A Hybrid Control Strategy based on the Equilibria of the Continuous Dynamics

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Abstract

This paper presents a model for hybrid systems based on timed Petri nets. The dynamics of the system depend on temporal and routing data associated respectively with the transitions and the places of the Petri net. The hybrid plant is controlled by a supervisor which determines the temporal and the routing policy to satisfy the specifications. The discrete specifications are expressed by linear constraints on the marking vector, while the control objective is to drive the continuous state to prescribed regions of the state space. A DES control method, namely supervisory control of Petri nets based on place invariants is applied to satisfy the discrete specifications. We assume that the continuous subsystems admit a family of equilibria and each equilibrium has a domain of attraction associated with it. The hybrid system switches from a subsystem to another in a way that the state gradually progresses from one equilibrium to another towards the final equilibrium. A control algorithm based on the notion of a common flow region is presented. It is shown that in the case when the continuous dynamics are described by affine systems, such a control algorithm can be designed easily. Finally, a temperature control application is described to illustrate the method.

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1 Introduction

Hybrid systems incorporate both continuous components, described by differential or difference equations, and digital components governed by digital control programs. The key features that characterize these systems are the mixture of continuous and discrete variables, and the coupling of the *time-driven* and *event-driven* dynamics of the continuous and discrete-event parts respectively. In this paper, the discrete-event part of the hybrid plant is modeled using timed Petri nets. For DES control, Petri nets modeling formalism offers advantages over finite automata, the exploitation of which seems also useful for hybrid systems control. The hybrid plant is controlled by a supervisor which determines a timed sequence of events to be followed by the discrete event part of the system. The explicit introduction of time in the discrete event part of the hybrid plant and in the supervisor is necessary because of the coupling of time-driven and event-driven dynamics. Petri nets are known to be effective for modeling both *untimed* and *timed* DESs and can capture the discrete behavior of the hybrid system and relate it to the evolution of time. Petri nets have been used before for modeling of hybrid systems, for example in [1, 2]. The proposed model has been used for the case when the continuous dynamics are described by first order integrators in [3].

The paper is organized as follows. In Section 2, the general model describing the hybrid plant is presented. In Section 3 the supervisor and its tasks are presented. Finally, Section 4 presents the special case when the continuous dynamics are described by affine systems and outlines the design procedure for the supervisor, illustrated by an example describing a temperature contol system.

2 Hybrid Plant

The hybrid plant consists of the continuous part which describes the dynamics of the system at each mode, the discrete event part that models the switching rules between the physical modes of the system, and the interface consisting of an actuator and a generator that governs the interaction between the two parts.

2.1 Continuous Part

The continuous state of the hybrid plant evolves in $X \in \Re^n$ and is described by

$$\dot{x}(t) = f_i(x(t)), \quad T_k \leq t \leq T_{k+1}, \quad i = 1, \ldots, N$$

where T denotes the global time, and $f_i \in \Sigma$ a finite set of globallly Lipschitz vector fields.

2.2 Discrete-Event Part

An *ordinary Petri net structure* [4, 5, 6] is a triple, $N = (P, T, \mathcal{E})$, where P is a finite set of places, T is a finite set of transitions, and $\mathcal{E} \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs connecting places to transitions and vice versa. A *marking* is a map μ from the set of places onto the nonnegative integers and is represented by a number of tokens at each place.The *preset* and *postset* of a place p or a transition t are denoted by $\bullet p$, $p\bullet$ and $\bullet t$, $t\bullet$ respectively.

A *labeled Petri net* is a 5-tuple $G = (N, \Sigma, \ell, \mu_0, F)$ where $N = (P, T, \mathcal{E})$ is an ordinary Petri net, Σ is a finite set of events, $\ell : T \to \Sigma$ is a labeling function that assigns an event to each transition, μ_0 is an initial marking, and F is a finite set of final markings. The *L-type language* (also called *marked behavior*) represents all evolutions of the labeled net that reach a terminal state or states.

The introduction of time to Petri nets can be done by assigning time either to transitions or to places, resulting to *timed transition Petri nets* (TTPNs) or to *timed place Petri nets* (TPPNs). Both TPPNs and TTPNs have been studied in [7] to model manufacturing processes. In [8] a timed Petri net, called *programmable timed Petri net*, has been proposed as a modeling formalism for hybrid systems with clocks. In this preliminary work we consider TTPNs, and we attach a deterministic time delay to each transition which can be controlled. Because of the time constraints in the operation of the system, lower and upper bounds may be applied to this time delay. It is the supervisor's task to compute a valid time delay for each transition so that the continuous specifications of the hybrid plant will be satisfied. The computation of the time delays will be performed by solving an optimization problem.

The temporal information is described by the data $\sigma^t(n)$, $n \in N$, where $\sigma^t(k)$ represents the duration of the k^{th} firing of transition t. During the time interval $\sigma^t(k)$, the tokens of the input places of transition t do not change, these tokens are put into the output places of t upon the completion of firing of the transition, according to the firing rule of the untimed Petri net.

To model a hybrid plant we think of the transitions as jobs described by differential equations. Let $\sigma^{t_i}(k)$ denote the operation time associated with the k^{th} firing of the transition t_i. Note that during this time interval the continuous state of the hybrid plant evolves according to the differential equation $\dot{x} = f_i(x)$. This assignment is defined by the labeling function $\ell : T \to \Sigma$ which associates with each transition a vector field f_i and it may assign the same vector field to different transitions of the Petri net.

We assume that at each time instant exactly one transition is firing. In addition, we assume that $0 < \Delta \leq \sigma^t(n) < \infty$ for all firings n and transitions t. We may easily incorporate in our model instantaneous transitions, but these will correspond to jumps of the continuous state and will not be considered here. The assumption $\sigma^t(n) \geq \Delta > 0$ eliminates the possibility of infinitely many switchings in a finite time interval.

The control algorithm for the mode selection problem will be based on the routing data. Routing policy for timed Petri nets is used usually for resolution of conflicts and was introduced in [9]. In our case, routing data $\nu^p(n)$ are attached to each place, where $\nu^p(k)$ gives the transition $t \in p\bullet$ to which the k^{th} token to enter place p is to be routed. If the k^{th} token is routed to $t \in p\bullet$, then the transition t wins the token, which after a firing time of $\sigma^t(k)$ is routed to $t\bullet$, the output places of the transition. Since each transition of the timed Petri net corresponds to a vector field, we require that at each time instant exactly one transition is firing. The temporal data $\sigma^t(n)$ and the routing data $\nu^p(n)$ are to be determined by the supervisor by implementing a control algorithm, which takes into consideration the previous essential assumption.

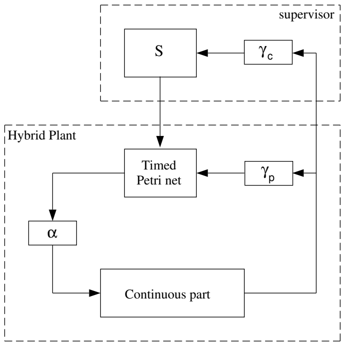

Figure 1: The hybrid plant and the supervisor

The markings and constraints on the markings of the Petri net represent the available resources and the logical constraints that have to be satisfied during the operation of the hybrid system. In the context of the hybrid plant we think of the labeled TTPN as an input-output system. The inputs come from the supervisor representing the time instants for mode changes and from the continuous part through the generator representing uncontrollable plant events. The output of the Petri net is applied through the actuator to the continuous part affecting the evolution of the continuous state. The state (marking) of the Petri net do not affect directly the continuous part of the system; the evolution in time of the discrete state represents the structural properties the hybrid system must possess during its operation.

2.3 Interface

In [10] an interface was used to provide means for communication between the continuous plant and the DES controller. In this approach a similar interface is necessary in the hybrid plant to convert continuous time signals to events and vice versa. The interface consists of two subsystems, the generator γ_p and the actuator α. The generator γ_p issues plant events depending on the continuous state. In particular, a set of hypersurfaces is specified and whenever the plant state crosses a hypersurface a plant event is issued. From the view of the discrete event part, the plant events will be considered as uncontrollable events.

A generator γ_c will be used also to provide means of communication between the continuous part of the hybrid plant and the supervisor. The generators γ_p and γ_c are in general different. For example, consider the situation of high temperature in a chemical process. The lack of the plant resources to keep the temperature at a high level may be modeled by an uncontrollable plant event. If the system is not allowed to switch to a control vector that will further increase the temperature, the supervisor must be also informed, but the event will not necessarily be the same as the above plant event.

Finally, the actuator α converts the timed sequence of events generated by the supervisor to a piecewise constant function of time applied to the continuous part. We assume that the actuator and the generators respond instantaneously to their inputs.

3 Supervisor

The supervisor has two main tasks. The first task is to determine a sequence of events that satisfy specifications imposed on the discrete-event part of the hybrid plant and the second, to compute the inter-event durations to satisfy specifications on the continuous part of the plant. In untimed Petri nets one can prohibit controlled transitions from firing, but cannot force the firing of a transition at a particular instant. In a timed Petri net controlled transitions are forced to fire, as this can be accomplished by considering the firing vectors to be functions of a global timer T.

Here, we consider the discrete event part of the hybrid plant and the supervisor to be a controlled timed Petri net. The supervisor has to solve the mode selection problem and determine the duration each mode will be active. We will show that for a special class of problems, we can determine the routing policy by requiring that the temporal data will not violate certain conditions imposed on the continuous dynamics.

3.1 Supervisory Control of Discrete Event Systems

The first step is to satisfy the discrete specifications of the hybrid plant by applying DES control methods. We assume that the discrete specifications are described by linear inequalities of the form $L\mu \leq b$ where μ is the marking vector of the Petri net, L is an $n_c \times n_p$ integer matrix, b is an $n_c \times 1$ integer vector , $n_p = |P|$ is the number of places, and n_c is the number of 1-dimensional constraints of the type $\sum_{i=1}^{n_p} l_i \mu_i \leq \beta$. Specifications that can be transformed into linear constraints on the marking vector include sequential behaviors of defined by a marked language, linear inequalities containing firing vectors, logical propositions containing marking and firing vectors [11, 12].

A methodology for DES control based on Petri net place invariants has been proposed in [13, 14]. A feedback controller based on place invariants is implemented by adding control places and arcs to existing transitions in the Petri net structure. Although the method was developed for ordinary Petri nets, the introduction of time delays associated to each transition will not affect the controlled behavior of the Petri net with respect to the discrete specifications.

3.2 Hybrid Strategy based on Equilibria

In the nonlinear control literature, switching has been used to expand the domain of attraction of a control system [15, 16]. In the hybrid systems case, we assume that the continuous part admits a family of equilibria corresponding to different symbolic inputs generated by the discrete event part. Each equilibrium has a domain of attraction associated with it. The idea is to switch at discrete time instants from a symbolic input to another in a way that the system gradually progresses from one equilibrium to another towards the final equilibrium.

This idea can be formalized using an *invariant based approach* for hybrid systems proposed in [17, 18]. A *common flow region* for given target region, is defined as a set of states which can be driven to the target region with the same control policy. Common flow regions are bounded by invariant manifolds and an exit boundary, so that the state trajectory can leave the common flow region only through the exit boundary.

Definition 1 For the continuous part of the hybrid plant, the set B is a common flow region for a given region T if

$$\forall x(t_0) \in B,\ \exists t_1, t_2,\ t_0 < t_1 < t_2$$

such that

$$x(t) \in B,\ t \le t_1$$

and

$$x(t) \in T,\ t_1 < t < t_2$$

subject to

$$\dot{x}(t) = f_i(x(t))$$

The following two propositions are presented in [17] and give sufficient conditions for the hypersurfaces bounding B and T to ensure that all state trajectories in B will reach T.

Proposition 1 *Given the following:*

1. *A flow generated by a smooth vector field,* f
2. *A target region,* $T \subset X$
3. *A set of smooth hypersurfaces,* $h_i,\ i \in I_B \subset 2^I$
4. *A smooth hypersurface (exit boundary),* h_e

such that $B = \{\xi \in X :\ h_i(\xi) < 0,\ h_e(\xi) > 0,\ \forall i \in I_B\} \neq \emptyset$. *For all* $\xi \in B$ *there is a finite time,* t, *such that* $x(0) = \xi,\ x(t) \in T$, *if the following conditions are satisfied:*

1. $\nabla_\xi h_i(\xi) \cdot f(\xi) = 0,\ \forall i \in I_B$
2. $\exists \epsilon > 0, \nabla_\xi h_e(\xi) \cdot f(\xi) < -\epsilon,\ \forall \xi \in B$
3. $B \cap \mathcal{N}(h_e) \subset T$

The following proposition uses in addition, a cap boundary in order to obtain a common flow region which is bounded.

Proposition 2 *Given the following:*

1. *A flow generated by a smooth vector field,* f
2. *A target region,* $T \subset X$

3. *A set of smooth hypersurfaces,* $h_i,\ i \in I_B \subset 2^I$
4. *A smooth hypersurface (exit boundary),* h_e
5. *A smooth hypersurface (cap boundary),* h_c

such that $B = \{\xi \in X :\ h_i(\xi) < 0,\ h_e(\xi) > 0,\ h_c(\xi) < 0,\ \forall i \in I_B\} \neq \emptyset$ *and* $\bar{B}$ *(closure of* B*) is compact. For all* $\xi \in B$ *there is a finite time,* t*, such that* $x(0) = \xi,\ x(t) \in T$*, if the following conditions are satisfied:*

1. $\nabla_\xi h_i(\xi) \cdot f(\xi) = 0,\ \forall i \in I_B$
2. $\nabla_\xi h_c(\xi) \cdot f(\xi) < 0,\ \forall \xi \in B \cap \mathcal{N}(h_c)$
3. $B \cap \mathcal{N}(h_e) \subset T$
4. *There are no limit sets in* $\bar{B}$

Each of the two proposition gives sufficient conditions for a set of hypersurfaces to form a common flow region. We will show that for a special class of problems, under certain assumptions, common flow regions can be guaranteed to exist, and furthermore the set of invariant hypersurfaces can be determined based on approximation of the domain of attraction of each equilibrium point.

The switching sequence can be represented as

$$s(t_0, x_0) = \left\{(i, \sigma^{t_i}(k))\right\}_{i=1,\ldots,N\ ,\ k=0,1,\ldots},\quad k_0 = t_0$$

where the pair $(i, \sigma^{t_i}(k))$ means that at the k^{th} firing of the net, the transition t_i is forced to fire, and hence the continuous part evolves according to

$$\dot{x}(t) = f_i(x(t)),\ \text{for } t_k \leq t < t_{k+1}$$

$$\sigma^{t_i}(k) = t_{k+1} - t_k$$

We also define the *event projection* and the *timed projection* of the swithcing sequence as

$$\begin{aligned} \pi_1(s) &= i_0, i_1, i_2, \ldots \\ \pi_2(s) &= \sigma^{t_{i_0}}(k_0), \sigma^{t_{i_1}}(k_1), \ldots \end{aligned}$$

Corollary 1 *Let* $f_{i_1}, f_{i_2} \in \Sigma$ *satisfying the following assumptions*

- *Each* f_i *is globally Lipschitz and admits an isolated equilibrium point* $\bar{x}_i$*, and further,* $\bar{x}_i$ *is asymptotically stable w.r.t.* f_i*.*
- *For each* f_i *there exists an appropriate Lyapunov function* $V_i :\ \Re^n \to \Re$ *and*

$$\Omega_i = \{x \in \Re^n |\ V_i(x) \leq c\}$$

denotes the domain of attraction of the equilibrium point $\bar{x}_i$ *w.r.t.* f_i*.*

In addition, assume that $\Omega_{i_1} \cap \Omega_{i_2} \neq \emptyset$ and $\bar{x}_{i_1} \in T' = int(\Omega_{i_1} \cap \Omega_{i_2})$, then for every $x_0 \in \Omega_{i_1}$ there exists a switching sequence

$$s(x_0, t_0) = (i_1, \sigma^{t_{i_1}}(k_0)), (i_2, \sigma^{t_{i_2}}(k_1))$$

which drives the state to a region T of the equilibrium point $\bar{x}_{i_2}$.

Proof: First, assume that $\Omega_{i_1}, \Omega_{i_2}$ are bounded. Let $\Omega = \Omega_{i_1} \setminus \Omega_{i_2}$ and define the hypersurfaces $h_i(x) = \partial\Omega \cap \partial\Omega_{i_1}$ and $h_e(x) = \partial\Omega \cap \partial\Omega_{i_2}$. Since $\bar{x}_{i_1} \in T'$ is an asymptotically equilibrium point for f_{i_1}, from Proposition 1, Ω is a common flow region for $T' = int(\Omega_{i_1} \cap \Omega_{i_2})$. Let $\Omega' = \Omega_{i_2} \setminus T$ and define the hypersurfaces $h_i(x) = \partial\Omega_{i_2}$ and $h_e(x) = \partial T$, then Proposition 1 holds and Ω' is a common flow region for the target region T. For the case, Ω_i is unbounded, a cap boundary can be chosen appropriety to bound the common flow region and Proposition 2 is used.

The supervisor will determine the routing policy $\nu^p(n)$ and the temporal data $\sigma^t(n)$ to drive the continuous state to a target region of one of the equilibria.

Corollary 2 *Suppose there exists a switching sequence with timed projection $\pi_1(s) = i_0, i_1, \ldots, i_m$ such that every pair $(f_{i_j}, f_{i_{j+1}})$ satisfies Corollary 1 and further, $\pi_1(s) \in \mathcal{L}_m$, the marked behavior of the controlled Petri net. Given a target region T such that $\bar{x}_{i_m} \in int(T)$, there exists routing policy to drive the continuous state from any initial condition $x_0 \in \Omega_{i_0}$ to the region T in finite time. The temporal data $\sigma^t(n)$ will be chosen so that the switchings occur while $x \in int(\Omega_{i_j} \cap \Omega_{i_{j+1}})$.*

Remarks (1) When the timed projection $\pi_1(s)$ is an infinite sequence that can be generated by the controlled Petri net, the hybrid system exhibits a periodic behavior in the sense that the continuous state is visiting periodically neighborhoods of the equilibria. (2) In certain cases, the condition that every pair $(f_{i_j}, f_{i_{j+1}})$ satisfies Corollary 1 can be relaxed by allowing intermediate transitions which will keep the continuous state in the domain of attraction of $\bar{x}_{i_{j+1}}$. (3) A current research goal is the formulation of an efficient optimization algorithm to determine the temporal data $\sigma^t(n)$.

4 Affine Systems

Consider the case the continuous dynamics are described by

$$\dot{x} = Ax + c_i, \quad T_k \leq t < T_{k+1}$$

where $c_i \in W \subset \Re^n$ a finite set of control vectors and the matrix $A \in \Re^{n \times n}$ is Hurwitz.

Let $f_i(x) = Ax + c_i$, then $\bar{x}_i = -A^{-1}c_i$ is a globally asymptotically stable equilibrium point for $\dot{x} = f_i(x)$. In view of the global asymptotic stability of each equilibrium point, it is clear that Corollary 1 holds for every pair of control inputs.

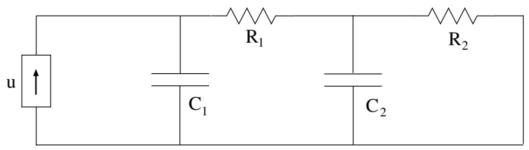

Figure 2: The temperature control system

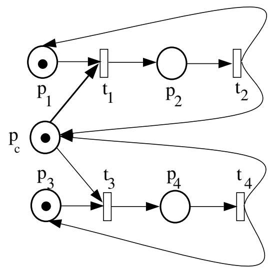

Figure 3: Discrete event part of the temperature control system

4.1 Temperature Control Systems [19]

Let a typical temperature control system be described by the electrical circuit shown in Fig. 2. The control objective is by applying the heat at one point in the system to control the temperature at a different point. Usually, an electrical analog of the temperature control system is used by considering the temperature analogous to electric voltage, heat quantity to current, heat capacity to capacitance, and thermal resistance to electrical resistance. The temperature control example is used in [19] to illustrate PID control design. Here, we assume that only discrete levels are available for the current (heat) input.

Let x_1 and x_2 denote the voltage across the capacitor C_1 and C_2 respectively, and consider that the numerical values of the electrical elements are

$$\begin{array}{c} R_1 = R_2 = 1 \\ C_1 = C_2 = 1 \end{array}$$

Then (using Kirchhoff's laws) the system is described by the state-space equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

Assume that $q_i,\ i = 1, \ldots, N$ are the available discrete levels for the input that correspond to the available control policies (outputs of the actuator α). Then it can be shown $\bar{x}_i = -A^{-1}c_i$ where

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}, \quad c_i = \begin{bmatrix} q_i \\ 0 \end{bmatrix}$$

is a globally asymptotically stable equilibrium point for the system $\dot{x} = Ax + c_i$.

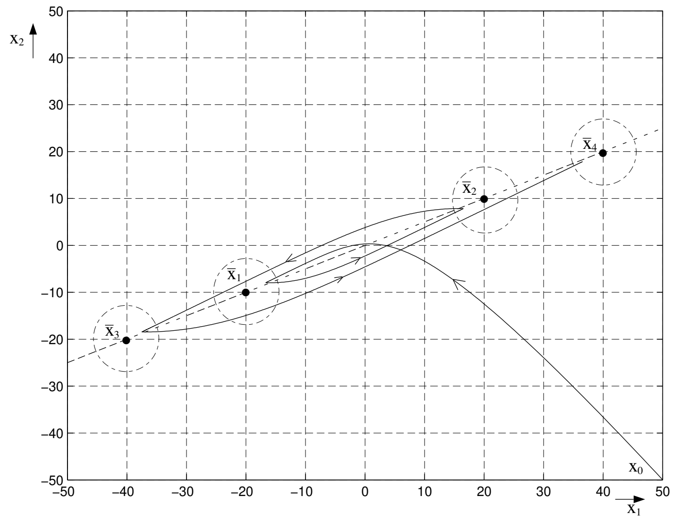

Figure 4: Periodic behavior of the hybrid system

We assume that the discrete levels of the heat input are $q_1 = -10, q_2 = 10, q_3 = -20$, and $q_4 = 20$. In addition, we suppose that there is a conflict because of resource contention and by applying the DES control methods we obtain the controlled Petri net shown in Fig. 3 where transition t_i corresponds to the control input c_i. In this case, the supervisor will determine the routing policy and the temporal data so that the continuous state of the hybrid system visits periodically neihborhoods of the equilibria (Fig. 4).

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