Lyapunov Stability of Continuous-Valued Systems Under the Supervision of Discrete-Event Transition Systems

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1 Introduction

This paper examines the Lyapunov stability of equilibrium points for switched control systems [Morse95]. A switched control system is a continuous-valued system whose control law is switched in a discontinuous manner as the system state evolves over a continuous-valued subset of $\Re^n$. Of particular interest in this paper are switched systems in which the switching logic is generated by a discrete-event transition system that can be represented as either a finite automaton or bounded Petri net.

There are a variety of prior results identifying sufficient conditions for such systems to be Lyapunov stability. In [Pel91] and [Sav96] a single positive definite functional is found which is Lyapunov for all control systems in the collection. Multiple Lyapunov function approaches in [Bran94] and [Hou96] have been presented which should be applicable to a larger set of systems than the single Lyapunov function methods. In certain cases, where the switched systems are linear time invariant and the switching regions are defined by conic sectors, it has been suggested that candidate Lyapunov functionals can be numerically computed by finding feasible points of a linear matrix inequality [Pet96] [Rant97].

While these prior results have provided great insight into the Lyapunov stability of switched systems, they do not account for the actual switching laws used by the system. In the case of the computational methods proposed in [Pet96] and [Rant97] this leads to LMI's which are extremely large and hence provide an overly restrictive sufficient condition for switched system stability. This paper examines the numerical question and asks what sort of information about the switching law can be used to significantly reduce the computational complexity and conservatism associated with finding candidate Lyapunov functions of switched systems. The principal result of this paper states that if the switching law can be represented as a discrete-event transition system such as a finite automaton or Petri net, then it suffices to examine the fundamental cycles of the directed graph associated with such structures to assess switched system stability. In particular, the results and viewpoints suggested in this paper provide a way in which the traditional control theoretic methods cited above can be combined with results from computer science [Alur94] [Alur96] concerned with the behaviour of timed transition systems.

The remainder of the paper is organized as follows. In section 2, we first introduce a formal model for switched control systems which are supervised by a discrete-event transition system. Section 3 states recent results [Bran94] [Pet96] providing sufficient conditions for switched system stability using a multiple Lyapunov function approach. Section 4 motivates, states, and proves the paper's principal result. Section 5 presents an example illustrating the value of using fundamental cycles in assessing switched system stability. Section 6 concludes with topics and directions for further study.

2 Problem Statement

Let $X \subset \Re^n$ be a smooth n-dimensional manifold and let I be a finite set of N integers. Let Δ be a constant dimensional distribution,

$$\Delta = \{f_1, \ldots, f_N\} \tag{1}$$

where $f_i : X \to X$ for $i = 1, \ldots, N$ are locally Lipschitz vectorfields over X. We consider **switched dynamical system** to be described by the following set of

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equations.

$$\dot{x}(t) = f_{i(t)}(x(t)) \quad (2)$$
$$i(t) = q(x(t), i(t^-)) \quad (3)$$

where $x : \Re \to X$, $i : \Re \to I$, and $q : X \times I \to I$. $i(t^-)$ refers to the lefthand limit of the function $i(t)$ at point t. In the sequel, we refer to each f_i as a **subsystem** of the switched system. The preceding model is essentially that used in [Tav87].

A **trajectory** of the switched system is the ordered pair, (x, i), where $x : \Re \to X$ and $i : \Re \to I$ which **solves** the system equation. The value taken by the trajectory at time $t \in \Re$ is denoted by $(x(t), i(t))$. We say that (x, i) **solves** the system equation if and only if the equations are satisfied by $x(t)$ and $i(t)$ for all $t \in \Re$. This paper does not treat questions concerned with the existence of solutions. In general, however, solutions (when they do exist) will not be unique due to the nondeterminism in the switching law.

Let (x, i) be the trajectory generated by a switched dynamical system. The set of **switching times**, Ω, of a trajectory (x, i) will be

$$\Omega = \left\{ t : \lim_{\tau \to t^+} i(\tau) \neq \lim_{\tau \to t^-} i(\tau) \right\} \quad (4)$$

The set of **switching events**, $\mathcal{E}$, of trajectory (x, i) is denoted as

$$\mathcal{E} = \left\{ (i, t) \in I \times \Re : t \in \Omega, i = \lim_{\tau \to t^+} i(\tau) \right\} \quad (5)$$

We define the **timed projection** $P_t : \mathcal{E} \to \Re$ by the equation $P_t[(i, \tau)] = \tau$ and the **event projection**, $P_e : \mathcal{E} \to I$ by the equation $P_e[(j, \tau)] = j$.

The **switching sequence**, is a mapping $\lambda : Z \to \mathcal{E}$ such that

$$P_t[\lambda(n)] < P_t[\lambda(n+1)] \quad (6)$$

for all $n \in Z$. Suppose λ is a switching sequence. Let I^* be the set of all strings formed from I. We let $\lambda_e = P_e[\lambda] \in I^*$ and $\lambda_t = P_t[\lambda]$ denote the event and time projections of λ, respectively. Let the subsequence of times when system j is turned on and off be denoted as $\lambda_{t,j} \in I^*$. In other words,

$$\lambda_{t,j} = \lambda_t(n_1), \lambda_t(n_1+1), \cdots \lambda_t(n_k), \lambda_t(n_k+1), \cdots \quad (7)$$

where n_k is a subsequence of Z such that $P_e[\lambda(n_k)] = j$. Define the interval completion $I(\lambda_{t,j})$ as the set obtained by taking the union of all open intervals in which system j is active. In other words,

$$I(\lambda_{t,j}) = \bigcup_{k=1}^{\infty} (\lambda_t(n_k), \lambda_t(n_k+1)) \quad (8)$$

Denote $E(\lambda_{t,j})$ as a subsequence of $\lambda_{t,j}$ when the subsystem j is turned on. In other words,

$$E(\lambda_{t,j}) = \lambda_t(n_1), \lambda_t(n_2), \cdots, \lambda_t(n_k), \cdots \quad (9)$$

The preceding model of a switched system assumes a very general switching function, q. To obtain more precise results, however, we need to specify the nature of the switching function. A common choice is to associate a **discrete-event** transition system such as a finite automaton or Petri net with the switching system. In this paper we limit our scope to finite automata. An automaton is tied to the switched system by associating the vertices to the switched system's subsystems and by associating the arcs with *switching sets* called *guards*. The timed automaton [Alur94] and hybrid automaton [Alur96] provide tangible examples of this approach. In this paper we begin by considering a discrete-event transition system that is represented by a finite automaton, (V, A).

A **finite automaton** associated with the switched system is the directed graph (V, A) where $V = I$ is a set of vertices and $A \subset V \times V$ is a set of directed arcs. By definition, the automaton associates a subsystem f_i with each vertex of the (V, A). We define the **guard**, Ω_{ij} of arc $(i, j) \in A$ as

$$\Omega_{ij} = \{x \in X : j = q(x, i)\} \quad (10)$$

The ordered pair (i, j) is an arc of A if and only if $\Omega_{ij} \neq \emptyset$. The guard therefore represents a subset of the switched system's state space in which a switch can occur. The guard set Ω_{ii} will sometimes be denoted as Ω_i and represents the set in which subsystem f_i remains active.

The preceding paragraph characterized the switching logic by a finite automaton (V, A). It is straightforward to generalize this approach to consider more complex switching logics. In particular, let's consider how this might be done for a switching logic generated by a Petri net. A Petri net is represented by a directed graph (V, A) where the vertex set consists of two types of vertices, **places**, P, and **transitions**, T. The vertex set, therefore, takes the form $P \times T = V$. We associate this directed graph structure with the switched system by letting $P = I$. We therefore associate a subsystem with each place of the Petri net. The guards, Ω_{ij}, are associated with the transition $t \in T$ which connect the ith and jth places of the network. Petri nets provide natural structures for modeling concurrency and synchronization in parallel systems. In general, a Petri net can provide a more expressive characterization of a system's switching logic than can be provided by a finite automaton.

Let (x, i) be the trajectory generated by a switched dynamical system. The trajectory is said to be **deadlock**

free if the event projection of the switching sequence $P_e[\lambda]$ is not finite. We say that the trajectory is **live** if the event projection of the switching sequence $P_e[\lambda]$ contains an infinite number of each index, $i \in I$. In other words any subsystem can be switched an infinite number of times in a switching sequence. We say that the trajectory is **nonZeno** if the timed projection of the switching sequence $P_t[\lambda]$ satisfies

$$\sum_{n=1}^{\infty} P_t[\lambda(n)] > \infty \tag{11}$$

We say that the switched system is live, deadlock free, or nonZeno if all of its trajectories are live, deadlock free, or nonZeno, respectively.

An important issue which is not addressed in this paper concerns necessary and sufficient conditions for a switched system to be live, deadlock free, or nonZeno. In this paper, we assume that the switched system is live and nonZeno.

3 Prior Results

This section briefly discusses prior results on switched system stablity. Let (x, i) be any trajectory generated by the switched dynamical system. Assume that $f_i(0) = 0$ for all $f_i \in \Delta$. The equilibrium point $x = 0$ is said to be **stable in the sense of Lyapunov** if and only if for all $\epsilon > 0$ there exists $\delta > 0$ such that $\|x(t_0)\| < \delta$ implies $\|x(t)\| < \epsilon$ for all $t \geq t_0$.

In the following we will denote the open ball of radius r centered at the origin as

$$B(r) = \{x \in \Re^n \ : \ \|x\| < r\} \tag{12}$$

The sphere, $S(r)$, of radius r centered at the origin is the set

$$S(r) = \{x \in \Re^n \ : \ \|x\| = r\} \tag{13}$$

Let λ be a switching sequence for a switched dynamical system where λ_t is its time projection. we say that a continuously differentiable function $V : \Re^n \to \Re^+$ is **Lyapunov-like function** over sequence λ_t if and only if $\dot{V}(x(t)) \leq 0$ for all $t \in \mathcal{I}(\lambda_t)$ and V is monotonically nonincreasing on $E(\lambda_t)$. Using this definition of a Lyapunov like function, the following sufficient condition for Lyapunov stability was proven in [Bran94]. The proof uses standard techniques employed in proving Lyapunov stability for nonautonomous systems. A significant generalization of this result will be found in [Hou96].

Theorem 1 *Suppose we have candidate Lyapunov functions V_j ($j \in I$) and suppose that the switched system is nonZeno and satisfies $f_i(0) = 0$ for all $j \in I$. If V_j is a Lyapunov like function for switching sequence $\lambda_{t,j}$ for all $j \in I$. then the equilibrium point $x = 0$ of the switched system is stable in the sense of Lyapunov.*

The preceding theorem provides a sufficient condition for Lyapunov stability of switched systems. The condition requires that a set of Lyapunov like functions be determined for all possible switching sequences λ that can be generated by the system. The determination of Lyapunov like functions may not be possible in general. For switched systems in which each subsystem is a linear time invariant system and the guard sets are represented by conic sectors in $\Re^n$, a method for determining the Lyapunov like functions was presented in [Pet96] and [Rant97]. Assume that each subsystem can be written as

$$\dot{x}(t) = A_i x(t) \tag{14}$$

where $A_i \in \Re^{n \times n}$ and $i \in I$. Assume that the guard sets can be bounded by conic sectors parameterized by symmetric matrices Q_{ij}. In other words, consider sets,

$$\Omega_{ij} \subseteq \{x \in \Re^n | x' Q_{ij} x \leq 0\} \tag{15}$$

Ω_{ii} represents the set in which the ith subsystem is free to operate and Ω_{ij} (where $i \neq j$) denotes the guard set for the transition between the ith and jth vertices. If we can find real matrices, $P_i = P_i' > 0$ for all $i \in I$ and real constants $\alpha_i > 0$ and $\alpha_{ij} > 0$ such that

$$\begin{aligned} A_i' P_i + P_i A_i + \alpha_i Q_{ii} &\leq 0 && (16)\\ P_i - P_j + \alpha_{ij} Q_{ij} &\leq 0 \ , && (17) \end{aligned}$$

then the functionals, $V_j = x' P_j x$ are Lyapunov like functions of the switched system. This particular conditions is more restrictive than that formulated in [Bran94]. But it can be readily reformulated as a linear matrix inequality (LMI) which can be solved using interior-point methods for convex optimization.

4 Main Result

The sufficient conditions presented in [Bran94] [Hou96] and used in [Pet96] [Rant97] to compute candidate Lyapunov functionals provide a very conservative approach for testing switched system stability. In the first place, the stability theorems in [Bran94] [Hou96] require that V_j be Lyapunov like for all possible switching sequences. These papers place no assumptions on the nature of the switching laws used so that when the computational methods of [Pet96] and [Rant97] are employed, the worst case switching law has to be considered. The worst case switching law is one in which every switch is possible. This assumption can result in an extremely high dimensional linear matrix inequality which may be more restrictive than it needs to be.

In this section, we present and prove a result which shows that when the switching logic can be characterized by a finite automaton, then we only need to search for Lyapunov like functions over a restricted set of **fundamental cycles** in the finite automaton. Essentially, the following result shows that rather than having to examine whether a set of candidate functions are Lyapunov like for all possible switching sequences, we only need consider whether the candidate functions are Lyapunov like over a potentially smaller sized set of fundamental cycles. In section 5, this result is used to significantly reduce the computational complexity and conservatism of the LMI method suggested in [Pet96] and [Rant97].

Let the directed graph (V, A) have $n+1$ vertices, $i_0, i_1, \cdots, i_n$. The sequence of arcs

$$(i_0, i_1), (i_1, i_2), \cdots, (i_{n-1}, i_n) \tag{18}$$

is called a path of length n. A **cycle** of a directed graph is any path such that $i_0 = i_n$. A cycle of length n

$$(i_0, i_1), (i_1, i_2), \cdots, (i_{n-1}, i_0) \tag{19}$$

is said to be **fundamental** if $i_j \neq i_k$ for all j, k not equal to zero or n and for all $j \neq k$. The following results are basic facts from graph theory. In any fundamental cycle, any two vertices are connected by one and only one path. An arc of a directed graph that is in a cycle is also in a fundamental cycle. For any cycle, C, in a directed graph, there exists a set of fundamental cycles $C_1, C_2, \cdots, C_N$ such that

$$\mathrm{Arcs}(C) = \bigcup_{i}^{N} \mathrm{Arcs}(C_i) \tag{20}$$

Finally, the fundamental cycles of a directed graph can be determined in polynomial time by constructing a minimal spanning tree for the graph.

To state and prove the main result of this paper, we first need to establish some facts about fundamental cycles generated by live switched systems. The first principal lemma is a result saying that any event sequence generated by a switched system can be constructed by recursively inserting fundamental cycles into a legal switching sequence. We then introduce a sufficient condition for a fundamental cycle to be **uniformly bounded** with respect to time. These two results are then combined to establish the Lyapunov stability of the entire switched system.

Lemma 1 *In the automaton associated with a live switched system, every arc is in at least one fundamental cycle*

Proof: Let (V, A) denote the finite automaton associated with a switched system. Assume that there exists an arc $(i, j) \in A$ which is not in any cycle of (V, A). Therefore, once we go through arc (i, j) then there is no path back to vertex $i \in V$. Therefore in any switching sequence λ that contains arc (i, j) the number of times when vertex i is reached will be reached is finite which contradicts the definition of a live transition system. Therefore every arc of a live automaton is in a cycle. Furthermore from the fundamental results about cycles in directed graphs, we know that every arc is in at least one fundamental cycle, so the the lemma is proven. •

Lemma 2 *Any switching sequence λ generated by a live switched system can be decomposed as*

$$\lambda_e = \sigma_1 \sigma_2 \sigma_3 \tag{21}$$

where σ_1 is a prefix of λ_e, σ_3 is a suffix of λ_e, and σ_2 is a fundamental cycle of the switched system's automaton.

Proof: Assuming there exists a switching sequence λ with event projection λ_e such that the decomposition doesn't exist. This means that there is no substring in λ_e which is a fundamental cycle. But from the definition of a live switched system, we know that every arc must be in a cycle. Let i_1 be the vertex where such a cycle starts. If the cycle is fundamental, then we have a contradiction and the proof is finished. But if the cycle is not fundamental, then there is a vertex i_2 which is crossed more than once in the cycle. Consider the cycle starting from i_2. Either this cycle is fundamental, or not. If not, then we can repeat the above argument to find a smaller cycle within this one. However, because the automaton is finite, this recursion has to terminate in a fundamental cycle. We therefore have a contradiction and the lemma is proven. •

Proposition 1 *Given a switching sequence λ generated by a live switched system, let $\Lambda : Z \rightarrow I^*$ be a sequence of sequences in I^* constructed by the recursive procedure:*

1. *$\Lambda[0]$ is a fundamental cycle C_0*
2. *$\Lambda[n] = \sigma_1 C_n \sigma_2$ where $\sigma_1 \sigma_2 = \Lambda[n-1]$ and C_n is a fundamental cycle.*

Then there exists a set of C_i such that $\Lambda[n]$ is a prefix of λ for all n.

Proof: From lemma 2 we know that any switching sequence can be decomposed to $\sigma_1 \sigma_2 \sigma_3$ where σ_2 is a fundamental cycle. Note that if we pull out σ_2 from the switching sequence, then $\sigma_1\sigma_3$ is still a possible switching sequence. We can now decompose the resulting sequence $\sigma_1\sigma_3$ using lemma 2 to pull out another fundamental cycle of the automaton. Since the switching sequence is countable, we can repeat this process to pull out a countable sequence of fundamental cycles. This sequence is the set of C_i referred to in the above proposition. •

A given sequence of events can be generated in various ways by a switched system. What we'd like to do is ensure that the cycle is well-behaved in some appropriate sense. In particular, we'll require that the continuous-state trajectory over the cycle is uniformly bounded with respect to time. The following lemma provides sufficient conditions for the system to be uniformly bounded.

Lemma 3 *Let λ_e be any cycle generated by the live switched system consisting of events*

$$\lambda_e = j_1, \ldots, j_K \tag{22}$$

where $j_{K+1} = j_1$ with switching times

$$t_0, t_1, \cdots t_K \tag{23}$$

So that t_i is the time when the ith system is switched off and the $i+1$st system is switched on.

If there exist a set of continuously differentiable functions $V_j : \Re^n \to \Re$ for $j \in I$ such that $\dot{V}_j(x(t)) \leq 0$ for all $t \in [t_{j-1}, t_j)$, then for any $\epsilon > 0$ there $\delta(\epsilon) > 0$ such that for all $\|x(t_0)\| < \delta(\epsilon)$, $\|x(t)\| < \epsilon$ for all $t \in [t_0, t_K]$.

Proof: Consider an arbitrary $\epsilon > 0$ and let

$$\beta_K = \min_{x \in S(\epsilon)} V_{j_K}(x) \tag{24}$$

Define the closed set,

$$\Omega_K = \{x \in B(\epsilon) : V_{j_K}(x) \leq \beta_K\} \tag{25}$$

Choose ρ_K such that for all $x \in B(\rho_K)$, we have $V_{j_K}(x) < \beta_K$. We now define

$$\beta_{K-1} = \min_{x \in S(\rho_K)} V_{j_{K-1}}(x) \tag{26}$$

and introduce the closed set,

$$\Omega_{K-1} = \left\{x \in B(\rho_K) : V_{j_{K-1}}(x) \leq \beta_{K-1}\right\} \tag{27}$$

Choose ρ_{K-1} as was stated above and continue this process to construct a monotone sequence of sets

$$\Omega_1 \subseteq \Omega_2 \subseteq \cdots \subseteq \Omega_{K-1} \subseteq \Omega_K \tag{28}$$

Note that Ω_j is invariant with respect to subsystem f_j because of the condition on $\dot{V}_j$. Therefore, we expect that if we start in $B(\rho_0)$, we should stay in set $B(\epsilon)$, which is sufficient to establish the lemma's conclusion. •

A cycle for which such functionals can be found will be said to be **uniformly bounded**. We now state and prove the main result of this section. This result uses the preceding proposition to show by induction that each of the sequences in the supersequence of lemma 2 is uniformly bounded if each fundamental cycle is uniformly bounded.

Theorem 2 *Consider a live nonZeno switched system where $f_j(0) = 0$ for all $j \in I$. Let λ be a switching sequence generated by the system. Let μ denote a subsequence of contiguous switches in λ such that $P_e[\mu]$ is a fundamental cycle of the system's automaton. Let $\bar{\mu}$ denote the infinite sequence formed by concatenation of μ with itself.*

If there exist a set of continuously differentiable functions $V_j : \Re^n \to \Re$ which are Lyapunov like over sequence $\bar{\mu}_{t,j}$ for all $j \in I$, then the system is stable in the sense of Lyapunov.

Proof: From our earlier lemma, we know that any switching sequence can be constructed by inserting fundamental cycles into a legal switching sequence. Let

$$\Lambda = \lambda[0], \lambda[1], \cdots \lambda[n], \cdots \tag{29}$$

By definition $\lambda[0]$ is a fundamental cycle and under the theorem's hypothesis this is uniformly bounded.

Now assume that the sequence $\Lambda[n]$ is uniformly bounded. By assumption the fundamental cycle inserted into $\Lambda[n]$ is uniformly bounded. Note also, however, that since V_j is Lyapunov like we require that if $x(t_0) \in \Omega_1$, then it must return to that set. Hence the addition of the fundamental cycle does not change the boundedness of the original sequence $\Lambda[n]$. We can therefore conclude that $\Lambda[n+1]$ is uniformly bounded.

We now consider the limit as $n \to \infty$. Since the δ determined for uniform boundedness is indepedent of time, we can conclude that it holds for sequences of arbitrary length and hence the system is stable in the sense of Lyapunov.•

5 Example

In this section, we present some examples illustrating the application of the result in the preceding section to the computation of Lyapunov-like functionals using the LMI methods of [Pet96] and [Rant97].

Consider a live switched system whose automaton is shown in figure 1. Associated with each vertex is an LTI subsystem of the form

$$\dot{x} = A_i x \tag{30}$$

where $i = 1, 2, \ldots, 6$. In addition to $A_i \in \Re^{2\times 2}$, we associate the "self-switching" set characterized by the symmetric matrix Q_i. Figure 1 shows the given automaton and the assumed matrices associated with each vertex. Each arc (i, j) in the automaton has a matrix Q_{ij} associated with it. The arcs are shown in figure 1 also.

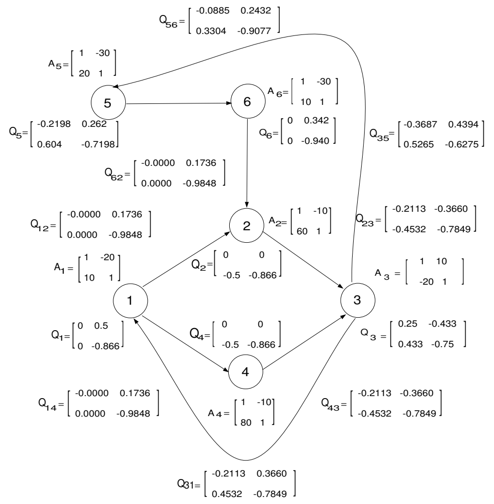

Figure 1: The automaton of the example live switched system

From the automaton we can identify a set of three fundamental cycles. These fundamental cycles are obtained by determining a minimal spanning tree for the automaton's directed graph. This directed graph is shown in figure 2 and the resulting fundamental cycles are $1-2-3$, $1-4-3$ and $5-6-2-3$, respectively.

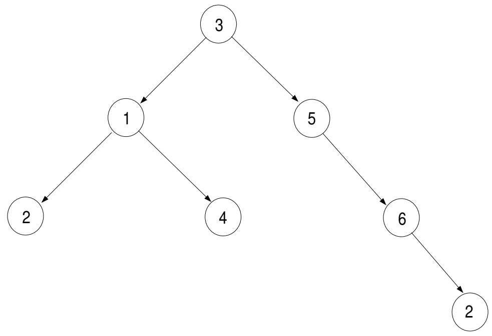

Figure 2: Spanning Tree Identifying Switched System's Fundamental Cycles

From the theorem proven above, we know that it suffices to find a set of continuously differentiable functions, V_j, which are Lyapunov-like for each fundamental cycle in the automaton. Determining such Lyapunov-like functions can now be done using the method suggested in [Pet96] and [Rant97]. We establish three sets of matrix inequalities corresponding to the three fundamental cycles. For cycle $1-2-3$, we have the set of inequalities,

$$A_i' P_i + P_i A_i + \alpha_i Q_i \leq 0 \; i = 1, 2, 3$$

$$P_2 - P_1 + \alpha_{12} Q_{12} \leq 0$$

$$P_3 - P_2 + \alpha_{23} Q_{23} \leq 0$$

$$P_1 - P_3 + \alpha_{31} Q_{31} \leq 0$$

A similar set of inequalities can be formed for the other three cycles. To find the Lyapunov like functions, $V_i = x' P_i x$, we want to make sure that all fundamental cycles are stable, so we build a large LMI which includes all the matrix inequalities associated with the three fundamental cycles. For this example, there are a total of 14 matrix equations.

The 14 equation LMI is still a large LMI, but it can be readily solved using the LMI toolbox. If we had proceeded using the technique originally proposed in [Pet96], then we would need to build an LMI which accounted for all individual transitions that could possibly happen. If the automaton had N vertices, then we would have N^2 equations in our linear matrix inequality. On the otherhand, if the automaton had M fundamental cycles, then the total size of the LMI would be bounded by $(M+1)N$ since each cycle can have no more than N vertices. In many cases, this bound is much larger than could be seen. For our particular example, we would have a 36 equation LMI to solve.

The implication of increasing LMI size is that it represents an overly restrictive sufficient condition for system stability. In our case, we can see this quite easily by solving the 14 equation LMI obtained by examining the fundamental cycles of the system versus the 36 equation LMI obtained by using the methods in [Pet96]. The P matrices obtained in both cases for our example are shown in figure 3

The existence of these P matrices indicates that the given system is stable. A simulation of the example system's trajectory is shown in figure 4. This trajectory is clearly stable.

In computing the first table, the LMI toolbox required 33478 flops to determine the P matrices for the original method. The simplified method developed in this paper only required a total of 13310 flops. So our method clearly has a lower computational complexity than the original method of [Pet96]. More important than this, however, is the difference between the matrices. As can be clearly seen above, the singular values for the P matrices obtained from the simplified approach are around 50. For the original approach in [Pet96], however, these

	simplified method	original method
P_1	$\begin{bmatrix} 38.3443 & -5.7616 \\ -5.7616 & 65.2333 \end{bmatrix}$	$\begin{bmatrix} 0.0869 & -0.0134 \\ -0.0134 & 0.1470 \end{bmatrix}$
P_2	$\begin{bmatrix} 27.6904 & -1.7744 \\ -1.7744 & 5.7280 \end{bmatrix}$	$\begin{bmatrix} 0.0610 & -0.0042 \\ -0.0042 & 0.0128 \end{bmatrix}$
P_3	$\begin{bmatrix} 35.6272 & 16.0261 \\ 16.0261 & 21.2592 \end{bmatrix}$	$\begin{bmatrix} 0.0761 & 0.0276 \\ 0.0276 & 0.0436 \end{bmatrix}$
P_4	$\begin{bmatrix} 31.3473 & -1.3288 \\ -1.3288 & 4.9189 \end{bmatrix}$	$\begin{bmatrix} 0.0707 & -0.0032 \\ -0.0032 & 0.0112 \end{bmatrix}$
P_5	$\begin{bmatrix} 40.3580 & -3.3383 \\ -3.3383 & 46.6225 \end{bmatrix}$	$\begin{bmatrix} 0.0799 & -0.0075 \\ -0.0075 & 0.0894 \end{bmatrix}$
P_6	$\begin{bmatrix} 36.0113 & -14.3852 \\ -14.3852 & 87.6743 \end{bmatrix}$	$\begin{bmatrix} 0.0740 & -0.0288 \\ -0.0288 & 0.1771 \end{bmatrix}$

(31)

Figure 3: P matrices for example

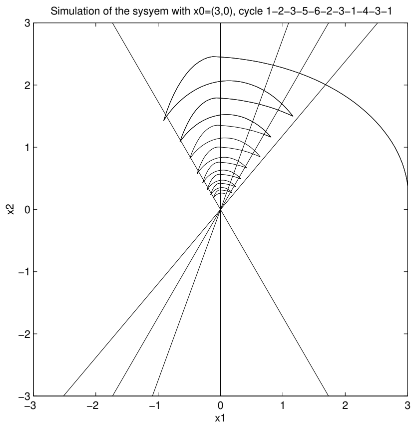

Figure 4: The trajectory of the sample system under switching cycle

values are about .1. Since the singular value is a measure of how close the matrix is to being singular, this means that the original method was almost unable to determine the candidate Lyapunov functions. With minor changes in the Q matrices it is quite possible to generate examples in which the original method is unable to find the required P matrices, but our method would find such matrices.

6 Future Work

This paper has presented a sufficient method for switched system stability which takes advantage of prior knowledge of the system's switching logic. In particular, it was shown that if the switching logic can be shown to be generated by a finite discrete-event transitions system such as a finite automaton or Petri net, then it suffices to determine Lyapunov-like functions only over the fundamental cycles of the state machine. This observation can greatly reduce the computational complexity involved in testing for switched system stability as well as providing a less conservative set of Lyapunov functions. These conclusions were validated by using them to compute the Lyapunov functions for an LTI switched system using the LMI method suggested in [Pet96] and [Rant97].

The preliminary results presented in this paper are encouraging and suggest several possible directions for future study. One future direction involves extending the concepts introduced here to study switching logics generated by Petri nets. The use of unfolding methods should allow the efficient identification of fundamental cycles in the Petri net's reachability tree, thereby providing a sufficient test for the stability of such systems. Another promising avenue of future study involves developing sufficient tests for uniform ultimate boundedness (bounded-amplitude) in switched systems. For important classes of systems, we can also formulate these sufficient conditions as matrix inequalities thereby allowing the efficient testing of switched system performance with respect to a specified ultimate bound.

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