z$ which determines the next possible discrete state $i(t)$ at time t given the current continuous state and the "previous" discrete state $i(t^-)$.

There are a variety of results providing sufficient conditions for the Lyapunov stability of such switched systems (assuming $w = 0$). In (Peleties and DeCarlo, 1991) a single positive definite functional is found which is a Lyapunov function for all subsystems of the switched system. Multiple Lyapunov function methods (Branicky, 1994; Hou *et al.*, 1996) have been developed which apply to a larger set of systems than the single Lyapunov function methods. In certain cases, where the switched system consists of linear time invariant subsystems, it has been suggested that multiple candidate Lyapunov-like functions can be determined by finding feasible points of a linear matrix inequality (LMI) (Pettersson and Lennartson, 1996) (Johansson and Rantzer, 1998). These last results are particularly important because they provide a computational method for checking the sufficient conditions for switched system stability provided in (Branicky, 1994).

The sufficient conditions presented in (Branicky, 1994; Hou *et al.*, 1996) and used in (Pettersson and Lennartson, 1996) to compute candidate Lyapunov functionals provide conditions for switched system stability, which may be very conservative, unless the structure of the switching law is explicitly accounted for. The purpose of this section is to show how such structural information can be extracted from Petri net models of switching logics and how such information is then used to formulate the LMI's whose feasibility provide a sufficient test for a switched system's qualitative behaviour. In particular, we examine a specific example from the power systems field in which we are interested in establishing uniform ultimate bounds on disturbed system behaviour.

4.1 PTPN Modeling of Switched Multi-agent Systems

The programmable timed Petri net provides a compact method of modeling switched systems consisting of several independently operating subsystems. Examples of such systems include networks of robots in a distributed manufacturing system, complex process control systems, distributed command and control systems. In this section we examine a distributed command and control system used to supervise the behaviour of a power distribution system.

Recall that a PTPN is a Petri net, $\mathcal{N}$, labeled with logical propositions defined over the times generated by a set $\mathcal{X}$ of local timers. A PTPN can be used to model a switched dynamical system in the following manner. The network, $\mathcal{N}$, is used to represent the logical dependencies between mode switches in the successor function. The timers, $\mathcal{X}$, of the PTPN are the dynamical equations associated with the continuous time dynamics of the system. The labels ℓ_P, ℓ_T, ℓ_I, and ℓ_O are chosen to represent conditions on the continuous state for mode switches as well as describing the various switching behaviour within the network.

Let $\mathcal{D} = \{f_1, \ldots, f_N\}$ be a set of N Lipschitz continuous vector fields and let $\mathcal{G} = \{h_1, \ldots, h_M\}$ be a set of smooth hypersurfaces in $\Re^n$. These functions are sometimes referred to as the *guards* of the system. Consider a network $\mathcal{N} = (P, T, I, O)$ and a set of timers $\mathcal{X}$ where the ith timer has rate $\dot{x}_i$, initial time x_{i0}, and reset time τ_{i0}. We label the places, transitions, and arcs of the Petri net $\mathcal{N}$ with wffs defined over the timer states, z_i. In particular, these labels are defined as follows.

- Let $J(p)$ be a subset of $\{1, \ldots, N\}$ associated with place $p \in P$ representing those clocks associated with place p. $\ell_P(p)$ is a wff of the form,

  $$\ell_P(p) = \bigvee_{i \in J(p)} ((\dot{x}_i = f_j) \wedge (\tau_{i0} = \tau)) \tag{7}$$

  This formula is interpreted as follows. When place p is marked, then the timer states, z_i, for all $i \in J(p)$ are reset to satisfy $\ell_P(p)$. In particular, this means that the initial time, τ_{i0}, and the clock rate ,$\dot{x}_i$, are reset to the values specified in the equation. The label $\ell_P(p)$ is therefore used to represent switching of the system's vector field when events occur (i.e. transitions fire).

- $\ell_T(t)$ is chosen to be a tautology in this section. This need not always be the case (see following sections), but in the specific example given below, we will only reset or restrict system states at places and arcs.

- Let $J(p, t)$ be a subset of $\{1, \ldots, M\}$ denote a set of hypersurfaces in $\mathcal{G}$ associated with the input arc, (p, t). $\ell_I((p, t))$ is chosen to be a wff whose truth commits the transition t to firing provided this transition is already enabled. In particular, we confine our attention to wffs of the form

  $$\ell_I((p, t)) = \bigwedge_{i \in J(p,t)} (g_i(x(\tau)) < 0) \tag{8}$$

  This condition allows t to be committed to firing when the continuous state (at time τ) satisfies the listed set of inequalities with respect to the hypersurfaces in $\mathcal{G}$. We refer to $\ell_i((p, t))$ as the input guard equation.

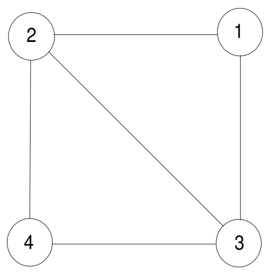

Figure 1: The example power system

- $\ell_O((p,t))$ is chosen as a wff whose truth completes the firing of the transition, t, assuming that transition t is enabled and committed. These conditions also take the same form as the input guard equations 8 labeling the network's input arcs.

The following example illustrates concretely the use of the PTPN in modeling multiagent systems. We consider the 4 node power system shown in figure 1. Each node in the figure represents a generator and the arcs denote the transmission lines between generators.

The continuous state of the ith generator is characterized by its rotor angle, θ_i, and the rotor angle's rate of change $\dot{\theta}_i$. Without loss of generality, we assume that node 3 is a reference node, so we can assume $\theta_3 = 0$ and $\dot{\theta}_3 = 0$ for all time. We therefore represent the continuous-state of the system as a vector in $\Re^6$ of the following form

$$x = \begin{bmatrix} \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 & \theta_4 & \dot{\theta}_4 \end{bmatrix}^T \tag{9}$$

The differential equation for the ith generator's angle, θ_i, is

$$\delta p_i = \frac{d^2\theta_i}{dt^2} + D_i \frac{d\theta_i}{dt} \tag{10}$$

where δp_i is the REAL power's variation about a specified operating level and D_i is a constant determined by the system's operating point and mass properties. From the power flow equation we know that

$$\delta p_i = \sum_j \left(B_{ij} \cos(\theta_i - \theta_j)\theta_i\right) + w_i \tag{11}$$

where B_{ij} is a constant based on the transmission line parameters and w_i is a bounded disturbance signal. From the preceding two equations, we obtain the following linearized system equations.

$$\begin{aligned} \dot{x} &= Ax + Bw && (12) \\ z &= Cx && (13) \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -B_{11} & -D_1 & -B_{12} & 0 & -B_{14} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -B_{21} & 0 & -B_{22} & -D_2 & -B_{24} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -B_{41} & 0 & -B_{42} & 0 & -B_{44} & -D_4 \end{bmatrix} \tag{14}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{16}$$

In this example, the setpoint was chosen to be $x_{set} = [0.1, 0, 0.1, 0, 0.1, 0]^T$. The transmission line parameters were chosen so that $B_{ij} = 10$ for all i, j.

The control objective is to regulate the variation of the generator angle to less than 0.1 for any disturbance $w \in \mathcal{BL}_\infty$. A switching policy is used to help achieve this goal. It is assumed that each generator has two winding ratios to choose from, D_{i0} and D_{i1}, for $i = 1, 2$ and 4. It is assumed that $D_{i0} = 5$ and $D_{i1} = 10$. The ith generator (node) is in discrete state 0 if the first winding ratio is used (i.e. $D_i = D_{i0}$) and is in state (mode) 1 otherwise. There are two conditions which the generators need to respond to.

- The generator must respond to a *fault* which could be caused by a large transient current. If such a transient is detected in the neighborhood of node i then we need to increase the generator winding ratio to protect the generator. The supervisory strategy therefore switches the generator from mode 0 to 1. In this example it is assumed that such a fault is detected when $|\theta_i| > 0.05$.
- At times, however, the generator will need to adjust its output in order to track changing load conditions. If a request to change the load is generated and the node is in mode 1, then we will switch the generator to mode 0. In the context of a detected fault, the switch from mode 1 to mode 0 will be constrained to reset the operating mode 5 seconds after the fault was tripped.

The strategy outlined above can be applied to each generator in a decoupled manner. We can construct a network, $\mathcal{N}$, to represent the logical states of the system as shown in figure 2. In this figure, we see that the switching logic for each generator is represented by a subnet consisting of two places and two transitions. The places are labeled with the system matrices and represent the two different modes that the system can operate in. The transitions represent the tripping of the fault alarm and the subsequent resetting of the generator. The conditions for tripping these alarms appear as logical labels on the input arcs from the places to the transitions in the subnets.

In practice, however, the simple strategy shown in figure 2 will not be able to meet the performance specification. This failure is due to the fact that the generators are coupled by the transmission lines shown in figure 1. We can readily verify the effect of generator coupling through simulation studies. Figure 3 shows a simulation result in which an impulsive input $\omega(t)$ satisfying $\|\omega(t)\| \leq 1$ occurs every 10 seconds within the system. The simulation shows that if two neighborhing generator nodes (i.e. nodes 1 and 2 or nodes 2 and 4) are both in mode 1, the rotor angle of the generators will exhibit large variations in excess of the performance requirement in the presence

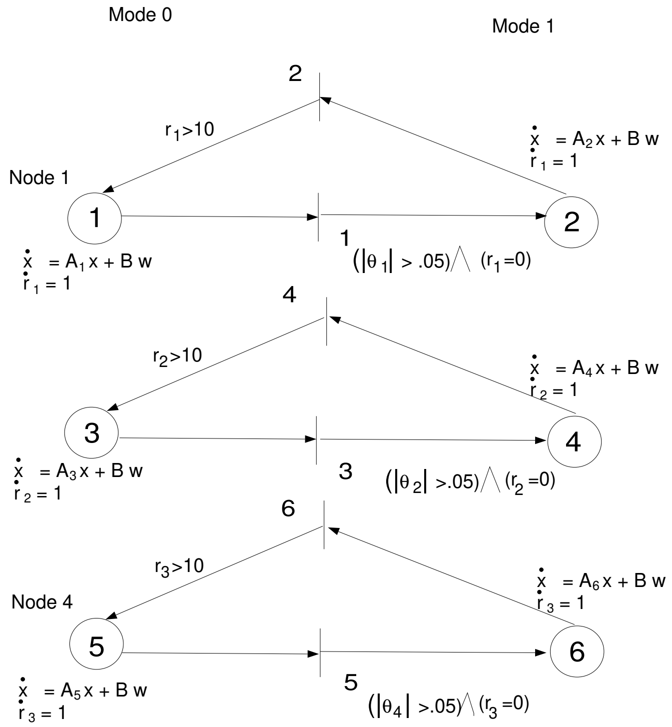

Figure 2: Original Petri-net model of example power system

of the disturbance. To achieve the control objective, we therefore implement a supervisory control that prevents adjacent generators from being in mode 1 at the same time. The Petri net model of this controlled system is shown in figure 4. This supervision introduces a control place (also called a monitor) to ensure that adjacent generators enter mode 1 in a mutually exclusive manner. It is the stability of this supervised system that will be studied in the following sections.

4.2 Uniform Ultimate Bounds of Switched Systems

The supervised controller represented in figure 4 attempts to ensure acceptable system behaviour in the presence of bounded faults. The determination of this supervisor was based on "simulation" methods which may not be able to check for all possible failures that can occur in the system. We therefore need to develop a more systematic method of identifying potential system faults that lead to violation of the performance specification without having to resort to exhaustive simulation. This subsection studies the qualitative behaviour of switched systems modeled as programmable timed Petri nets in which exhaustive simulations are not needed. In particular, we use recent results from (Bett and Lemmon, 1997) to compute uniform ultimate bounds for switched systems whose subsystems are linear time invariant (LTI) and whose switching regions form conic sectors in the continuous state space of the system. As noted above there has been considerable interest in studying the Lyapunov stability of such switched systems. In many applications, however, the ability to obtain uniform ultimate bounds on system behaviour may be more crucial. System switching occurs when state trajectories cross specified boundaries (guard conditions) in the state

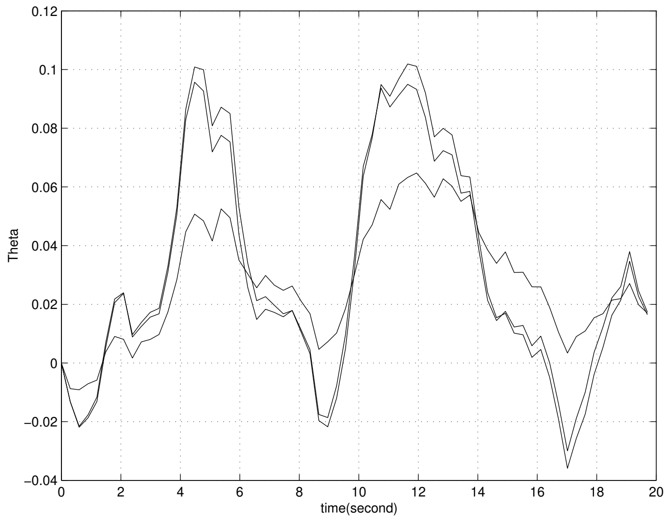

Figure 3: Simulation Results

space. From this standpoint, therefore, we are very concerned with being able to bound the *amplitude* of the state trajectory (as measured by the $\mathcal{L}_\infty$ signal norm) in order to control the system's switching behaviour.

Definition 3 Consider a disturbed system $\dot{x} = f(x, w)$ where $w \in \mathcal{BL}_\infty$. We say that x_0 is an *equilibrium* point of the undisturbed system if $f(x_0, 0) = 0$. We say that the disturbed system is *uniformly ultimately bounded* if and only if for all $\epsilon > 0$ there exists a time $T(\epsilon) > 0$ and $\delta > 0$ such that if $x(t_0) < \epsilon$, then $x(T) < \delta$ for all $t > T(\epsilon)$.

Computing the uniform ultimate bound δ and the dwell time $T(\epsilon)$ for switched systems is well understood. The key result here is a switching lemma stated in (Bett and Lemmon, 1997). Before stating this result we need to introduce some terminology. Consider a linear time invariant system of the form

$$\dot{x} = Ax + Bw \tag{17}$$
$$z = Cx + Dw \tag{18}$$

Let α and β be real non-negative constants. We say that a matrix $P \in \text{FeasRic}(A, B, \alpha, \beta)$, read as Feasible Riccati, if P satisfies the following Riccati inequality,

$$A'P + PA + (\alpha + \beta)P + \frac{1}{\alpha} PBB'P \leq 0 \tag{19}$$

Theorem 1 *(Bett and Lemmon, 1997). Consider two LTI systems $\Sigma_1 = (A_1, B_1, C_1, D_1)$ and $\Sigma_2 = (A_2, B_2, C_2, D_2)$ and consider any finite constants $r \in (0, 1]$ and $\gamma > 0$. Suppose there exists positive constants α, β, and ρ and positive definite matrices P_1 and P_2 such that*

$$rP_2 \leq P_1 \tag{20}$$

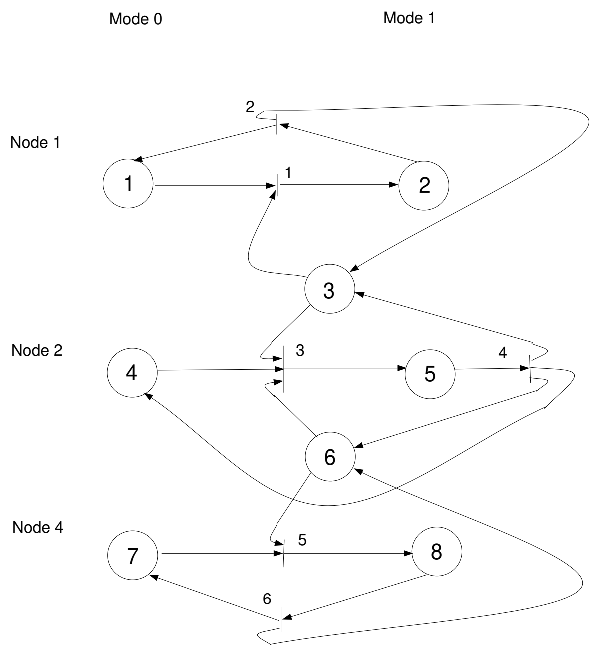

Figure 4: The controlled Petri-net model of the example power system

$$
\begin{aligned}
\gamma^2 P_1 &\geq C_1' C_1 && (21)\\
\gamma^2 P_2 &\geq C_2' C_2 && (22)\\
P_1 &\in \mathrm{FeasRic}(A_1, B_1, 2\beta + \frac{\alpha}{r}, \alpha) && (23)\\
P_2 &\in \mathrm{FeasRic}(A_2, B_2, \rho, \rho) && (24)
\end{aligned}
$$

Consider a time $t_s > 0$ and let w, x, and z, be the input, state, and output of the dynamical system which evolves according to system Σ_1 for $0 < t < t_s$ and which evolves according to Σ_2 for $t > t_s$. If

$$t_s > -\frac{1}{2\beta} \log r = T_d \tag{25}$$

then $\|z\|_\infty \leq \gamma$ for all $t > T_d$.

The preceding theorem provides conditions that the LTI subsystems must satisfy in order to ensure that switching transients do not violate the amplitude constraint $\|z\|_\infty < 1$. These results, therefore provide a convenient way of ensuring uniform ultimate boundedness of the switched system. Note that these conditions can also be used to formulate linear matrix inequalities similar in structure to those used by (Pettersson and Lennartson, 1996) and (Johansson and Rantzer, 1998) to ensure Lyapunov stability of switched systems. The obvious implication here is that we should be able to use the preceding theorem to construct linear matrix inequalities whose feasibility is sufficient for the switched system trajectory to possess a uniform ultimate bound.

Unfoldings (Engelfriet, 1991) can be used to study the stability and performance of a switched system represented by a PTPN. A sufficient condition (He and Lemmon, 1997) for the Lyapunov stability and ultimate bounded behaviour of a switched LTI system is that a set of LMI's associated with the fundamental cycles of the system's reachability graph be feasible. These fundamental cycles can be found by constructing the reachability graph, an inefficient approach, or they can be systematically constructed from the network unfolding. In the following example, we illustrate how the use of unfoldings can reduce the complexity of searching for fundamental cycles in the PTPN's reachability graph.

We first summarize some basic results on unfoldings. Given a Petri net, we say that two places p_1 and p_2 are in conflict if there exist distinct transitions t_1 and t_2 such that $\bullet t_1 \cap \bullet t_2 \neq \emptyset$ and (t_1, p_1) and (t_2, p_2) belong to the reflexive and transitive closure of the arcs. In other words, p_1 and p_2 are in conflict if two paths terminating at those places start at the same place and immediately diverge. An occurrence net is an acyclic conflict free net. A branching process of network N is a pair (N', h) such that N' is an occurrence net and h is a net homomorphism mapping N' to N in a way that preserves the behaviour of the original net (see (Engelfriet, 1991) or (Esparza *et al.*, 1996) for precise definitions). An unfolding is the maximal branching process associated with a Petri net N. Consider the occurrence net of a Petri net unfolding. A configuration C of this net is a set of transitions satisfying the following conditions,

- $t \in C$ implies for all $t' \leq t : t' \in C$
- for all transitions in C are conflict free.

An occurrence net may have several different configurations. Two configurations which can be marked at the same time are said to be "concurrent". Concurrency can be viewed as an equivalence relation over the set of all configurations of an occurrence net. In particular, this means that the set of configurations can be partitioned into equivalence classes.

Let's return to the example considered above. The unfolding of the controlled Petri-net is shown in figure 6. Three configurations are identified in this unfolding. These configurations are represented by the following three sets of transitions; (t_1, t_2), (t_3, t_4), and (t_5, t_6). We label these configurations as c_1, c_2 and c_3, respectively. Concurrency of these configurations induces two equivalence classes. Configuration c_2 forms one of the equivalence classes and configurations c_1 and c_3 form the other equivalence class. We have been able to develop a systematic algorithm (He and Lemmon, 1997) for constructing the fundamental cycles. In this particular example, the two configurational equivalence classes form 5 fundamental cycles represented by the sequence of firing of transitions t_1, t_2, t_5, and t_6. These fundamental cycles are $t_1 - t_2$, $t_5 - t_6$, $t_3 - t_4$, $t_1 - t_5 - t_6$, and $t_5 - t_1 - t_2$. For each fundamental cycle an LMI is constructed. The feasibility of the resulting LMI is easily checked using existing commercial software. In this example, the LMI is feasible thereby showing that the system satisfies the bounded amplitude objective, i.e. $\|z\|_\infty < 0.1$. Simulation results (fig. 5) shows that $\|\theta_i(t)\| \leq 0.6$, which clearly validates the correctness of this approach.

It is useful to examine the reachability tree for this example. Constructing the reachability graph requires a total number of 10 nodes to be created and 33 paths to be traced. The unfolding of the network, however, only requires 3 configurations to be identified and 3 paths to be traced. This observation demonstrates that unfolding provides a more efficient method for finding fundamental cycles in PTPN than direct construction of the reachability graph. This empirical finding supports

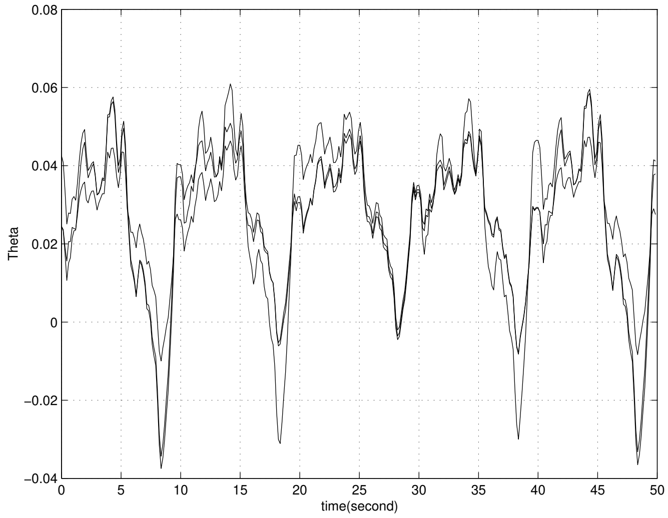

Figure 5: Controlled System Simulation Results

the claims made in (McMillan, 1992) where it was asserted that the computational complexity of constructing the reachability graph is exponential in the number of places and transitions. In contrast, the unfolding generally has a polynomial complexity. This difference is illustrated by first deriving the fundamental cycles using the reachability graph for this problem (fig. 7), where the labels of the nodes represent the markings of the net in hexadecimal notation, and comparing it to the relatively simple occurrence net for this problem (fig. 6).

5 Supervision of hybrid systems

In this section supervisory controllers are derived that determine the switching policy of the hybrid system. Here, again we consider the programmable timed Petri net model introduced in Section 3.

It is assumed that the switching policy is embedded in the dynamics of the underlying Petri net structure and that the supervisors are described by Petri nets. A two level supervisory control is derived. In the first level, the temporal information is disregarded and DES control methods to restrict the discrete event behavior are applied. In the second level, based on information about the continuous dynamics of the subsystems, conditions on the switching times are imposed to satisfy the continuous specifications.

In more detail, we assume that the dynamics of the system depend on temporal and routing data associated with the transitions and the places of the Petri net respectively. The hybrid plant is controlled by a supervisor which determines the temporal and the routing policy to satisfy the specifications. The discrete specifications are expressed in terms of linear constraints on the marking vector, while the control objective is to drive the continuous state to prescribed regions of the state space. A DES control method, namely supervisory control of Petri nets based on place invariants (Moody and Antsaklis, 1996), is applied to satisfy the discrete specifications. We

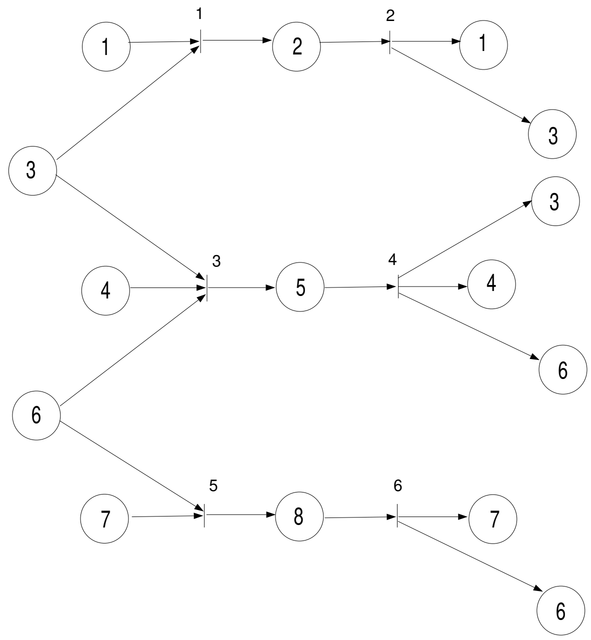

Figure 6: Occurrence Network of Problem

assume that the continuous subsystems admit a family of equilibria and each equilibrium has a domain of attraction associated with it. The hybrid system switches from a subsystem to another, in a way that the state gradually progresses from one equilibrium to another towards the desired target equilibrium. A control algorithm based on the notion of a common flow region is introduced in the case when the continuous dynamics are described by affine systems. When the continuous dynamics are described by first order integrators, the supervisor determines the routing policy and the temporal data by solving a linear programming problem.

The remaining of this section is organized as follows. First, the hybrid plant is modeled by a PTPN using well defined mathematical entities for the routing and temporal information to clarify the presentation of the approach. Then, the supervisor and its tasks are presented. A methodology for DES control based on the place invariants of the Petri net is used for the discrete specifications. Next, an algorithm based on the notion of a common flow region is presented for the continuous specifications. It is shown that affine systems satisfy the conditions and this algorithm can be applied. Finally, we study the special case when the continuous dynamics are described by first order integrators.

5.1 Hybrid Plant

In this section, we consider that the plant is a hybrid dynamical system described by equations (4,5) where we have set the disturbance $w(t)$ to zero. Then the continuous state of the hybrid plant evolves in $X \subset \Re^n$ and is described by

$$\dot{x}(t) = f_i(x(t)), \quad T_k \leq t \leq T_{k+1}, \quad i = 1, \ldots, N \tag{26}$$

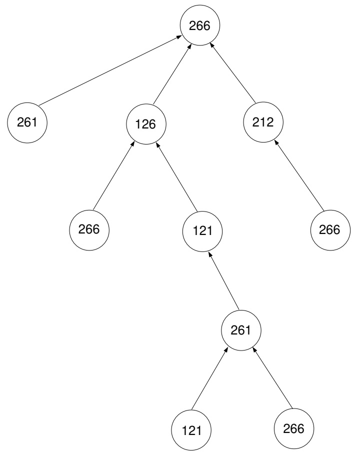

Figure 7: Reachability Graph

where T denotes the global time.

The switching function of equation (5) is implemented by the switching rules of the underlying Petri net structure. In the following, we formulate a mathematical description of the general switching function which can be naturally introduced to the Petri net structure.

Consider the sequence

$$s(t_0, x_0) = (i_0, \tau_0), (i_1, \tau_1), \ldots, (i_k, \tau_k), \ldots \tag{27}$$

constructed from the sequence of switching times $\{t_i\}$, $i = 0, 1, \ldots$ by defining $\tau_i = t_{i+1} - t_i$, $i = 0, 1, \ldots$ and implying that the system f_{i_k} is active during the time interval τ_k. The *event projection* and the *timed projection* of the switching sequence are define as

$$\pi_1(s) = i_0, i_1, i_2, \ldots \tag{28}$$

$$\pi_2(s) = \tau_0, \tau_1, \tau_2, \ldots \tag{29}$$

We will represent the event projection with routing data associated with the places and the timed projection with temporal data associated with the transitions of the Petri net. The supervisor considers initially the routing and temporal data as different mathematical entities. First, it determines the routing policy to generate a desirable event projection and then, it imposes conditions of the temporal data. In general, the coupling between the time-driven and the event-driven dynamics is very complicated to allow such a desing approach. Here, we want to identify problems for which such a decomposition of the switching function allows the development of efficient control algorithms.

The temporal information is described by the data $\sigma^t(n)$, $n \in N$, where $\sigma^t(k)$ represents the duration of the k^{th} firing of transition t. During the time interval $\sigma^t(k)$, the tokens of the input places of transition t do not change, these tokens are put into the output places of t upon the completion of firing of the transition, according to the firing rule of the untimed Petri net.

To model a hybrid plant we think of the transitions as processes described by differential equations. Let $\sigma^{t_i}(k)$ denote the operation time associated with the k^{th} firing of the transition t_i. During this time interval the continuous state of the hybrid plant evolves according to the differential equation $\dot{x} = f_i(x)$. This assignment is defined by the labeling function $\ell_T(t) : T \to \Sigma$ which is chosen to be an atomic rate formula of the form $\dot{x} = f_i(x)$.

We assume that at each time instant exactly one transition is firing. In addition, we assume that $0 < \Delta \le \sigma^t(n) < \infty$ for all firings n and transitions t. We may easily incorporate in our model instantaneous transitions, but these correspond to jumps in the continuous state and will not be considered here. The assumption $\sigma^t(n) \ge \Delta > 0$ eliminates the possibility of infinitely many switchings in a finite time interval.

The control algorithm for the mode selection problem will be based on the routing data. Routing policy for timed Petri nets is used usually for resolution of conflicts and was introduced in (Baccelli *et al.*, 1992). In our case, routing data $\nu^p(n)$ are attached to each place, where $\nu^p(k)$ gives the transition $t \in p\bullet$ to which the k^{th} token to enter place p is to be routed. If the k^{th} token is routed to $t \in p\bullet$, then the transition t wins the token, which after a firing time of $\sigma^t(k)$ is routed to $t\bullet$, the output places of the transition. Since each transition of the timed Petri net corresponds to a vector field, we require that at each time instant exactly one transition is firing. The temporal data $\sigma^t(n)$ and the routing data $\nu^p(n)$ are to be determined by the supervisor by implementing a control algorithm, which takes into consideration the previous essential assumption.

Using these definitions the switching sequence is represented as

$$s(t_0, x_0) = \left\{ (i, \sigma^{t_i}(k)) \right\}_{i=1,\dots,N\ ,\ k=0,1,\dots}, \quad k_0 = t_0 \tag{30}$$

where the pair $(i, \sigma^{t_i}(k))$ means that at the k^{th} firing of the net, the transition t_i is forced to fire, and hence the continuous part evolves according to

$$\dot{x}(t) = f_i(x(t)), \text{ for } t_k \le t < t_{k+1} \tag{31}$$

$$\sigma^{t_i}(k) = t_{k+1} - t_k \tag{32}$$

The *event projection* and the *timed projection* of the swithcing sequence are defined as

$$\begin{aligned} \pi_1(s) &= i_0, i_1, i_2, \dots && (33) \\ \pi_2(s) &= \sigma^{t_{i_0}}(k_0), \sigma^{t_{i_1}}(k_1), \dots && (34) \end{aligned}$$

5.2 Supervisor

The supervisor has two main tasks. The first task is to determine a sequence of events that satisfy specifications imposed on the discrete-event part of the hybrid plant and the second, to compute the inter-event durations to satisfy specifications on the continuous part of the plant. In untimed Petri nets one can prohibit controlled transitions from firing, but cannot force the firing of a transition at a particular instant. In a timed Petri net controlled transitions are forced to fire, as this can be accomplished by considering the firing vectors to be functions of a global timer T.

Here, we consider the discrete event part of the hybrid plant and the supervisor to be a controlled timed Petri net. The supervisor has to solve the mode selection problem and determine the duration each mode will be active. We will show that for a special class of problems, we can first determine the routing policy and then require that the temporal data will not violate certain conditions imposed on the continuous dynamics. The supervisor is implemented by labeling the places, transition, input and output arcs of the Petri net with well formed formulas.

5.2.1 Supervisor Control of Petri Nets Based on Place Invariants

The first step is to satisfy the discrete specifications of the hybrid plant by applying DES control methods. We assume that the discrete specifications are described by linear inequalities on the marking vector of the Petri net. Specifications that can be transformed into linear constraints on the marking vector include sequential behaviors of defined by a marked language, linear inequalities containing firing vectors, logical propositions containing marking and firing vectors. A methodology for DES control based on Petri net place invariants has been proposed in (Moody and Antsaklis, 1996; Yamalidou *et al.*, 1996). A feedback controller based on place invariants is implemented by adding control places and arcs to existing transitions in the Petri net structure. Although the method was developed for ordinary Petri nets, the introduction of time delays associated to each transition will not affect the controlled behavior of the Petri net with respect to the discrete specifications.

The supervisor is used to enforce a set of linear constraints on the discrete state of the hybrid plant. These constraints can decribe a broad variety of problems including forbidden state problems, mutual exclusion problems, a class of logical predicates on plant behavior, conditions involving the concurrence of events, and the modeling of shared resources.

The system to be controlled is the untimed Petri net $\mathcal{N} = (P, T, I, O)$, which is called the plant net. We assume that the plant net has n places and m transitions and its incidence matrix is D_p. The controller net is a Petri net with incidence matrix D_c made up of the transitions of the plant net and a separate set of places. The controlled net is the Petri net with incidence matrix D made up of both the plant and the controller net. The control objective is to enforce the discrete state to satisfy constraints of the form

$$L\mu_p \leq b \tag{35}$$

where μ_p is the marking vector of the plant net, L is an $n_c \times n$ integer matrix, b is an $n_c \times 1$ integer vector, and n_c is the number of 1-dimensional constraints of the type $\sum_{i=1}^{n_p} l_i \mu_i \leq \beta$.

This inequality constraint can be tranformed to an equality by introducing an external Petri net controller that contains places representing nonnegative slack variables. Then

$$L\mu_p + \mu_c = b \tag{36}$$

where μ_c is an n_c integer vector which represents the marking of the controller places. The structure of the controller net will be computed by observing that the introduction of the slack variables forces a set of place invariants on the controlled system.

A place invariant is defined by an integer vector x that satisfies

$$x^T \mu = x^T \mu_0 \tag{37}$$

where μ_0 in the initial marking. The place invariants of a net are elements of the kernel of the net's incidence matrix, and they can be computed by finding integer solutions to

$$x^T D = 0 \tag{38}$$

where D is an $n \times m$ incidence matrix. The matrix D_c contains the arcs that connect the controller places to the transitions of the plant net. The incidence matrix D of the closed loop system is given by

$$D = \begin{bmatrix} D_p \\ D_c \end{bmatrix} \tag{39}$$

and the marking vector μ and the initial marking μ_0 are given by

$$\mu = \begin{bmatrix} \mu_p \\ \mu_c \end{bmatrix} \quad \mu_0 = \begin{bmatrix} \mu_{p0} \\ \mu_{c0} \end{bmatrix} \tag{40}$$

Note that equation (36) is in the form of (37), thus the invariants defined by equation (36) on the system (39),(40) must satisfy equation (38).

$$X^T D = [L, I] \begin{bmatrix} D_p \\ D_c \end{bmatrix} = 0 \tag{41}$$

$$L D_p + D_c = 0 \tag{42}$$

Proposition 1 *(Moody and Antsaklis, 1997) The Petri net controller with incidence matrix D_c and initial marking μ_{c0}, which enforces the constraints $L\mu_p \leq b$ when included in the closed loop system (39) with marking (40) is defined by*

$$D_c = -L D_p \tag{43}$$

with initial marking

$$\mu_{c0} = b - L\mu_{p0} \tag{44}$$

assuming that the transitions with arcs from D_c are controllable, observable, and that $\mu_{c0} \geq 0$.

5.2.2 Hybrid Strategy based on Equilibria

In the nonlinear control literature, switching has been used to expand the domain of attraction of control systems (McClamroch *et al.*, 1997; Guckenheimer, 1995). In the hybrid systems case, we assume that the continuous part admits a family of equilibria corresponding to different symbolic inputs generated by the discrete event part. Each equilibrium has a domain of attraction associated with it. The idea is to switch at discrete time instants from a symbolic input to another in a way that the system gradually progresses from one equilibrium to another towards the final equilibrium.

This idea can be formalized using an *invariant based approach* for hybrid systems proposed in (Stiver *et al.*, 1995; Stiver *et al.*, 1996*a*). A *common flow region* for a given target region, is defined as a set of states which can be driven to the target region with the same control policy. Common flow regions are bounded by invariant manifolds and an exit boundary, so that the state trajectory can leave the common flow region only through the exit boundary.

Definition 4 For the continuous part of the hybrid plant, the set B is a *common flow region* for a given region T if

$$\forall x(t_0) \in B,\ \exists t_1, t_2,\ t_0 < t_1 < t_2$$

such that

$$x(t) \in B,\ t \leq t_1$$

and

$$x(t) \in T,\ t_1 < t < t_2$$

subject to

$$\dot{x}(t) = f_i(x(t))$$

The following two propositions are presented in (Stiver *et al.*, 1995) and give sufficient conditions for the hypersurfaces bounding B and T to ensure that all state trajectories in B will reach T.

Proposition 2 *Given the following:*

1. *A flow generated by a smooth vector field, f*
2. *A target region, $T \subset X$*
3. *A set of smooth hypersurfaces, h_i, $i \in I_B \subset 2^I$*
4. *A smooth hypersurface (exit boundary), h_e*

such that $B = \{\xi \in X :\ h_i(\xi) < 0,\ h_e(\xi) > 0,\ \forall i \in I_B\} \neq \emptyset$. For all $\xi \in B$ there is a finite time, t, such that $x(0) = \xi$, $x(t) \in T$, if the following conditions are satisfied:

1. $\nabla_\xi h_i(\xi) \cdot f(\xi) = 0,\ \forall i \in I_B$
2. $\exists \epsilon > 0, \nabla_\xi h_e(\xi) \cdot f(\xi) < -\epsilon,\ \forall \xi \in B$
3. $B \cap \mathcal{N}(h_e) \subset T$

The following proposition uses in addition, a cap boundary (bounding hypersurface) in order to obtain a common flow region which is bounded.

Proposition 3 *Given the following:*

1. *A flow generated by a smooth vector field, f*
2. *A target region, $T \subset X$*
3. *A set of smooth hypersurfaces, $h_i,\ i \in I_B \subset 2^I$*
4. *A smooth hypersurface (exit boundary), h_e*
5. *A smooth hypersurface (cap boundary), h_c*

such that $B = \{\xi \in X :\ h_i(\xi) < 0,\ h_e(\xi) > 0,\ h_c(\xi) < 0,\ \forall i \in I_B\} \neq \emptyset$ and $\bar{B}$ (closure of B) is compact. For all $\xi \in B$ there is a finite time, t, such that $x(0) = \xi,\ x(t) \in T$, if the following conditions are satisfied:

1. $\nabla_\xi h_i(\xi) \cdot f(\xi) = 0,\ \forall i \in I_B$
2. $\nabla_\xi h_c(\xi) \cdot f(\xi) < 0,\ \forall \xi \in B \cap \mathcal{N}(h_c)$
3. $B \cap \mathcal{N}(h_e) \subset T$
4. *There are no limit sets in $\bar{B}$*

Remark The set of smooth hypersurfaces, $\{h_i,\ i \in I_B \subset 2^I\}$ is a set of smooth functionals $\{h_i : \Re^n \to \Re,\ i \in I_B\}$, defined on the state space of the plant. Each functional must satisfy the condition

$$\nabla_x h_i(\xi) \neq 0,\ \forall \xi \in \mathcal{N}(h_i) \tag{45}$$

which ensures that the null space of the functional $\mathcal{N}(h_i) = \{\xi \in \Re^n :\ h_i(\xi) = 0\}$ forms an $n-1$ dimensional manifold separating the state space.

Each of the two propositions above gives sufficient conditions for a set of hypersurfaces to form a common flow region. These hypersurfaces can be either invariant under the vector field of the given control policy or cap boundaries for the given vector field.

The set of all invariant hypersurfaces can be found in terms of $n-1$ functionally independent mappings which form the basis for the desired set of functionals, $\{h_i\}$. This basis is obtained by solving the characteristic equation

$$\frac{dx_1}{f_1(x)} = \frac{dx_2}{f_2(x)} = \ldots \frac{dx_n}{f_n(x)} \tag{46}$$

where $f_j(x)$ is the jth element of $f(x)$ ($f(x)$ is used rather than $f_i(x)$ to avoid subscript confusion).

Consider the hypersurfaces $\{h_i,\ i \in I_B\}$ which must be invariant under the vector field f of the given control policy. This can be achieved by choosing them to be integral manifolds of an $n-1$ dimensional distribution which is invariant under f. An $n-1$ dimensional distribution, $\Delta(x)$, is invariant under f if it satisfies

$$[f(x), \Delta(x)] \subset \Delta(x) \tag{47}$$

where $[f(x), \Delta(x)]$ indicates the Lie bracket. Of the invariant distributions, those that have integral manifolds as we require, are exactly those which are involutive (according to Frobenius). This means that

$$\delta_1(x), \delta_2(x) \in \Delta(x) \Rightarrow [\delta_1(x), \delta_2(x)] \in \Delta(x)$$

Therefore by identifying the involutive distributions which are invariant under the vector field f, we have identified a set of candidate hypersurfaces. For details about the relationships between vector fields and invariant distributions, see (Isidori, 1996).

Consider the hypersurface $h_c(x)$ that forms a cap boundary for the common flow region B. Assume that there exists an appropriate Lyapunov function $V(x)$ for the vector fields f such that

$$\begin{aligned} V(x) > 0, \ \forall x \in B \\ V(x) \to \infty \text{ as } \|x\| \to \infty \\ \dot{V}(x) < 0, \ \forall x \in B \end{aligned} \tag{48}$$

then $\Omega_c = \{x \in \Re^n | \ V(x) \leq c\}$ is bounded and the hypersurface $h_c(x) = V(x) - c$ is a cap boundary candidate. The constant parameter c can be selected appropriately so that $h_c(x)$ bounds the common flow region B.

The task to determine suitable invariant hypersurfaces is very difficult in general. However, appropriate cap boundaries can be determined efficiently using Lyapunov theory. Furthermore, the design based on Lyapunov functions will exhibit desirable robustness properties. The next proposition gives sufficient conditions for the state to progress from one equilibrium point to another.

Proposition 4 *Let $f_{i_1}, f_{i_2} \in \Sigma$ satisfying the following assumptions*

- *Each f_i is globally Lipschitz and admits an isolated equilibrium point $\bar{x}_i$, and $\bar{x}_i$ is asymptotically stable w.r.t. f_i.*
- *For each f_i there exists an appropriate Lyapunov function $V_i: \Re^n \to \Re$ and $\Omega_i = \{x \in \Re^n | \ V_i(x) \leq c_i\}$ such that*

$$\begin{aligned} V(x) > 0, \ \forall x \in \Omega_i \\ V(x) \to \infty \textit{ as } \|x\| \to \infty \\ \dot{V}(x) < 0, \ \forall x \in \Omega_i \end{aligned} \tag{49}$$

In addition, assume that $\Omega_{i_1} \cap \Omega_{i_2} \neq \emptyset$ and $\bar{x}_{i_1} \in T' = int(\Omega_{i_1} \cap \Omega_{i_2})$, then for every $x_0 \in \Omega_{i_1}$ there exists a switching sequence

$$s(x_0, t_0) = (i_1, \sigma^{t_{i_1}}(k_0)), (i_2, \sigma^{t_{i_2}}(k_1))$$

which drives the state to a region T of the equilibrium point $\bar{x}_{i_2}$.

Proof: Let $\Omega = \Omega_{i_1} \setminus \Omega_{i_2}$ and define the hypersurface $h_c(x) = \partial\Omega \cap \partial\Omega_{i_1}$ and $h_e(x) = \partial\Omega \cap \partial\Omega_{i_2}$. Since $\bar{x}_{i_1} \in T'$ is an asymptotically equilibrium point for f_{i_1}, from Proposition 3 , Ω is a common flow region for $T' = int(\Omega_{i_1} \cap \Omega_{i_2})$. Let $\Omega' = \Omega_{i_2} \setminus T$ and define the hypersurface $h_c(x) = \partial\Omega_{i_2}$ and $h_e(x) = \partial T$, then Proposition 3 holds and Ω' is a common flow region for the target region T.

Remark The conditions of Proposition 4 are stronger than the condition of Proposition 3 but they provide a systematic way, the search of Lyapunov function to check the existence of the cap boundary $h_c(x)$.

Corollary 1 *Suppose there exists a switching sequence with event projection $\pi_1(s) = i_0, i_1, \ldots, i_m$ accepted by the controlled Petri net such that every pair $(f_{i_j}, f_{i_{j+1}})$ satisfies Proposition 1 . Given a target region T such that $\bar{x}_{i_m} \in int(T)$, there exists routing policy to drive the continuous state from any initial condition $x_0 \in \Omega_{i_0}$ to the region T in finite time. The temporal data $\sigma^t(n)$ will be chosen so that the switchings occur while $x \in int(\Omega_{i_j} \cap \Omega_{i_{j+1}})$.*

Remark The condition that every pair $(f_{i_j}, f_{i_{j+1}})$ satisfies Proposition 4 can be relaxed by allowing intermediate transitions which will keep the continuous state in the domain of attraction of $\bar{x}_{i_{j+1}}$.

The supervisor will determine the routing policy $\nu^p(n)$ and the temporal data $\sigma^t(n)$ to drive the continuous state to a target region of one of the equilibria. The supervisor is implemented by assigning well-formed formulas to the places, transitions, input and output arcs of the controlled Petri net. Let $\{h_i\}$, $i = 1, \ldots, n$ be the set of hypersurfaces that bound a region M of the state space X. We can use the following well-formed formulas to describe that $x \in M$,

$$\ell = p_1 \wedge p_2 \wedge \ldots \wedge p_n \tag{50}$$

where p_i is a constraint of the form $h_i(x) < 0$. Consider a pair of vector fields $(f_{i_j}, f_{i_{j+1}})$ that satisfy Proposition 4 and let B_{i_j}, T_{i_j} and $B_{i_{j+1}}, T_{i_{j+1}}$ be the corresponding common flow and target regions. From Proposition 4 we have that the target region T_{i_j} coincides with the common flow region $B_{i_{j+1}}$. The switching algorithm is implemented by the following labeling functions, where $p \in P$ is the place to connect the output arc of t_{i_j} to the input arc of $t_{i_{j+1}}$

- $\ell_P(p)$ is chosen to be a tautology.
- $\ell_T(t_{i_j}), \ell_T(t_{i_{j+1}})$ are chosen to be the atomic rate formulas $\dot{x} = f_{i_j}(x)$ and $\dot{x} = f_{i_{j+1}}(x)$ respectively.
- $\ell_O((t_{i_j}, p))$ is chosen to be a wff of the form (50) representing that $x \in T_{i_j}$.
- $\ell_I((p, t_{i_{j+1}}))$ is chosen to be a wff of the form (50) representing that $x \in B_{i_{j+1}}$

Assuming that transition t_{i_j} is firing, the next transition to fire, $t_{i_{j+1}}$ is determined by the routing policy $\nu^p(n)$ so that the pair $(f_{i_j}, f_{i_{j+1}})$ satisfies Proposition 4. Transition $t_{i_{j+1}}$ will fire only when the temporal data $\sigma^t(n)$ assign true values to the logic formulas $\ell_O((t_{i_j}, p))$ and $\ell_I((p, t_{i_{j+1}}))$. For the initialization of the hybrid system we assume that $\ell_I((p, t_{i_0}))$ is a tautology.

Remark In the case when the Petri net is live and the event projection generated by the controlled Petri net $\pi_1(s)$ is an infinite sequence that satisfies Corollary 1, the hybrid system exhibits a periodic behavior in the sense that the continuous state is visiting periodically neighborhoods of the equilibria.

5.2.3 Affine Systems

Consider the case the continuous dynamics are described by

$$\dot{x} = Ax + c_i, \quad T_k \leq t < T_{k+1} \tag{51}$$

where $c_i \in W \subset \Re^n$ a finite set of control vectors and the matrix $A \in \Re^{n \times n}$ is Hurwitz.

Let $f_i(x) = Ax + c_i$, then $\bar{x}_i = -A^{-1}c_i$ is a globally asymptotically stable equilibrium point for $\dot{x} = f_i(x)$. In view of the global asymptotic stability of each equilibrium point, it is clear that Proposition 4 holds for every pair of control inputs.

5.3 First Order Integrators

Recently, attention has focused on a particular class of hybrid systems in which the continuous dynamics are governed by the differential equation $\dot{x}(t) = c$, where $c \in \Re^n$ (Alur *et al.*, 1993; Tittus, 1995; Stiver *et al.*, 1996*b*; Lennartson *et al.*, 1996). In (Tittus, 1995) hybrid systems with continuous dynamics described by first order integrators are used for control of batch processes. In the case when the continuous dynamics are described by first order integrators, the previous algorithm cannot be applied since the continuous part does not admit any isolated equilibria. In the following, we present an algorithm on a similar idea as in the case of multiple equilibria. The continuous part admits a family of invariant sets corresponding to different symbolic inputs generated by the discrete event part. The switching from a symbolic input to another occur at discrete time instants in a way that the system gradually progresses towards the target region. We determine a sequence of events that drive the state to the precribed target region by solving a linear programming problem as in (Lennartson *et al.*, 1996).

5.3.1 Continuous Part

The continuous state of the hybrid plant evolves in $X \in \Re^n$ and is described by first order integrators

$$\dot{x}(t) = c_i, \quad T_k \leq \tau_i \leq T_{k+1} \tag{52}$$

where $c_i \in W \subset \Re^n$ a finite set of control vectors and T denotes the global time.

Definition 5 A set $\mathcal{C}$ is said to be a *finitely generated cone* if it has the form

$$\mathcal{C} = \left\{ x : \; x = \sum_{j=1}^{r} \tau_j c_j, \; \tau_j \geq 0, \; c_j \in \Re^n, \; j = 1, \ldots, r \right\}$$

In the following, we assume that the finitely generated cone by the set W coincides with the continuous state space X. This assumption guarantees that continuous specifications such as state targeting can be satisfied.

Definition 5 A set $M \subset X$ is said to be *invariant* with respect to the system $\dot{x} = f(x)$ if $x(t_0) \in M \Rightarrow x(t) \in M, \ \forall t \in \Re$.

The system $\dot{x} = c$ admits a family of invariant sets described by the equation $x = ct + x_0$, which represents a family of parallel lines parametrized in t. The path from the initial state to the target region will be found by solving a linear programming problem to determine the time interval the state of the system evolves in each particular invariant set.

5.3.2 Supervisor

The procedure for the design of the supervisor is similar to that of the subsection 5.2. First, applying DES control methods we construct a controlled Petri net which satisfies the discrete specifications. Assume that the control objective is to drive the state to the targer region $\mathcal{T} \subset X$. A systematic way to determine the temporal and routing data is first to identify all the periodic behaviors the controlled Petri net can generate and then assign temporal constraints to the switching times. Several methods have been proposed to determine the periodic behavior of Petri nets (reachability tree, transition invariants, unfolding). For this problem, the size of the Petri net depends on the number of available symbolic inputs, hence we can efficiently identify all the periodic behaviors by computing the fundamental cycles of the reachability tree. The following proposition can be shown easily.

Proposition 5 *Consider the switching sequence* $s(t_0, x_0) = \{(i, \sigma^{t_i}(k))\}_{i=1,\ldots,N \ , \ k=0,1,\ldots,m}, \quad k_0 = t_0$ *such that*

- *The event projection* $\pi_1(s) = i_0, i_1, i_2, \ldots, i_m$ *consists of the transitions which form all the fundamental cycles.*
- *The timed projection* $\pi_2(s) = \sigma^{t_{i0}}(k_0), \sigma^{t_{i1}}(k_1), \ldots, \sigma^{t_{im}}(k_m)$ *satisfies the following conditions* $\sum_k \sigma^{t_i}(k) = \tau_i$ *and* $0 < \Delta \leq \sigma^t(n) < \infty$ *for all transitions and firings;* $\tau = [\tau_1, \ldots, \tau_N]^T$ *is the solution of the following linear programming problem*

$$min \ a^T \tau$$

$$subject \ to: \left\{ \begin{array}{l} x_f = x_0 + \sum_{i=1}^{N} \tau_i c_i \in \mathcal{T} \\ \tau_i \geq \Delta, \ i = 1, \ldots, N \end{array} \right.$$

where τ *is the vector of operation times to be determined,* a *is a weighting vector,* x_f *is the response of the continuous part at the time instant when the switching sequence* $s(t_0, x_0)$ *is completed.*

Then, the continuous state $x \in \Re$ *is driven to the target region in finite time* $t = \sum_{i=1}^{N} \tau_i$.

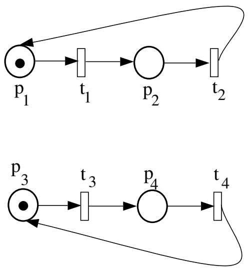

Figure 8: Petri net describing the switching policy of the hybrid plant.

In this case, the supervisor solves the above linear programming problem and labels the places, transitions, input and output arcs of the controlled Petri net as follows

- $\ell_P(p)$ is chosen to be a tautology.
- $\ell_T(t_i)$ is chosen to be the atomic rate formulas $\dot{x} = c_i$.
- $\ell_I((p, t_i))$ and $\ell_O((t_i, p))$ are chosen to be time constraints implement by a local timer so that $\sum_k \sigma^{t_i}(k) = \tau_i$.

5.4 Example: Hybrid System Describing Resource Contention

Consider the case of two different processes that use the same resource to carry out their operations. This is a conflict situation which stems from the resource contention. More specifically, assume that each process consists of two different operations which are described by ordinary differential equations and the switching policy is represented by the Petri net in figure 8.

The incidence matrix of the plant net is

$$D_p = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \tag{53}$$

and the initial condition of the marking vector $\mu_{p_0} = [1, 0, 1, 0]^T$. We consider the mutual exclusion constraint $L\mu_p \leq b$, where $L = [0, 1, 0, 1]$ and $b = 1$. Using Proposition 1 the closed loop system has the incidence matrix

$$D = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \tag{54}$$

and initial condition $\mu_0 = [1, 0, 1, 0, 1]$. The last row of the incidence matrix D represents the Petri net supervisor.

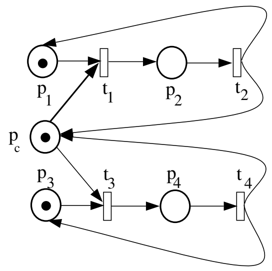

Figure 9: The controlled Petri net of the resource contention example

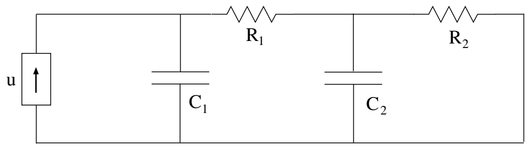

Figure 10: The temperature control system

The controlled Petri net is shown in figure 9 and describes the switching policy for the hybrid system. We will use this Petri net to describe the switching policy for two examples that follow. The first example is a temperature control system where the continuous dynamics are described by affine systems. In the second example, we consider a hybrid system with continuous dynamics described by first order integrators.

5.4.1 Temperature Control System

Let a typical temperature control system be described by the electrical circuit shown in Fig. 10. Here, an electrical analog of the temperature control system is used by considering the temperature being analogous to electric voltage, heat quantity to current, heat capacity to capacitance, and thermal resistance to electrical resistance. The control objective is to control the temperature at a point at the system by applying the heat input at a different point. The temperature control example is used in (Friedland, 1996) to illustrate PID control design. Here, we assume that only discrete levels are available for the current (heat) input.

Let x_1 and x_2 denote the voltage across the capacitor C_1 and C_2 respectively (representing the temperatures to be controlled), and consider that the numerical values of the electrical elements are

$$R_1 = R_2 = 1$$
$$C_1 = C_2 = 1$$

Then (using Kirchhoff's laws) the system is described by the state-space equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{55}$$

Assume that $q_i,\ i = 1, \ldots, N$ are the available discrete levels for the input that correspond to the

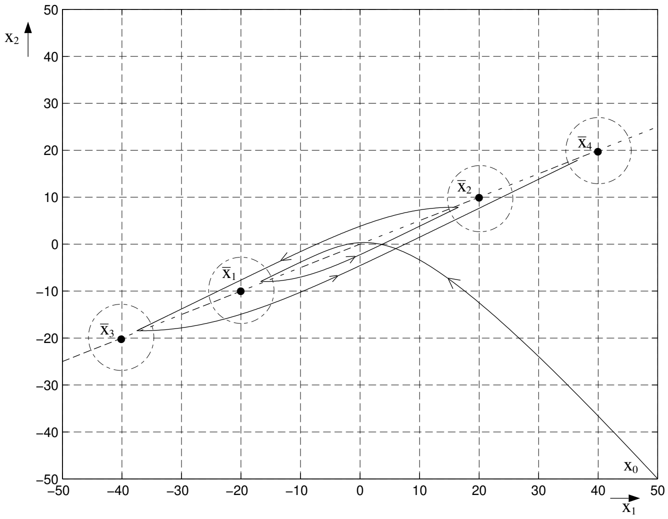

Figure 11: Periodic behavior of the hybrid system

available control policies. Then it can be shown $\bar{x}_i = -A^{-1}c_i$ where

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}, \quad c_i = \begin{bmatrix} q_i \\ 0 \end{bmatrix}$$

is a globally asymptotically stable equilibrium point for the system $\dot{x} = Ax + c_i$.

We assume that the discrete levels of the heat input are $q_1 = -10, q_2 = 10, q_3 = -20$, and $q_4 = 20$ and that the switching policy is described by the Petri net in figure 9 representing resource contention, where transition t_i corresponds to the control input c_i. In this case, the supervisor will determine the routing policy and the temporal data so that the continuous state of the hybrid system visits periodically neihborhoods of the equilibria (fig. 11).

5.4.2 First Order Integrators

We assume now that the continuous part of the hybrid plant consists of a set of first order integrators

$$\dot{x} = c_i \in \Re^2, \quad i = 1, 2, 3, 4. \tag{56}$$

$$C = [c_1, c_2, c_3, c_4] = \begin{bmatrix} 0.5 & -1 & 1 & -1 \\ 1 & 1 & 1 & -0.4 \end{bmatrix}$$

and we associate the differential equation $\dot{x} = c_i$ with the transition t_i of the Petri net in figure 8. The control objective is to drive the state from the initial condition $x_0 = [1, -1]^T$ to the convex region $\mathcal{T}$ of the state space where

$$\mathcal{T} = \left\{ x \in \Re^2 : \begin{bmatrix} 1 \\ 1 \end{bmatrix} \leq x \leq \begin{bmatrix} 1.1 \\ 1.1 \end{bmatrix} \right\} \tag{57}$$

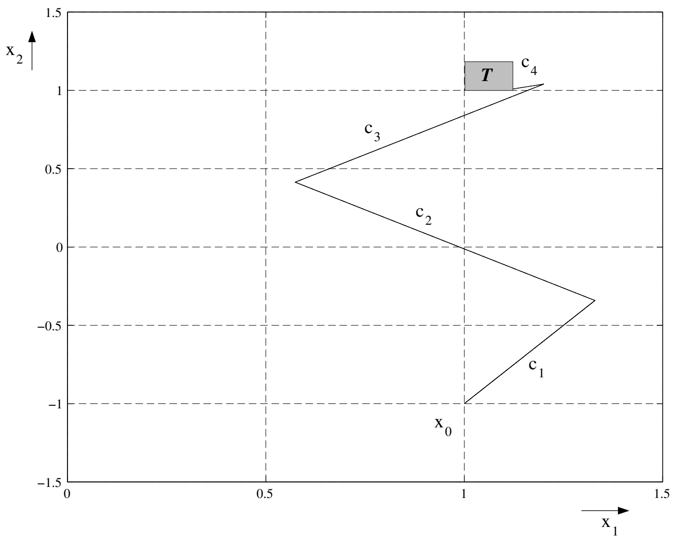

Figure 12: The trajectory of the continuous state.

According to Proposition 5 we formulate the following linear programming problem

$$min\ (\tau_1 + \tau_2 + \tau_3 + \tau_4)$$

$$subject\ to : \begin{cases} x_f = x_0 + \tau_1 c_1 + \tau_2 c_2 + \tau_3 c_3 + \tau_4 c_4 \in \mathcal{T} \\ \tau_i \geq \Delta \end{cases}$$

where $\Delta = 0.1$. The solution of the linear programming problem gives

$$\tau_1 = 0.6585, \quad \tau_2 = 0.7554; \quad \tau_3 = 0.6261, \quad \tau_4 = 0.1$$

and we can drive the state from x_0 to $\mathcal{T}$ with one firing of each transition by setting $\sigma^{t_i} = \tau_i$, $i = 1, 2, 3, 4$. The trajectory of the continuous state is shown in figure 12.

6 Conclusions

In this paper, uniform ultimate boundness and supervisory control of hybrid systems were addressed using a class of timed Petri nets named programmable timed Petri nets. New methodologies were introduced and efficient algorithms were derived to address these issues. Using Petri nets to model hybrid systems offers advantages over finite automata when concurrency and complexity issues are of concern. Sufficient conditions for uniform ultimate boundness and supervisory control were presented. For the case when the plant is a collection of linear systems with switching logic generated by a programmable timed Petri net, efficient algorithms for stability testing and supervisory control synthesis were developed.

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